Uniform Labelled Transition Systems

Rocco De Nicola

based on joint work with:
D. Latella, M. Loreti, M. Massink and, more recently, M. Bernardo

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and
IMT - Lucca

PaCo - L'Aquila
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Contribution by Firenze

DSIUF participants

- M. Loreti
- R. De Nicola
- L. D'Errico
- F. Calzolai

ISTI-CNR participants

- D. Latella
- M. Massink

Research Topics

- ULTRAS with Urbino - This talk
- Stochastic Model Checking with ISTI - - next talk
- Compositional Reasoning
- Tools for Verification

Outline…

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Stochastic Process Algebras - incomplete list

- TIPP (N. Glotz, U. Herzog, M. Rettelbach - 1993)
- Stochastic π-calculus (C. Priami - 1995, later with P. Quaglia)
- PEPA (J. Hillston - 1996)
- EMPA (M. Bernardo, R. Gorrieri - 1998)
- IMC (H. Hermanns - 2002)
- ...
- STOKLAIM
- MarCaSPiS
- ...

More Calculi will come: Besides qualitative aspects of distributed systems it more and more important that performance and dependability be addressed to deal, e.g., with issues related to quality of service.

CTMC Semantics of Stochastic PA

Randomized Actions

- It is assumed that action execution takes time
- Execution times is described by means of random variables
- Random Variables are assumed to be exponentially distributed
- Random Variables are fully characterised by their rates.

CTMC for SPA

CTMC model the stochastic behaviour of processes, and a CTMC is associated to each process term;

To get a CTMC from a term, one needs to...

- compute *synchronizations rate* ...
- ...while taking into account transition multiplicity, for determining correct execution rate

Motivations for our work

LTS and CTMC

The stochastic process algebras proposed in the last two decades are based on:

1. Labeled Transition Systems (LTS)
   - for providing compositional semantics of languages
   - for describing *qualitative properties*
2. Continuous Time Markov Chains (CTMC)
   - for analysing *quantitative properties*

However, ...

- there is no general framework for modelling the different formalisms
- it is rather difficult to appreciate differences and similarities of such semantics.

Outline...

Semantics of stochastic process calculi

We introduce a variant of Rate Transition Systems (RTS), proposed by Klin and Sassone(FOSSACS 2008), and use them for defining stochastic behaviour of a few process algebras.

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Like most of the previous attempts we take a two step approach: For a given term, say T, we define an enriched LTS and then use it to determine the CTMC to be associated to T.

- Our variant of RTS associates terms and actions to functions from terms to rates
- The *apparent rate* approach, originally developed by Hillston for multi-party synchronisation (à la CSP), is generalized to deal "appropriately" also with binary synchronisation (à la CCS).

Semantics of stochastic process calculi

Stochastic semantics of process calculi is defined by means of a transition relation $\longrightarrow$ that associates to a pair (P, α) - consisting of process and an action - a total function ($\mathscr{P}$, $\mathscr{Q}$,...) that assigns a non-negative real number to each process of the calculus. Value 0 is assigned to unreachable processes.

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$P \xrightarrow{\alpha} \mathscr{P}$ means that, for a generic process Q:

- if $\mathscr{P}(Q) = x$ $(\neq 0)$ then Q is reachable from P via the execution of α with rate/(weight) x
- if $\mathscr{P}(Q) = 0$ then Q is not reachable from P via α

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We have that if $P \xrightarrow{\alpha} \mathscr{P}$ then

- $\oplus\mathscr{P} = \sum_Q \mathscr{P}(Q)$ represents the total rate/weight of α in P.

Rate transition systems

Definition

A rate transition system is a triple $(S, A, \longrightarrow)$ where:

- S is a set of states;
- A is a set of transition labels;
- $\rightarrow \subseteq S \times A \times [S \rightarrow \mathbb{R}_{\geq 0}]$

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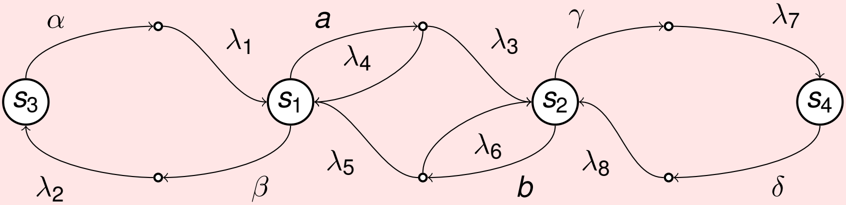
An example of RTS



