Improving the algorithms for performance evaluation in FASE

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FASE performance improvement

$$\exists\ a \in \mathbb{R}\ s.t.\ rp_P(n)\ = an + \Theta(1)$$

- FASE implements the quantitative characterization of the performance based on the response processes as stated before. In particular FASE calculates:
  1. **catastrophic cycles** – correctness of the process
  2. **n-critical paths** – exact value of $rp_P(n)$ for a given n
  3. **bad cycle(s)** – coefficient "a" of $rp_P(n)$

- Improving the used algorithms would be an important goal to increase FASE *usability* in *current* usage

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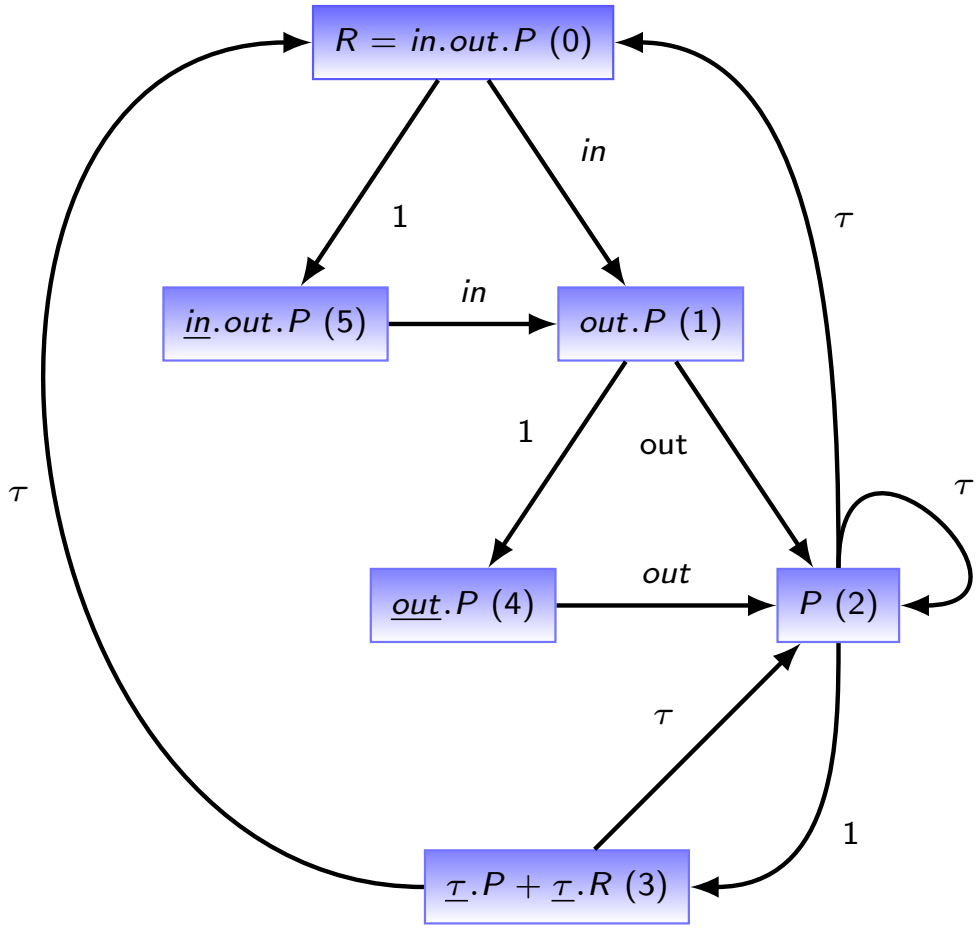
- It is crucial to ensure that the process is free from these cycles

Catastrophic cycles - Original algorithm

$R = in.out.P;$
$P = \tau.P + \tau.R;$



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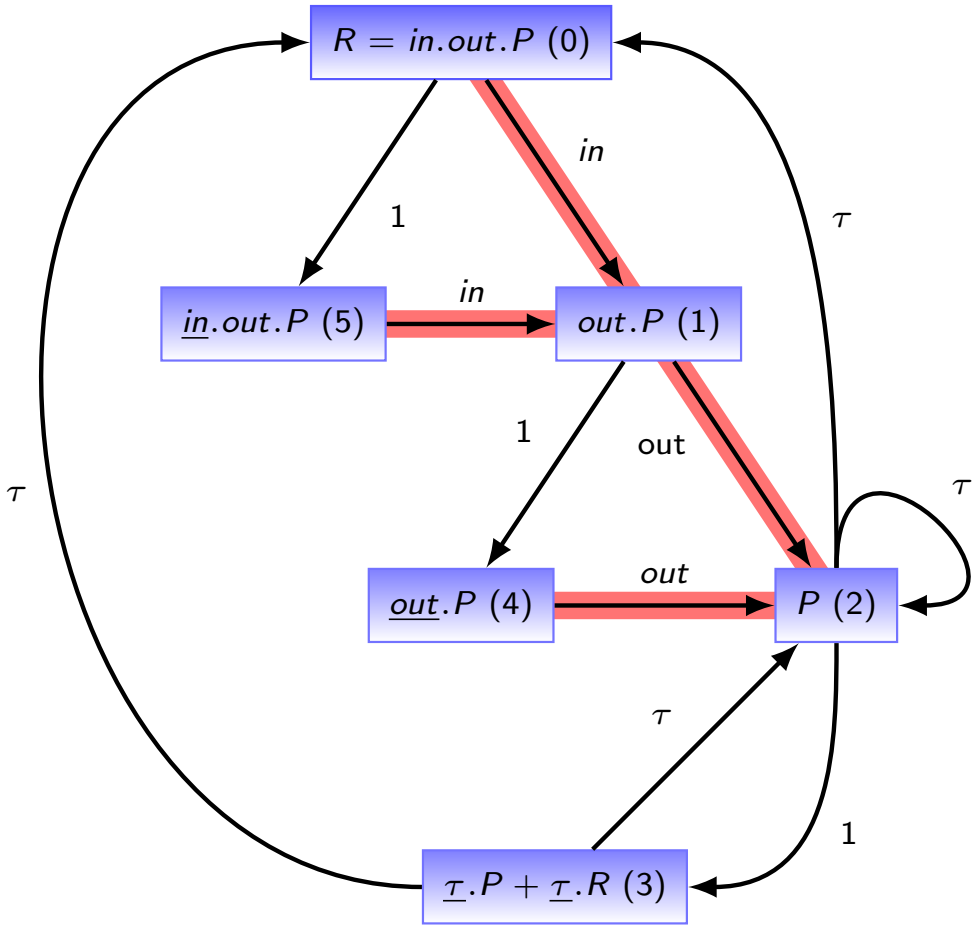

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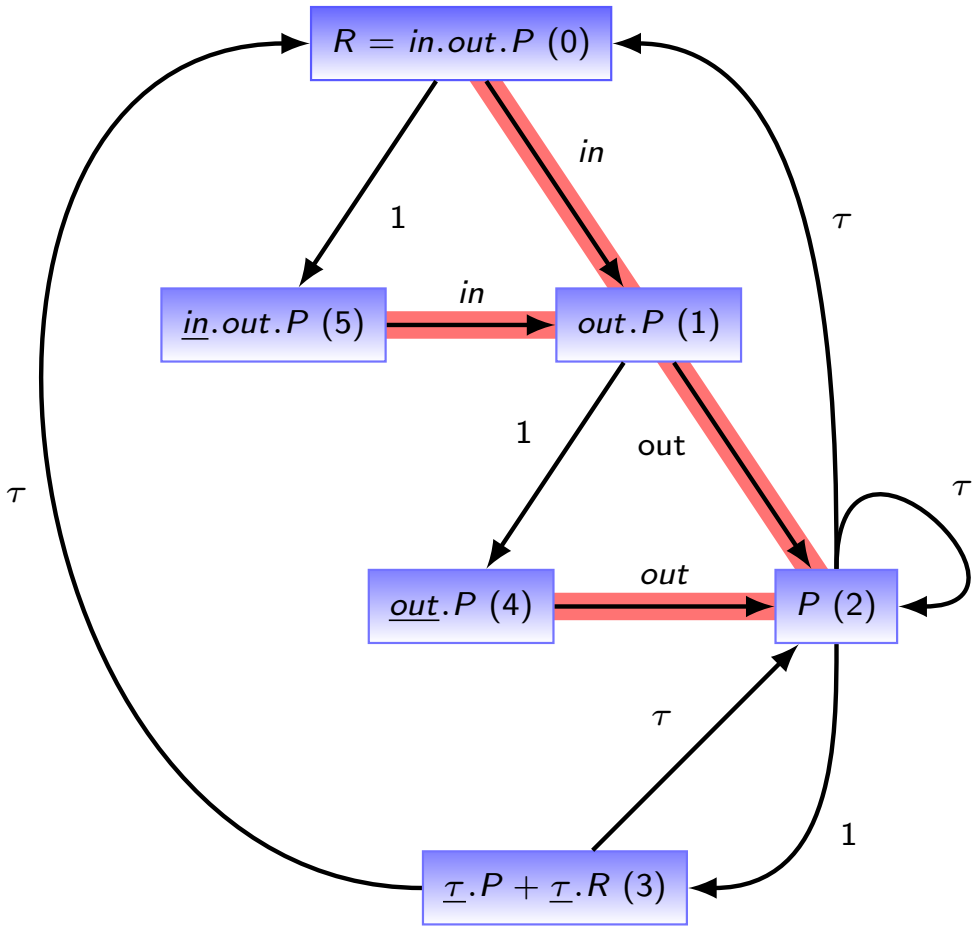

Algorithm Steps:

1. *in* and *out* edges removal
2. Floyd-Warshall shortest paths matrix calculation
   -1 for time step
   0 for τ's