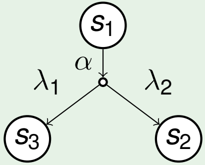
Rate transition systems

Definition

Let $\mathcal{R} = (S, A, \rightarrow)$ be an RTS, then:

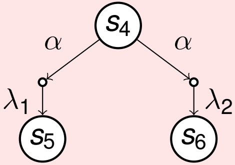
- $\mathcal{R}$ is *functional* if and only if for each $s \in S$, $\alpha \in A$, $\mathscr{P}$ and $\mathscr{Q}$ we have: $s \xrightarrow{\alpha} \mathscr{P}, s \xrightarrow{\alpha} \mathscr{Q} \implies \mathscr{P} = \mathscr{Q}$
- $\mathcal{R}$ is *image finite* if and only if for each $s \in S$, $\alpha \in A$ and $\mathscr{P}$ such that $s \xrightarrow{\alpha} \mathscr{P}$ we have: $\{s' | \mathscr{P}(s') > 0\}$ is finite

A functional RTS



that leads to a CTMC.

A general RTS



that leads to a CTM Dec. Proc.

From RTS to CTMC...

Reachable Sets of States

For sets $\mathcal{S}' \subseteq \mathcal{S}$ and $A' \subseteq A$, the set of derivatives of $\mathcal{S}'$ *through* A', denoted $Der(\mathcal{S}', A')$, is the smallest set such that:

- $\mathcal{S}' \subseteq Der(\mathcal{S}', A')$,
- if $s \in Der(\mathcal{S}', A')$ and there exists $\alpha \in A'$ and $\mathscr{Q} \in \Sigma_{\mathcal{S}}$ such that $s \xrightarrow{\alpha} \mathscr{Q}$ then $\{s' \mid \mathscr{Q}(s') > 0\} \subseteq Der(\mathcal{S}', A')$

Mapping $(\mathcal{S}, A, \rightarrow)$ into $(Der(\mathcal{S}', A'), \mathbf{R})$

Let $\mathcal{R} = (\mathcal{S}, A, \rightarrow)$ be a *functional* RTS, for $\mathcal{S}' \subseteq \mathcal{S}$, the CTMC of $\mathcal{S}'$, when one considers only actions $A' \subseteq A$ is defined as

$CTMC[\mathcal{S}', A'] \stackrel{def}{=} (Der(\mathcal{S}', A'), \mathbf{R})$ where for all $s_1, s_2 \in Der(\mathcal{S}', A')$:

$$\mathbf{R}[s_1, s_2] \stackrel{def}{=} \sum_{\alpha \in A'} \mathscr{P}^{\alpha}(s_2) \qquad \text{with } s_1 \xrightarrow{\alpha} \mathscr{P}^{\alpha}.$$

A translation from an RTS to a CTMC

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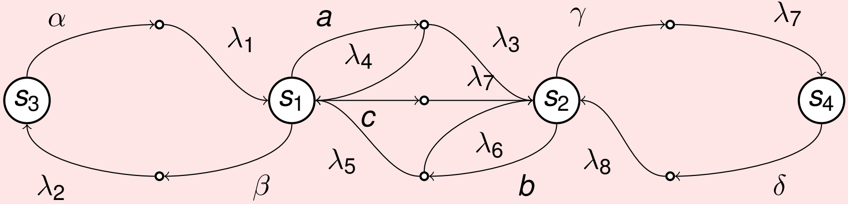
An RTS:



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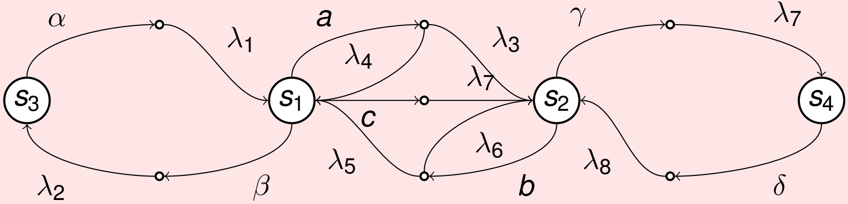
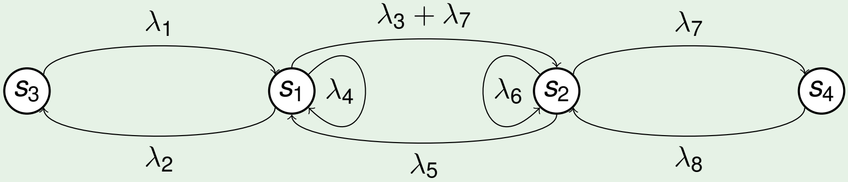


The corresponding CTMC:



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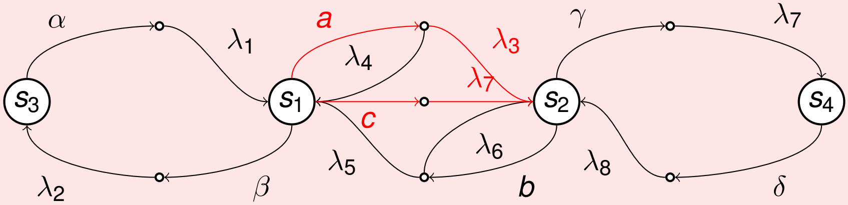

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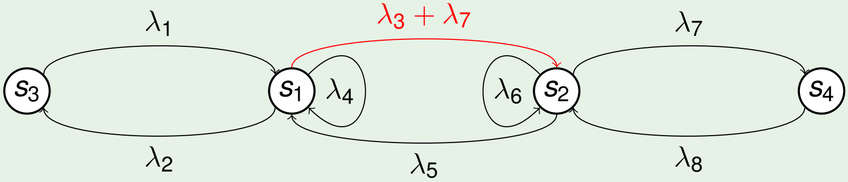

Strong Markovian Bisimilarity

Definition (Bisimulation)

Given a generic CTMC $(S, \mathbf{R})$

- An equivalence relation $\mathcal{E}$ on S is a Markovian bisimulation on S if and only if for all $(s_1, s_2) \in \mathcal{E}$ and for all equivalence classes $C \in S_{/\mathcal{E}}$ the following condition holds: $\mathbf{R}[s_1, C] \;=\; \mathbf{R}[s_2, C]$.

Definition (Bisimilarity)

Given a generic CTMC $(S, \mathbf{R})$

- Two states $s_1, s_2 \in S$ are strongly Markovian bisimilar, written $s_1 \sim_M s_2$, if and only if there exists a Markovian bisimulation $\mathcal{E}$ on S with $(s_1, s_2) \in \mathcal{E}$.

Rate aware bisimulation

Definition (Rate Aware Bisimilarity)

Let $\mathcal{R} = (\mathcal{S}, A, \rightarrow)$ be a RTS:

- An equivalence relation $\mathcal{E} \subseteq \mathcal{S} \times \mathcal{S}$ is a *rate aware* bisimulation if and only if, for all $(s_1, s_2) \in \mathcal{E}$, and $\underline{S} \in \mathcal{S}_{/\mathcal{E}}$, and for all α and $\mathscr{P}$:

$$s_1 \xrightarrow{\alpha} \mathscr{P} \Longrightarrow \exists \mathscr{Q} : \ s_2 \xrightarrow{\alpha} \mathscr{Q} \wedge \mathscr{P}(\underline{S}) = \mathscr{Q}(\underline{S})$$

- Two states $s_1, s_2 \in S$ are *rate aware bisimilar* $(s_1 \sim s_2)$ if there exists a rate aware bisimulation $\mathcal{E}$ such that $(s_1, s_2) \in \mathcal{E}$.

Theorem

Let $\mathcal{R} = (\mathcal{S}, A, \longrightarrow)$*, for each* $A' \subseteq A$ *and for each* $s_1, s_2 \in \mathcal{S}$ *and* $(S, \mathbf{R}) = CTMC[\{s_1, s_2\}, A']$*:* $s_1 \sim s_2 \Longrightarrow s_1 \sim_M s_2$

Notice that *rate aware bisimilarity* and *strong bisimilarity* coincide when one does not take into account actions.

Outline...

PEPA: Performance Process Algebra

PEPA Systems

PEPA systems are the result of *components* interaction via *activities*:

- Components reflect the behaviour of relevant parts of the system,
- activities capture the actions that the components perform.

PEPA Activities

Each PEPA activity consists of a pair (α, λ) where:

- α symbolically denotes the performed action;
- $\lambda > 0$ is the rate of the (negative) *exponential* distribution.