	0	1	2	3	4	5
0	∞	∞	∞	∞	∞	-1
1	∞	∞	∞	∞	-1	∞
2	-1	∞	-1	-1	∞	-2
3	0	∞	0	-1	∞	-1
4	∞	∞	∞	∞	∞	∞
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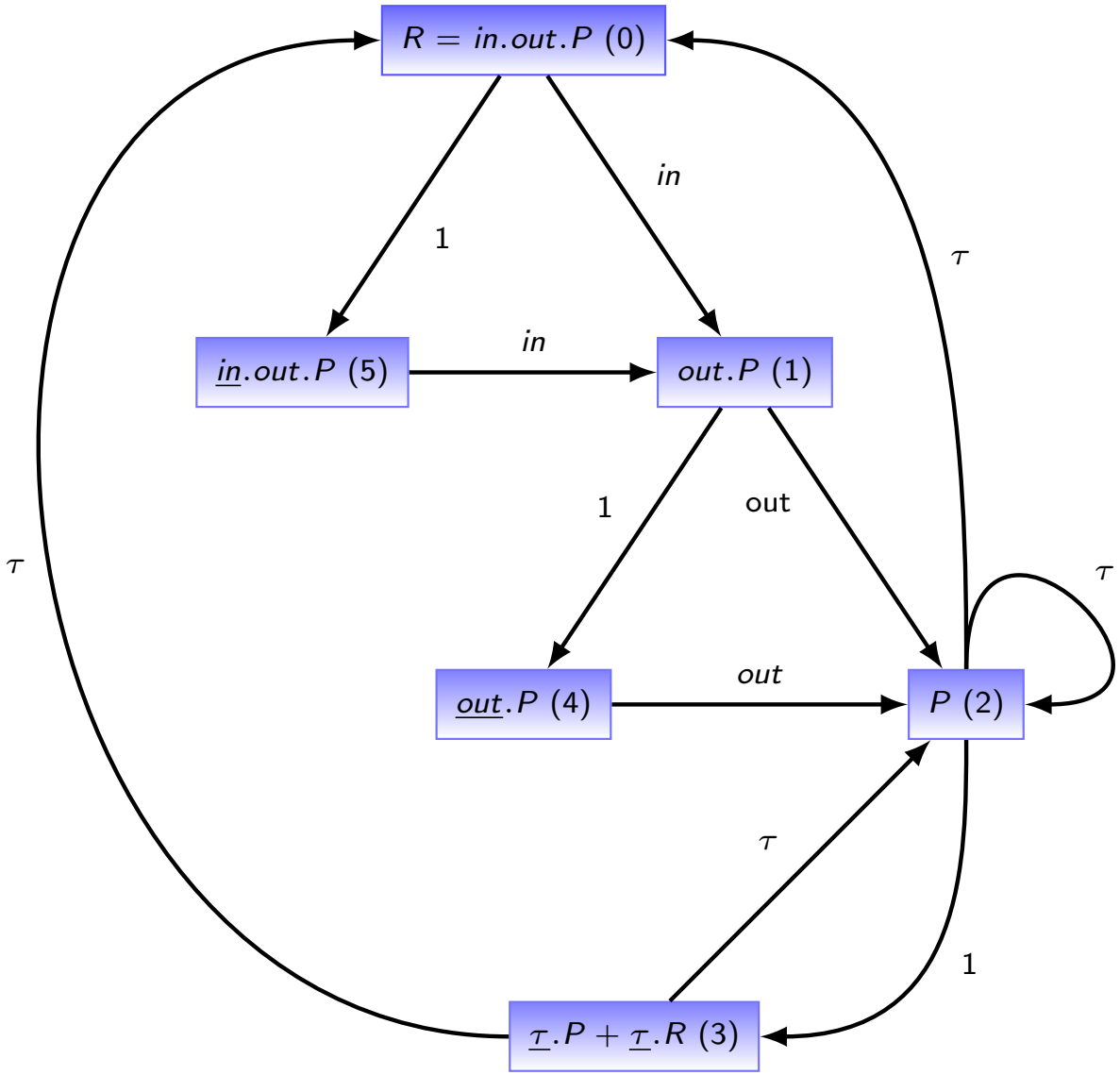
1. We consider a graph G obtained from $rRTS(P)$ with all *in* and *out* actions removed
2. We compute all the SCCs in G and if one of them contains some time steps then there is a catastrophic cycle in $rRTS(P)$

SCCs in the example process



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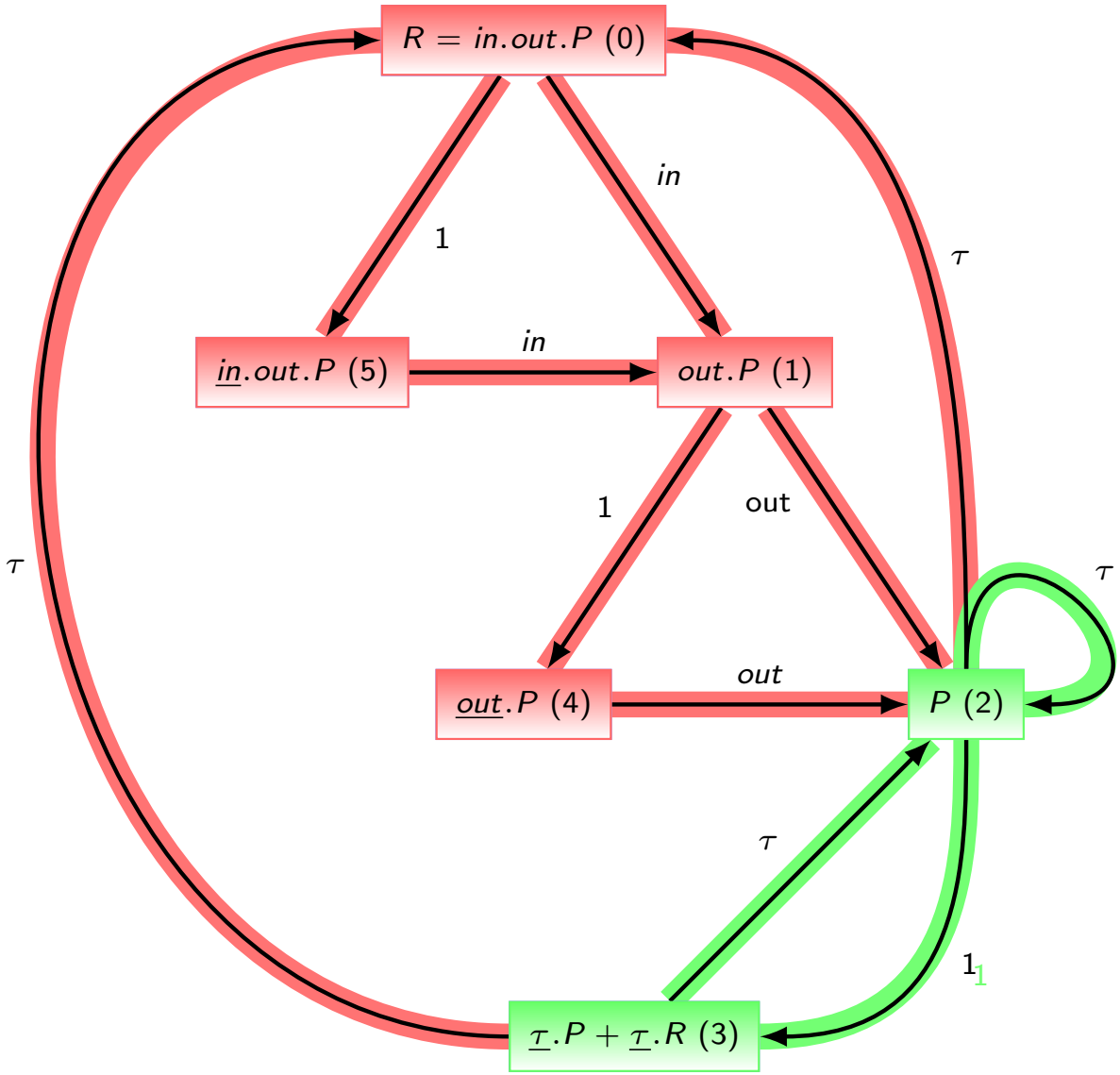


Catastrophic cycles algorithms performance comparison

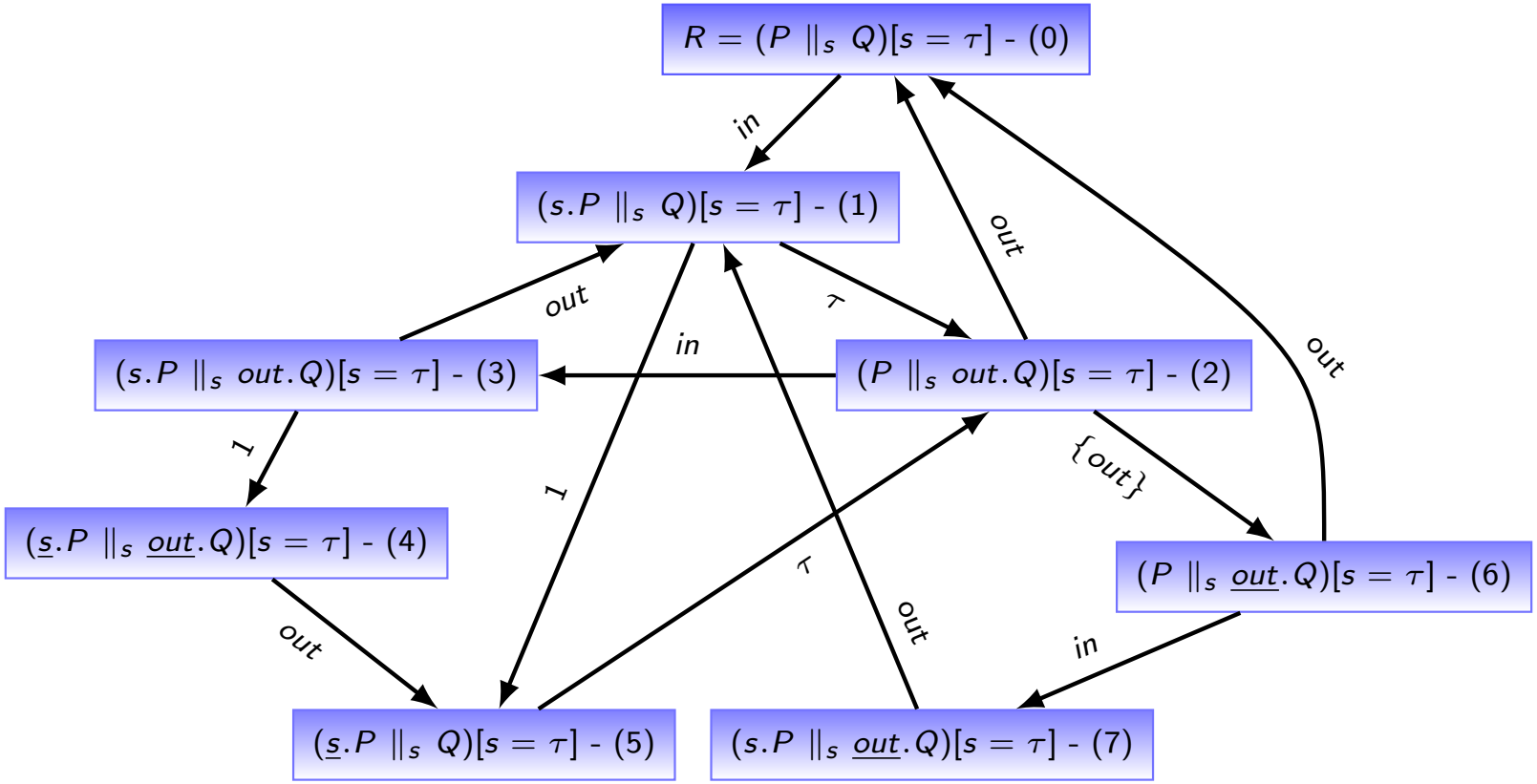
	nodes/edges	*Floyd*	*SCC*	*Gain*
$Fifo_5$	13/28	0	0	–
$Pipe_5$	114/292	62	16	74.1%
$Buff_5$	96/216	16	15	0%
$Fifo_7$	17/38	0	16	0%
$Pipe_7$	648/1958	11484	296	97.4%
$Buff_7$	240/560	390	47	87.9%
$Fifo_9$	21/48	0	0	–
$Pipe_9$	3680/12902	–	9687	100%
$Buff_9$	448/1064	2922	109	96.2%

Table: Catastrophic cycles detection time (expressed in *ms*)

n-critical paths

- Useful to calculate exactly the value rp of P for a given n
- We have not found a suitable algorithm that works in polynomial time to solve this problem
- Current algorithm calculates all n-critical path starting from the root node of process P to return the worst one (i.e. the longest in term of time steps)
- Clearly this approach has an **exponential complexity**

PIPE - 2-critical example



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Time:

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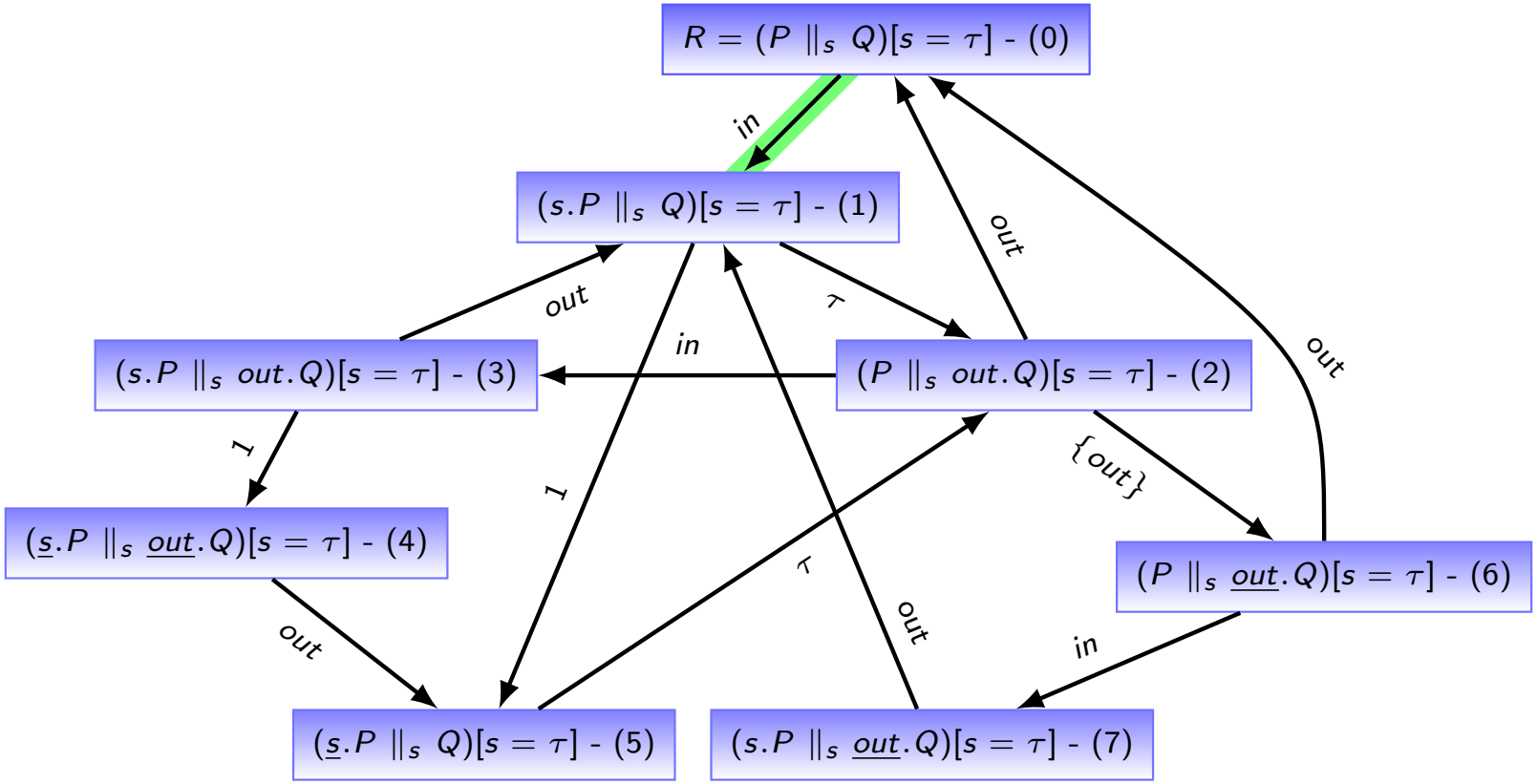

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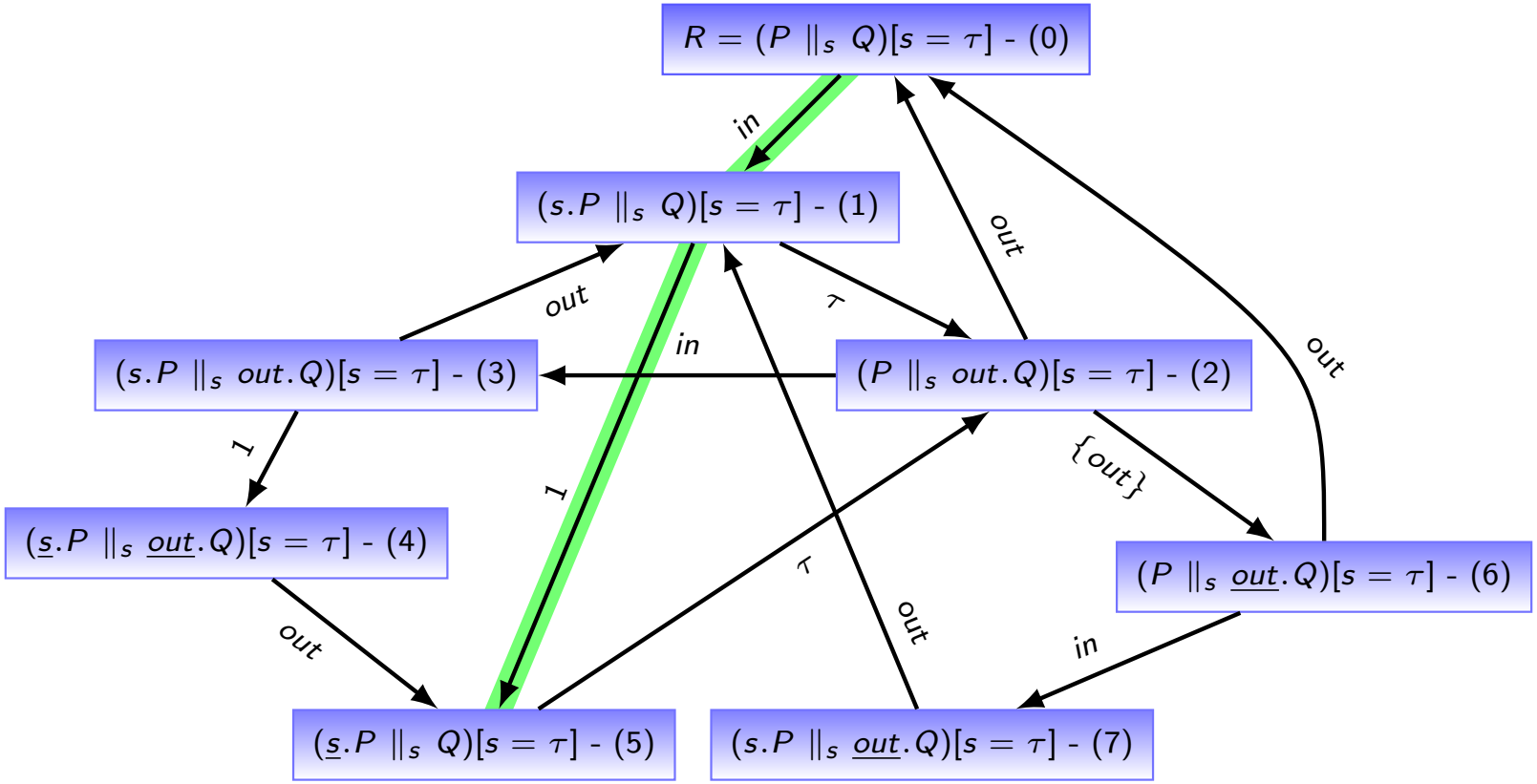