PEPA Syntax

If $\mathcal{A}$ is a set of *actions*, ranged over by $\alpha, \alpha', \alpha_1, \ldots$, then $\mathcal{P}_{PEPA}$ is the set of process terms $P, P', P_1, \ldots$ defined by:

$$P ::= (\alpha, \lambda).P \mid P + P \mid P \parallel_L P \mid P/L \mid A$$

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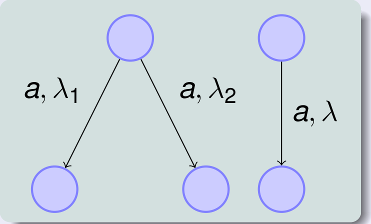

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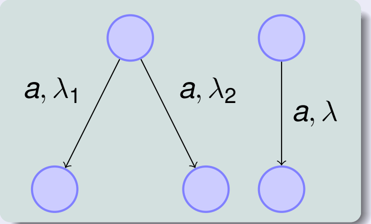

- Two synchronizations can occur with rates:

$$\frac{\lambda}{\lambda} \cdot \frac{\lambda_1}{\lambda_1 + \lambda_2} \cdot \min\{\lambda, \lambda_1 + \lambda_2\}$$

$$\frac{\lambda}{\lambda} \cdot \frac{\lambda_2}{\lambda_1 + \lambda_2} \cdot \min\{\lambda, \lambda_1 + \lambda_2\}$$

PEPA Stochastic semantics...

$$\frac{}{(\alpha,\lambda).P \xrightarrow{\alpha} [P \mapsto \lambda]} \text{(ACT)} \qquad \frac{\alpha \neq \beta}{(\alpha,\lambda).P \xrightarrow{\beta} \emptyset} \text{($\emptyset$-ACT)}$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q}}{P + Q \xrightarrow{\alpha} \mathscr{P} + \mathscr{Q}} \text{(SUM)} \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \notin L}{P \,||_L\, Q \xrightarrow{\alpha} \mathscr{P} \,||_L\, \chi_Q + \chi_P \,||_L\, \mathscr{Q}} \text{(INT)}$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \in L}{P \,||_L\, Q \xrightarrow{\alpha} \mathscr{P} \,||_L\, \mathscr{Q} \cdot \frac{\min\{\oplus\mathscr{P}, \oplus\mathscr{Q}\}}{\oplus\mathscr{P} \cdot \oplus\mathscr{Q}}} \text{(COOP)}$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad \alpha \notin L}{P/L \xrightarrow{\alpha} \mathscr{P}/L} \text{(P-HIDE)} \qquad \frac{\alpha \in L}{P/L \xrightarrow{\alpha} \emptyset} \text{($\emptyset$-HIDE)}$$

$$\frac{P \xrightarrow{\tau} \mathscr{P}_\tau \quad \forall \alpha \in L. P \xrightarrow{\alpha} \mathscr{P}_\alpha}{P/L \xrightarrow{\tau} \mathscr{P}_\tau/L + \sum_{\alpha \in L} \mathscr{P}_\alpha/L} \text{(HIDE)} \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad A \triangleq P}{A \xrightarrow{\alpha} \mathscr{P}} \text{(CALL)}$$

A couple results for our PEPA semantics

Theorem

$\mathcal{R}_{PEPA}$ is functional and image finite.

Theorem

For all $P, Q \in \mathcal{P}_{PEPA}$ and $\alpha \in \mathcal{A}$ the following holds:

$$P \xrightarrow{\alpha} \mathscr{P} \wedge \mathscr{P}(Q) = \lambda > 0 \Leftrightarrow P \xrightarrow{\alpha,\lambda}_{P} Q$$

where $\longrightarrow_P$ stands for the transition relation defined by Hillstone in [Hil96].

Binary Sinchronization

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- output activities are enriched with rates modeling their duration, input activities are enriched with weights characterizing the relative selection probability. The rate of a binary synchronization mainly depends on the one of the triggering *activity*.

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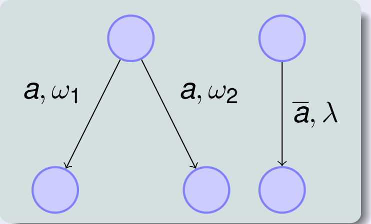