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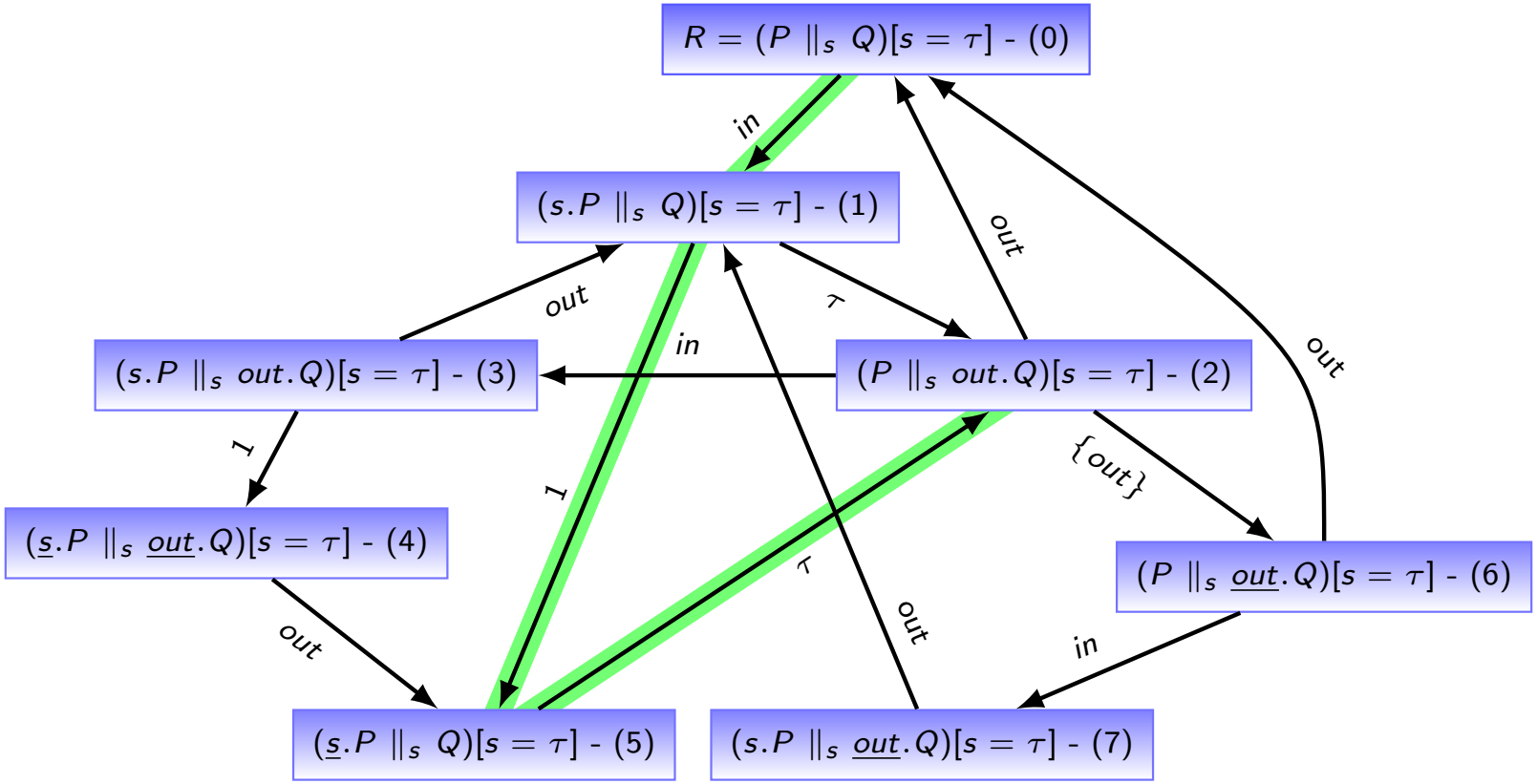

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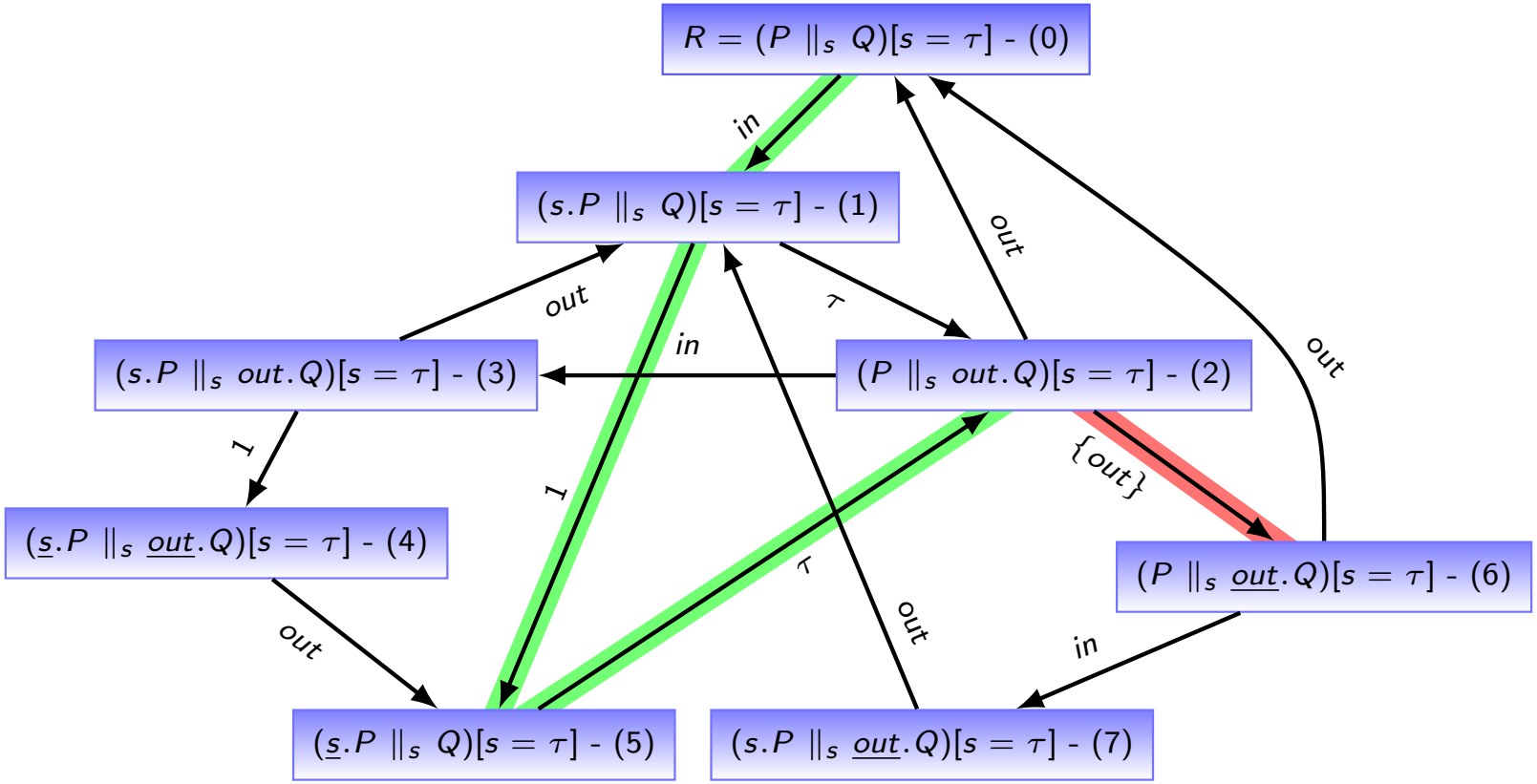

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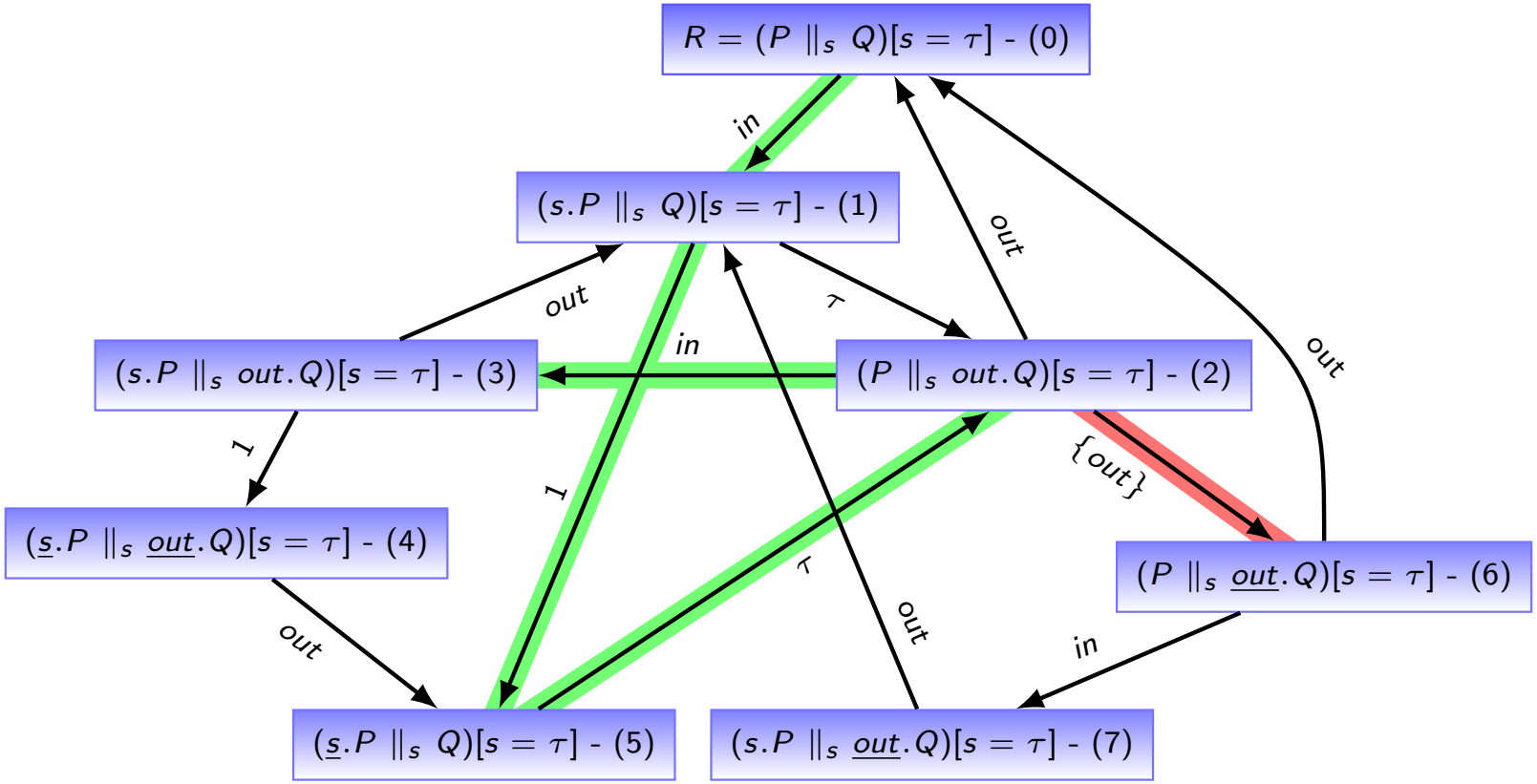

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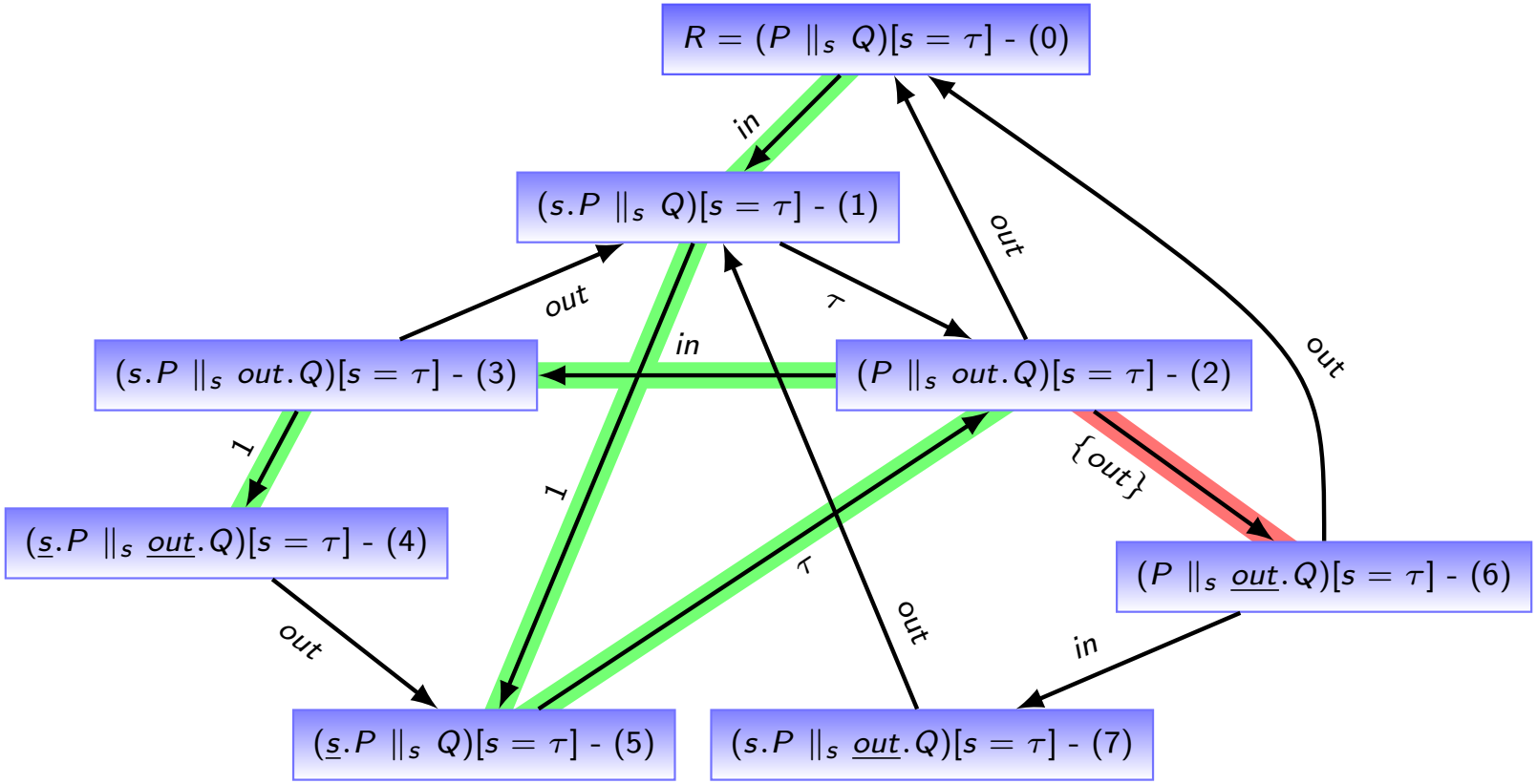

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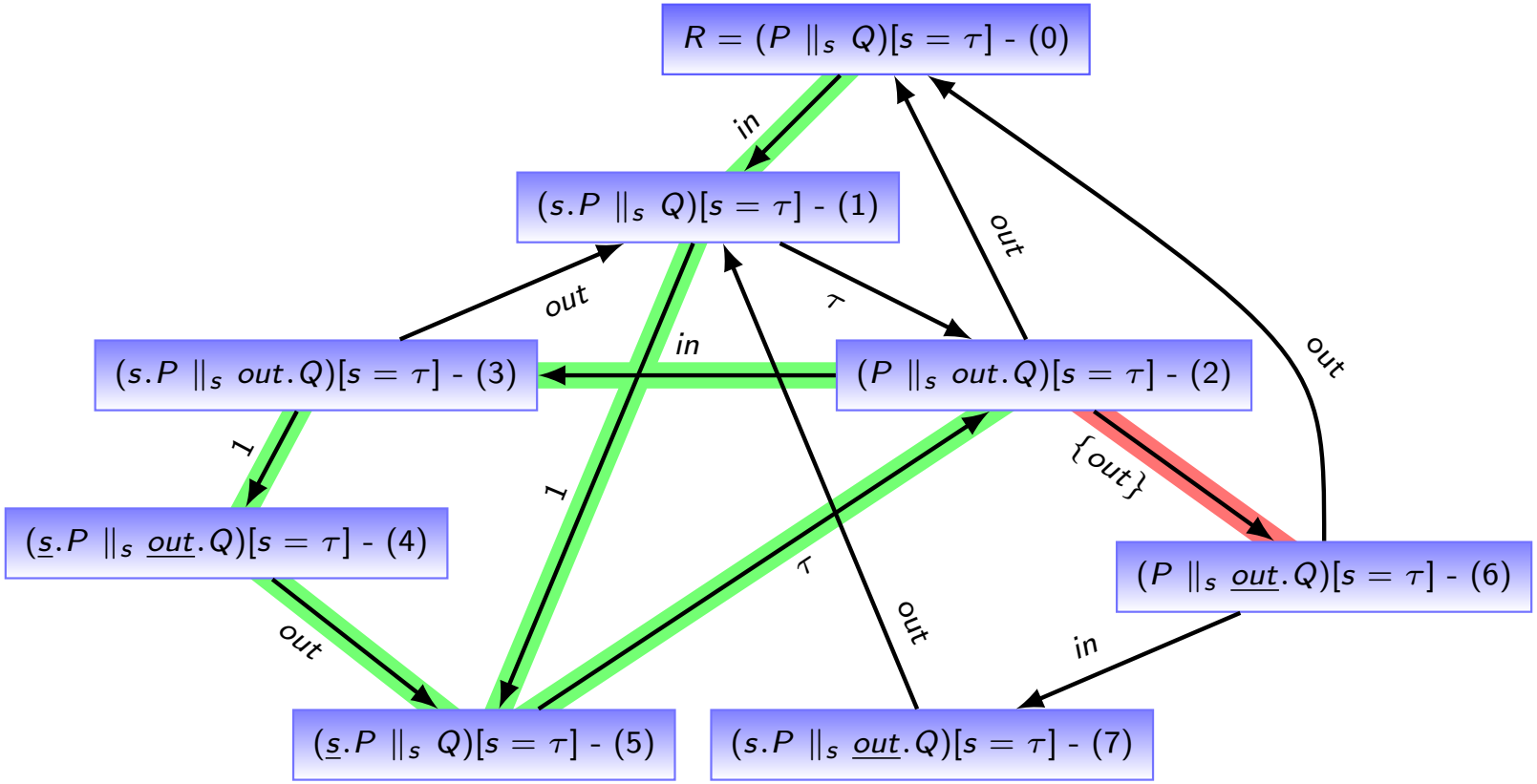