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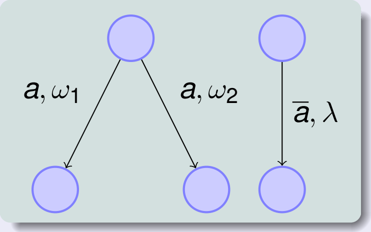
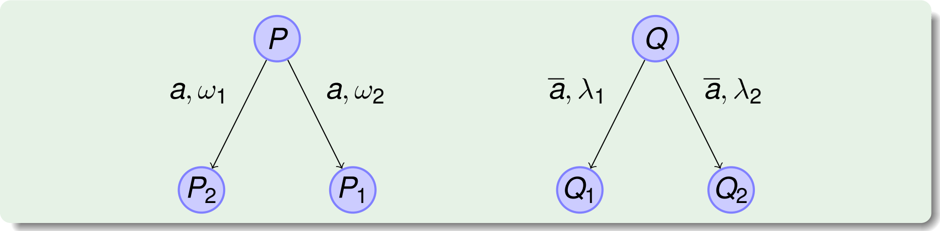


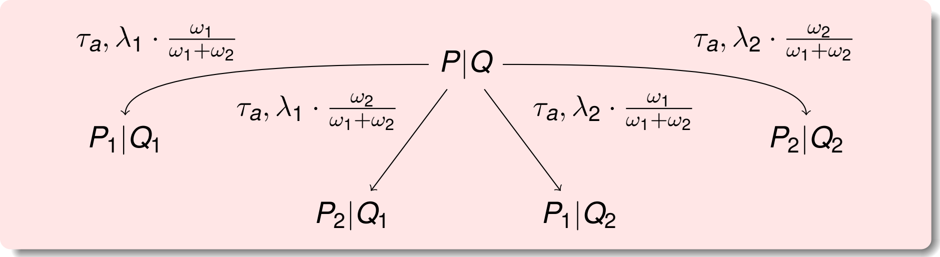
- Two synchronizations can occur with rates:

$$\lambda \cdot \frac{\omega_1}{\omega_1 + \omega_2} \qquad \lambda \cdot \frac{\omega_2}{\omega_1 + \omega_2}$$

- The overall sum of the synchronization rates is the same as the one of the output.

STOCCS: Transitions rates





Outline...

ULTRAS: Uniform Labelled Transition Systems

Uniform Labelled Transition Systems

Let D be a complete partial order (cpo) with least element $\perp$, a Uniform Transition System on D, that we call D – ULTRAS, is a triple

$$(S, A, \longrightarrow)$$

where:

- S is a set of states,
- A a set of transition labels,
- $\longrightarrow$ a subset of $S \times A \times [S \rightarrow D]$.

ULTRAS: Uniform Labelled Transition Systems

Annotated computations for ULTRAS

Let $\mathcal{R} = (S, A, \longrightarrow)$ be a D-ULTRAS, and s a state in S, an *annotated computation* is a sequence $\alpha \in (A \times D)^*$, where

- ε denotes the empty sequence;
- $(a, v) \cdot \alpha'$ denotes the sequence starting with (a, v) and continuing as α'.

$\mathcal{AC}(s)$ denotes the set of annotated computations starting from $s \in S$.

An annotated computation $\alpha \in \mathcal{AC}(s)$ if and only if: $\alpha = \varepsilon$ or $\alpha = (a, v) \cdot \alpha'$ and there exists $\mathscr{P}$ and s' such that:

- $s \xrightarrow{a} \mathscr{P}$
- $\mathscr{P}(s') \neq \bot$
- $\alpha' \in \mathcal{AC}(s')$.

ULTRAS: Weighting Functions

Weighting function:

Let W be a complete lattice where

- 0 denotes the least element of W
- 1 denotes the top element of W

$\mathcal{W}_D : S \times (A \times D)^* \times 2^S \rightarrow W$ is a weighting function for D-ULTRAS $(S, A, \longrightarrow)$ if and only if:

- for each $s \in S$ and $S' \subseteq S$ if $s \in S'$ then $\mathcal{W}_D(s, \varepsilon, S') = 1$;
  The weight of empty transitions is maximal
- for each $s \in S$ and $S' \subseteq S$ if $s \notin S'$ then $\mathcal{W}_D(s, \varepsilon, S') = 0$;
  The weight of transitions to non existing states is minimal
- for each $s \in S$ if $\alpha \notin Trace(s)$ then $\mathcal{W}_D(s, \alpha, S') = 0$;
  The weight of non existing transitions is minimal