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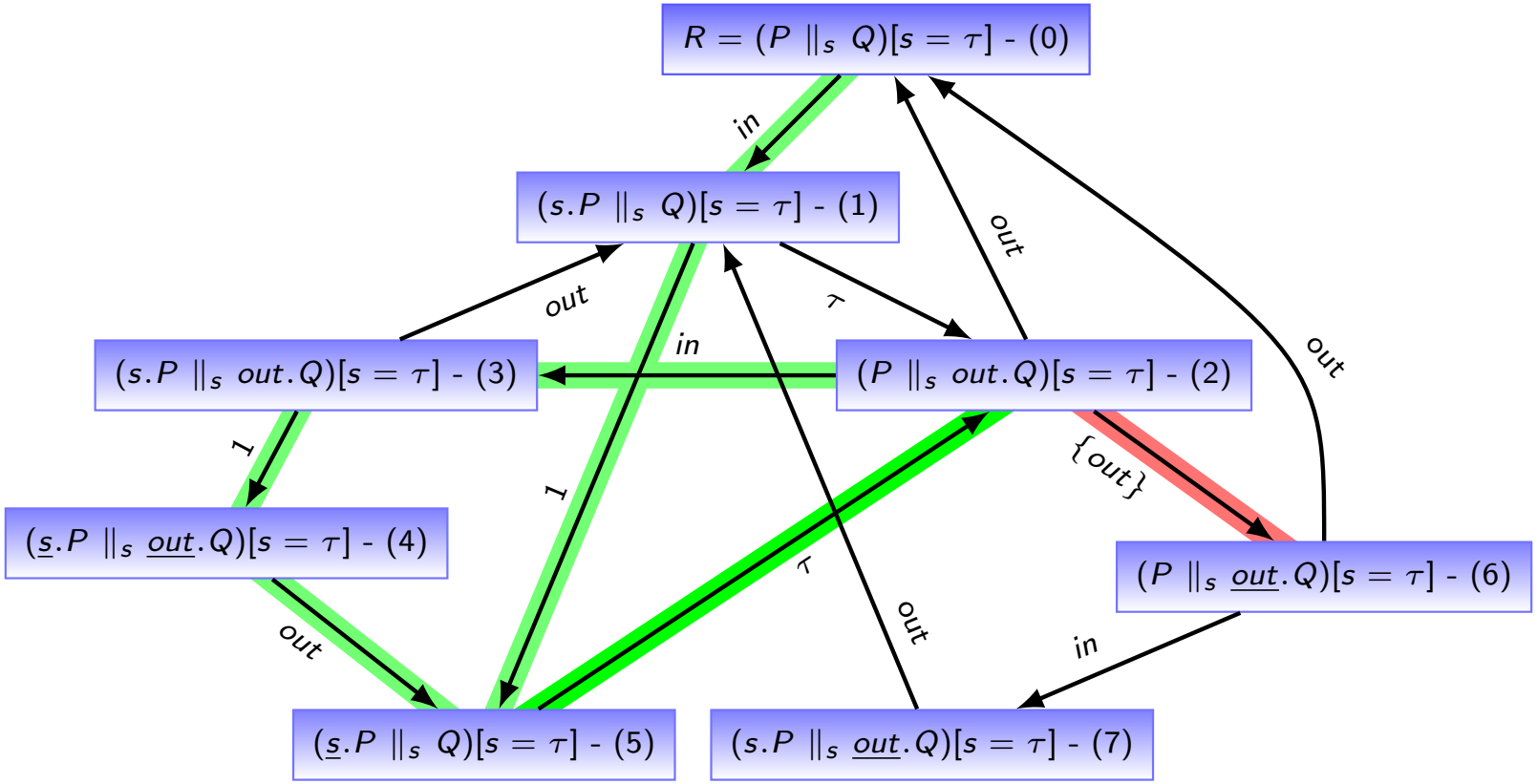

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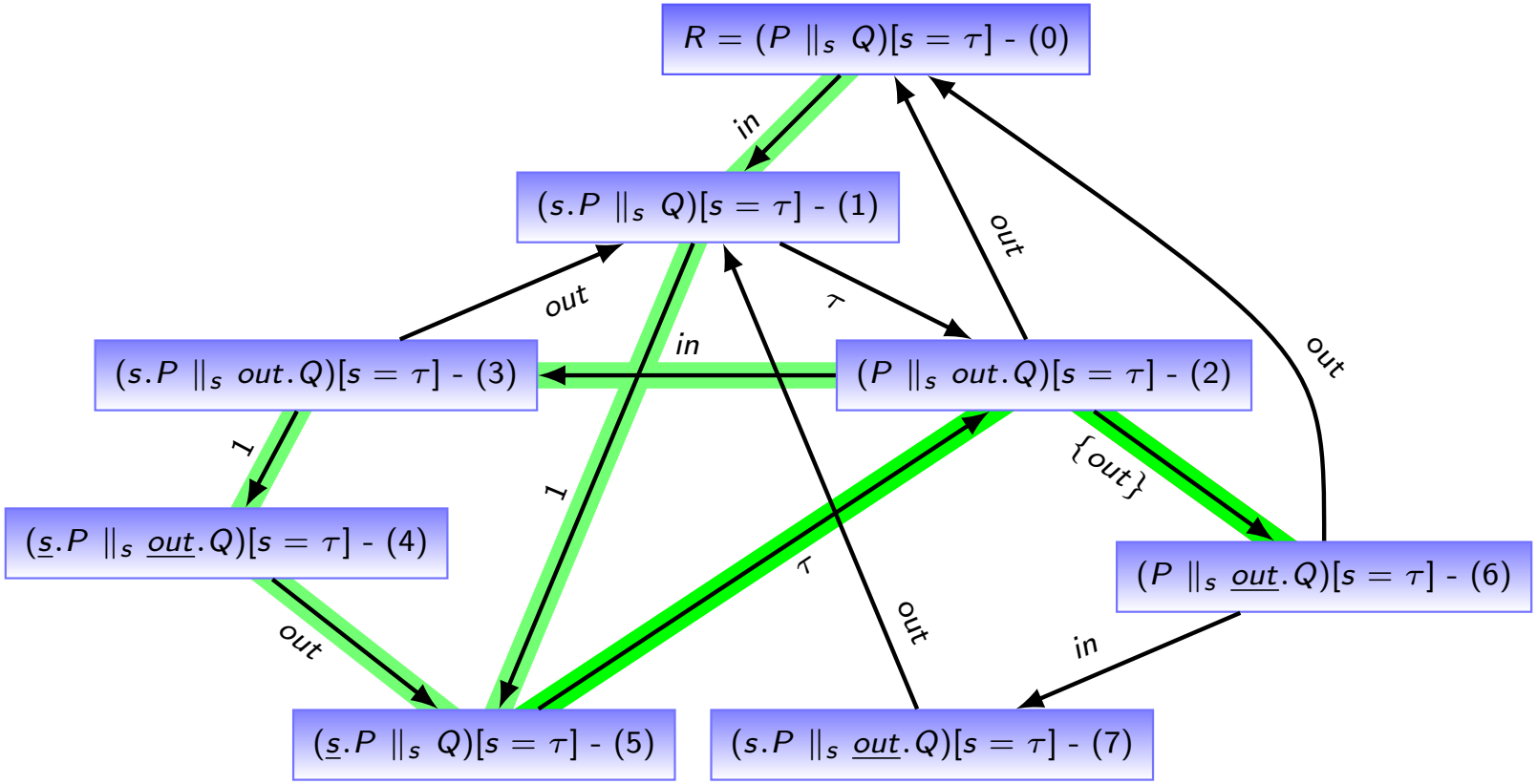

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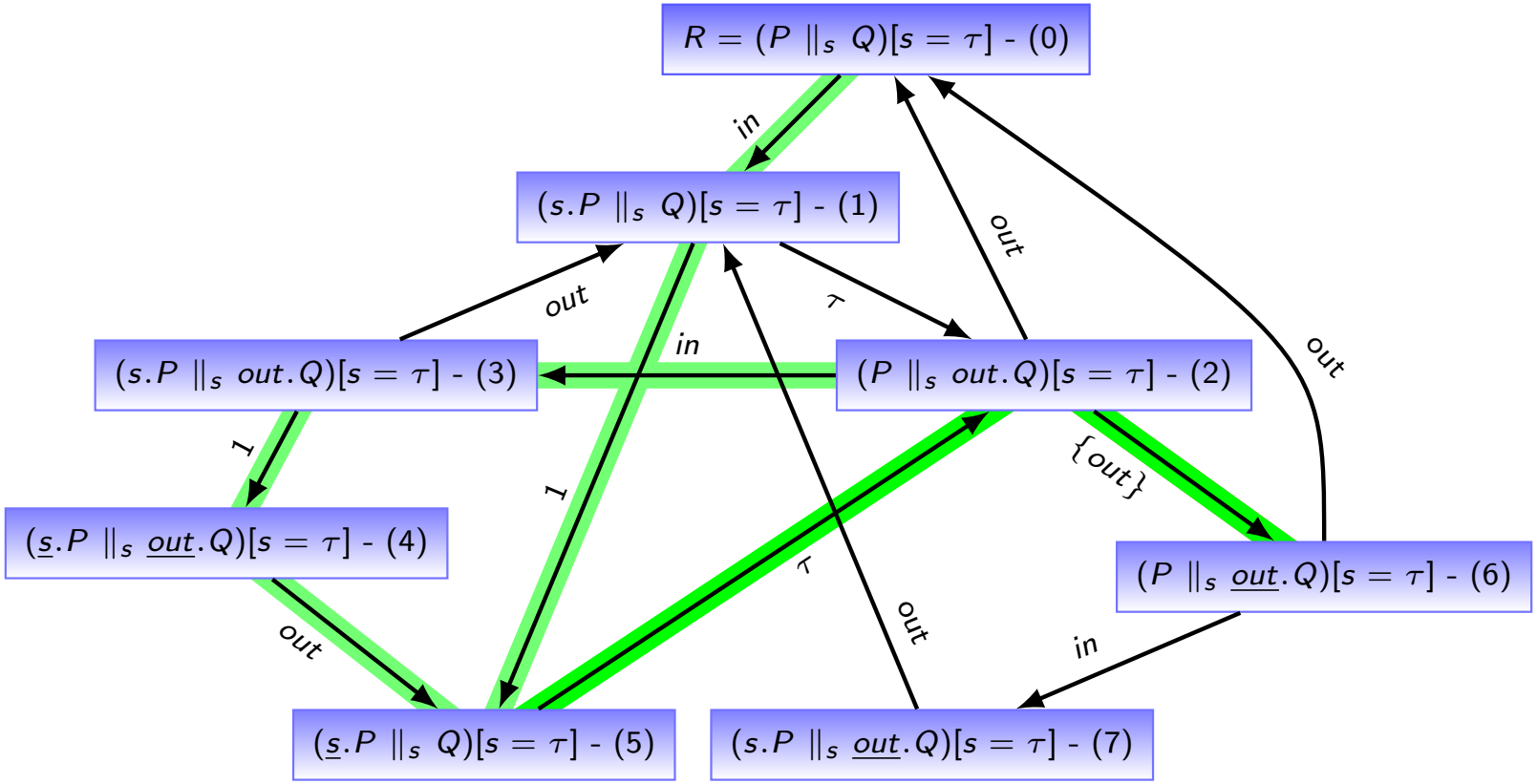

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Time: 1 1 1
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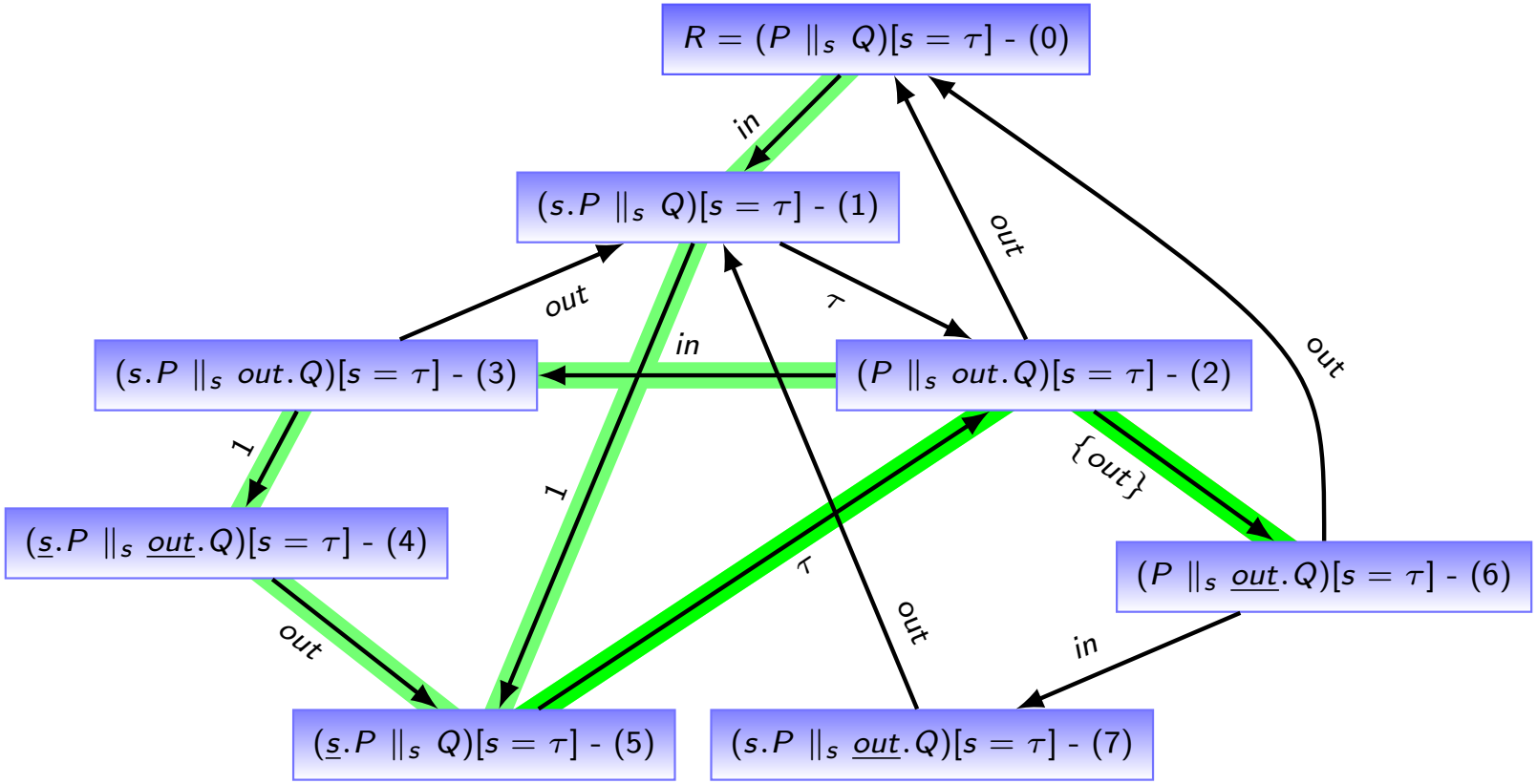

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Time: 1 1 1
$rp_{Pipe}(2) = 3$
It has been shown that $rp_{Pipe}(n) = n + 1$

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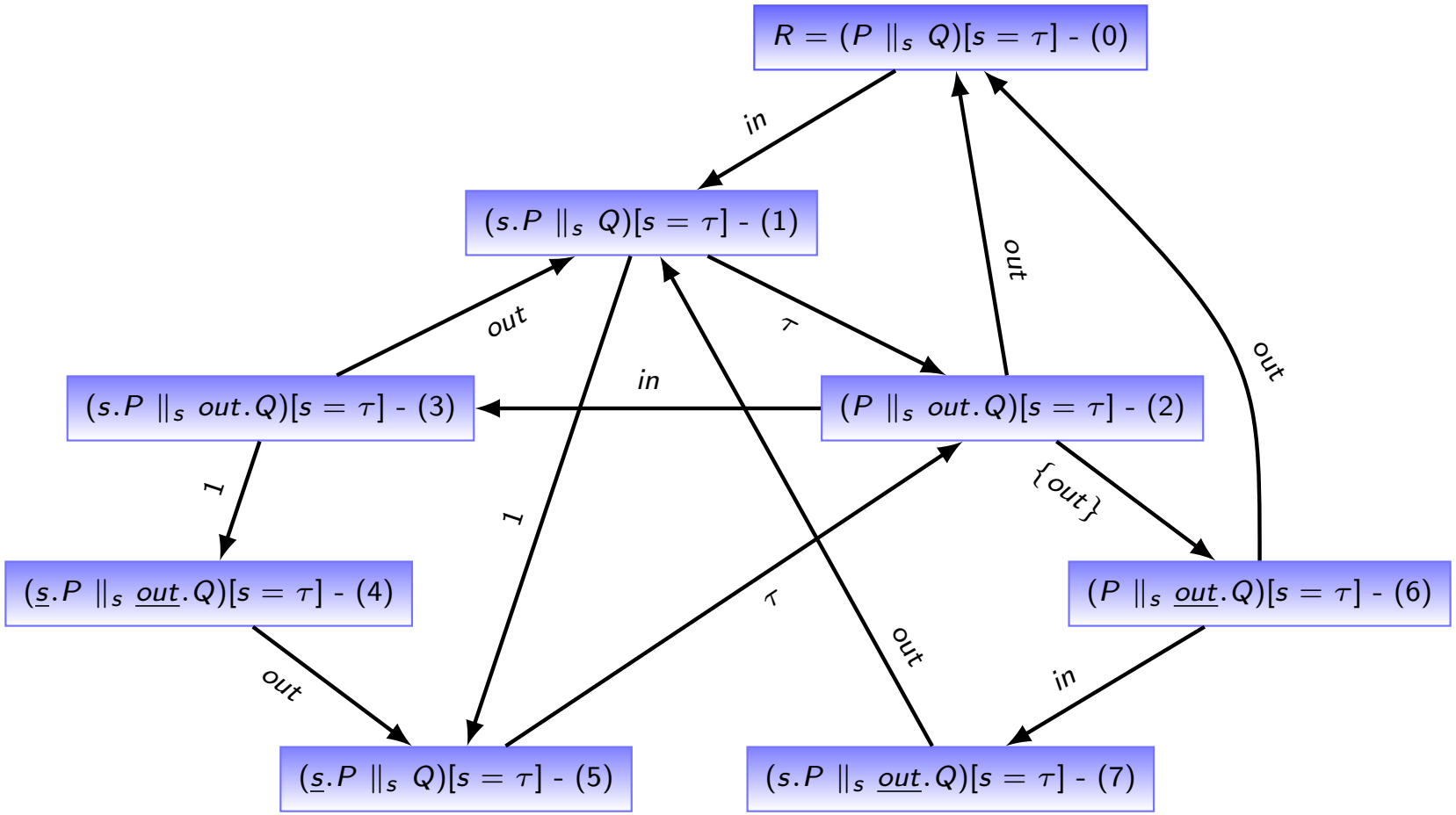
Average performance

$$\frac{|FTS|}{|in|}$$

- The cycle(s) with the *worst average cost*, i.e. the cycle(s) with the **maximum ratio** between the number of its (their) full time steps and *in* actions, is (are) called **bad cycle(s)**
- The average cost of the bad cycle(s) corresponds to the *asymptotic factor* of the response performance