Behavioural equivalences for ULTRAS

Trace equivalence

Let $(S, A, \longrightarrow)$ be a D-ULTRAS and $\mathcal{W}_D$ be a weighting function: Two states s_1, s_2 are *trace equivalent* if and only if, for each annotated computation $\alpha \in (A \times D)^*$:

$$\mathcal{W}_D(s_1, \alpha, S) = \mathcal{W}_D(s_2, \alpha, S)$$

Behavioural equivalences for ULTRAS

Bisimulation

Let $(S, A, \longrightarrow)$ be a D-ULTRAS and $\mathcal{W}_D$ be a weighting function:
An equivalence relation $\mathcal{E} \subseteq S \times S$ is a $\mathcal{W}_D$-*bisimulation* if and only if, for all $(s_1, s_2) \in \mathcal{E}$, for all $\alpha \in (A \times D)^*$ and $C \in S_{/\mathcal{E}}$:

$$\mathcal{W}_D(s_1, \alpha, C) = \mathcal{W}_D(s_2, \alpha, C)$$

Bisimilarity

Two states $s_1, s_2 \in S$ are $\mathcal{W}_D$-*bisimilar* ($s_1 \sim_{\mathcal{W}_D} s_2$) if there exists a $\mathcal{W}_D$-bisimulation $\mathcal{E}$ such that $(s_1, s_2) \in \mathcal{E}$.

Behavioural equivalences for ULTRAS

Testing equivalence:

Let $(S, A, \longrightarrow)$ be a D-ULTRAS and $\mathcal{W}_D$ be a weighting function.

Two states s_1, s_2 are *testing equivalent* if and only if, for each trace $\alpha \in (A \times D)^*$ and $A' \subseteq A$:

$$\mathcal{W}_D(s_1, \alpha, \mathcal{M}(A')) = \mathcal{W}_D(s_2, \alpha, \mathcal{M}(A'))$$

Must Sets:

Let $\mathcal{R} = (S, A, \longrightarrow)$ be a D-ULTRAS, $A' \subseteq A$, $s \in S$ and $a \in A$:

- s must a iff $\exists\ \mathscr{P} \neq \lambda x.\bot$ such that $s \xrightarrow{a} \mathscr{P}$
- s Must A' iff $\exists\ a \in A'$ such that s must a
- $\mathcal{M}(A') = \{s \in S \mid$ s *Must* A'$\}$

RTS as ULTRAS

Rate Transition Systems:

Rate transition systems are D-ULTRAS, where $D = \mathbb{R}_{\geq 0}$

Annotated computations in $\mathbb{R}_{\geq 0}$-ULTRAS are sequences of the form $(a_1, t_1) \cdots (a_n, t_n)$ where:

- a_i identifies the action executed at the i-th step of the computation;
- $t_i \in \mathbb{R}_{\geq 0}$ represents the rate associated to the action involved in the step.

Weighting function $\mathcal{W}_{\mathbf{R}_{\geq 0}}$ is defined in such a way that:

$$\mathcal{W}_{\mathbf{R}_{\geq 0}}(s, \alpha, S') = \text{probability to reach a state in } S' \text{ with trace } \alpha \text{ from } s$$

LTS as ULTRAS

Labelled Transition System:

A standard Labelled Transition System can be rendered as a $\mathbb{B}$-ULTRAS where:

- $\mathbb{B}$ is the set of boolean values ($\{\top, \bot\}$);

- Annotated computations have the form $(a_1, \top) \cdots (a_n, \top)$
- Weighting function $\mathcal{W}_{\mathbb{B}}$ is defined in such a way that:

$$\mathcal{W}_{\mathbb{B}}(s, \alpha, S') = \begin{cases} 1 & s \text{ reaches with } \alpha \text{ a state in } S' \\ 0 & \text{otherwise} \end{cases}$$

Correspondence Theorem

$$s_1 \sim_{LTS} s_2 \text{ if and only if } s_1 \sim_{\mathcal{W}_{\mathbb{B}}} s_2$$

Current Work

- Proving the correspondence theorem for RTS and $\mathbf{R}_{\geq 0}$-ULTRAS, with weight $\mathcal{W}_{\mathbf{R}_{\geq 0}}$
- Establishing correspondence theorems for other equivalences
- Using the RTS approach to model:
  - probabilitistic systems
  - truly-concurrent systems
  - timed systems
  - ...

Outline. . .

Summing Up

We have:

- introduced Rate Transition Systems and have used them as the basic model for defining stochastic behaviours of processes.
- introduced a natural notion of bisimulation over RTS that agrees with Markovian bisimulation.
- shown how RTS can be used to provide the stochastic operational semantics of PEPA and MarCaSPiS.
- introduced ULTRASas more general models of quantitative systems
- defined equivalence relations over ULTRAS
- shown that ULTRAScan be used for modelling other semantics (non-deterministic, stochastic, probabilitistic,. . .)

Thank you for your attention!