PIPE - Bad cycle(s) old algorithm



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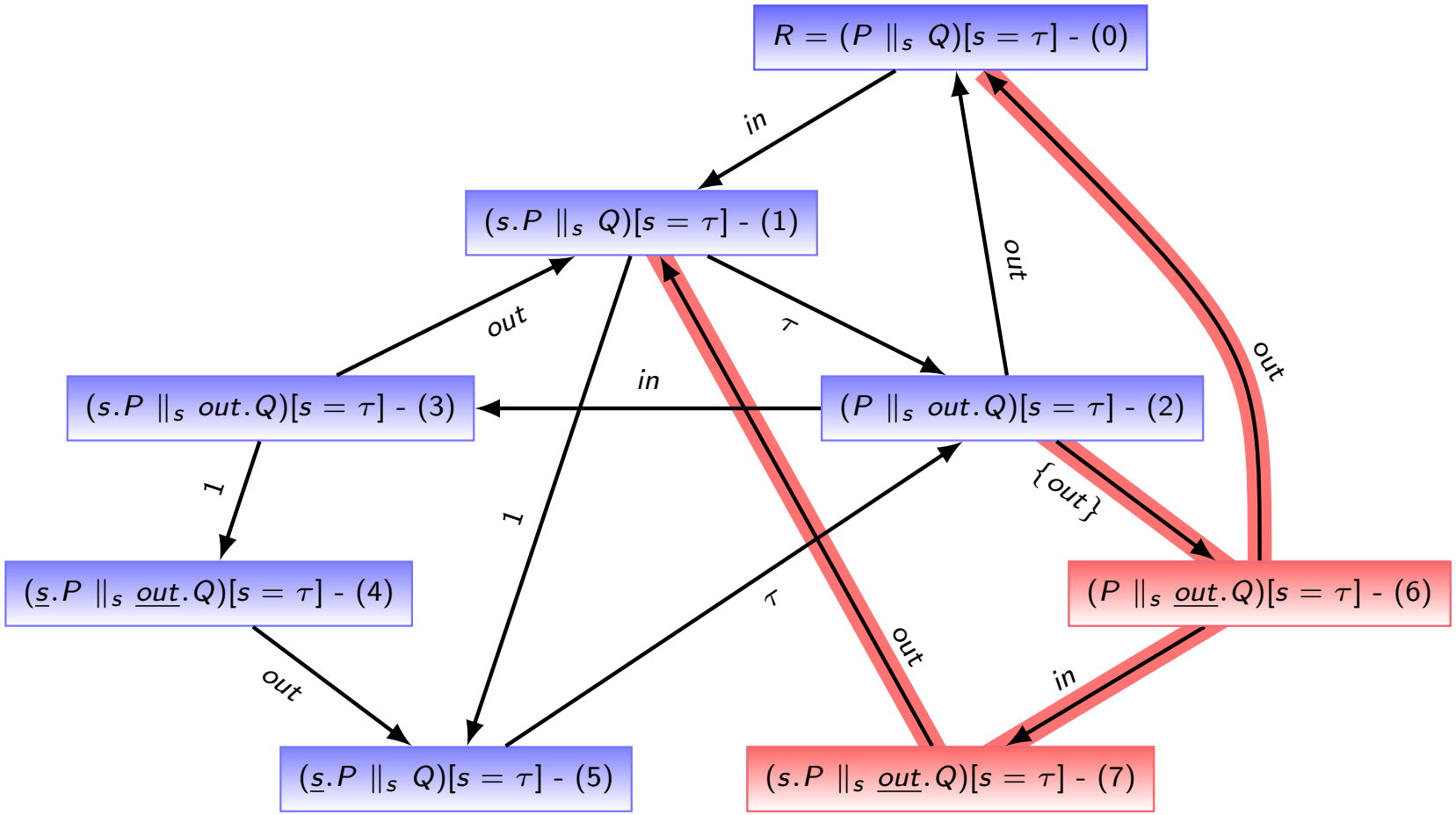

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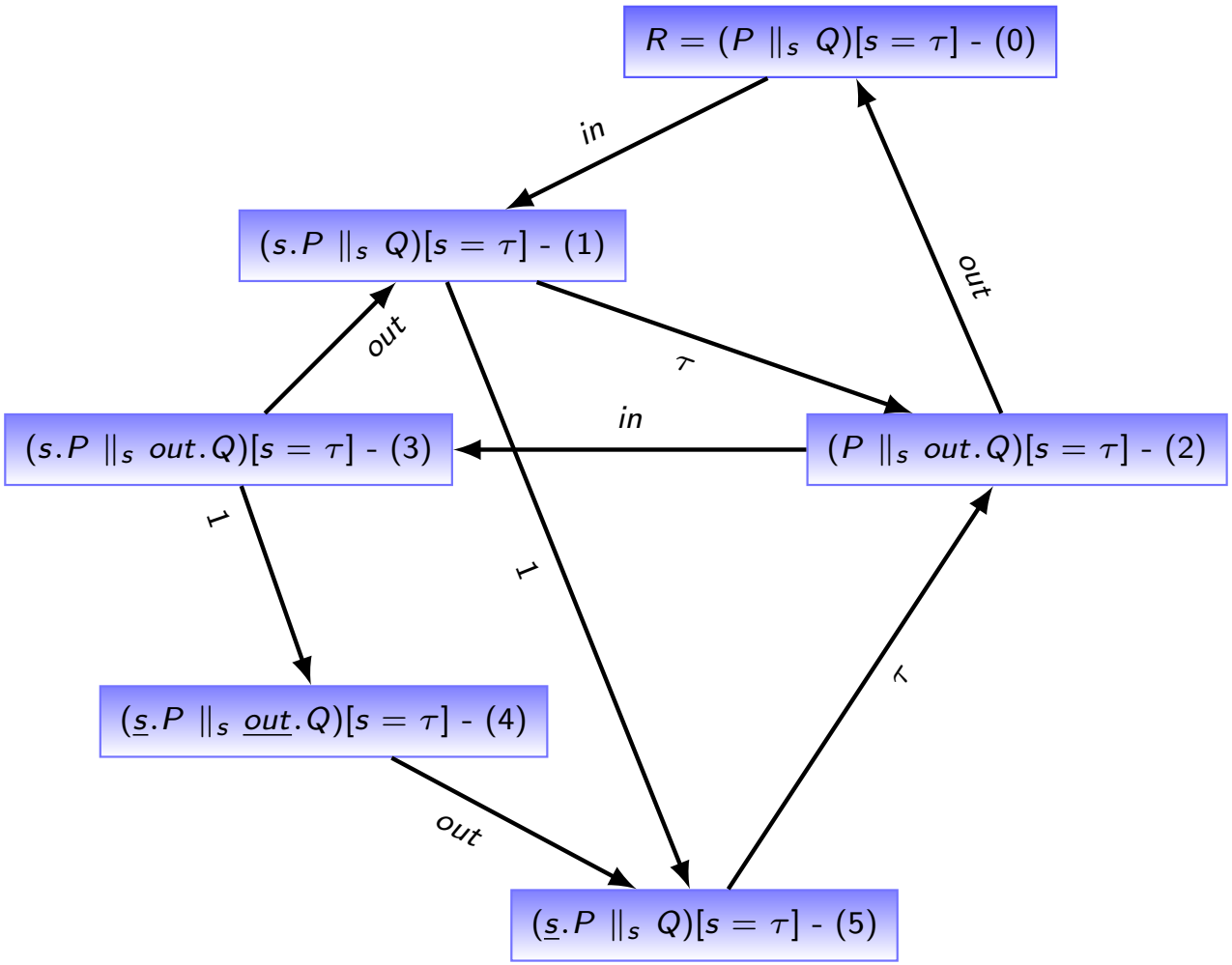

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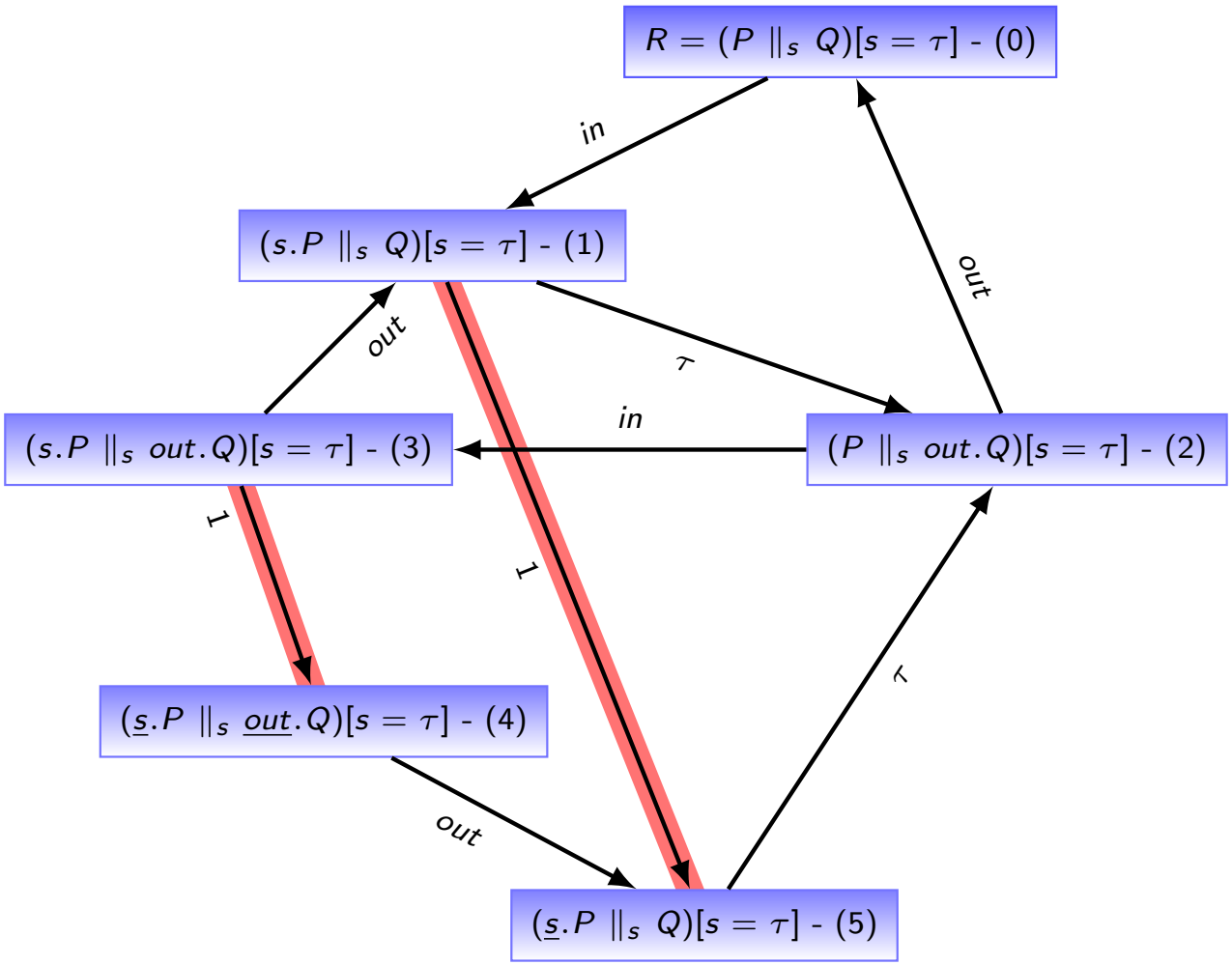

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Algorithm steps:

1. Temporal edge removal
2. Calculation of Floyd-Warshall shortest paths matrix
   1 - *in*'s
   0 - otherwise

	0	1	2	3	4	5
0	1	1	1	2	∞	∞
1	0	1	0	1	∞	∞
2	0	1	1	1	∞	∞
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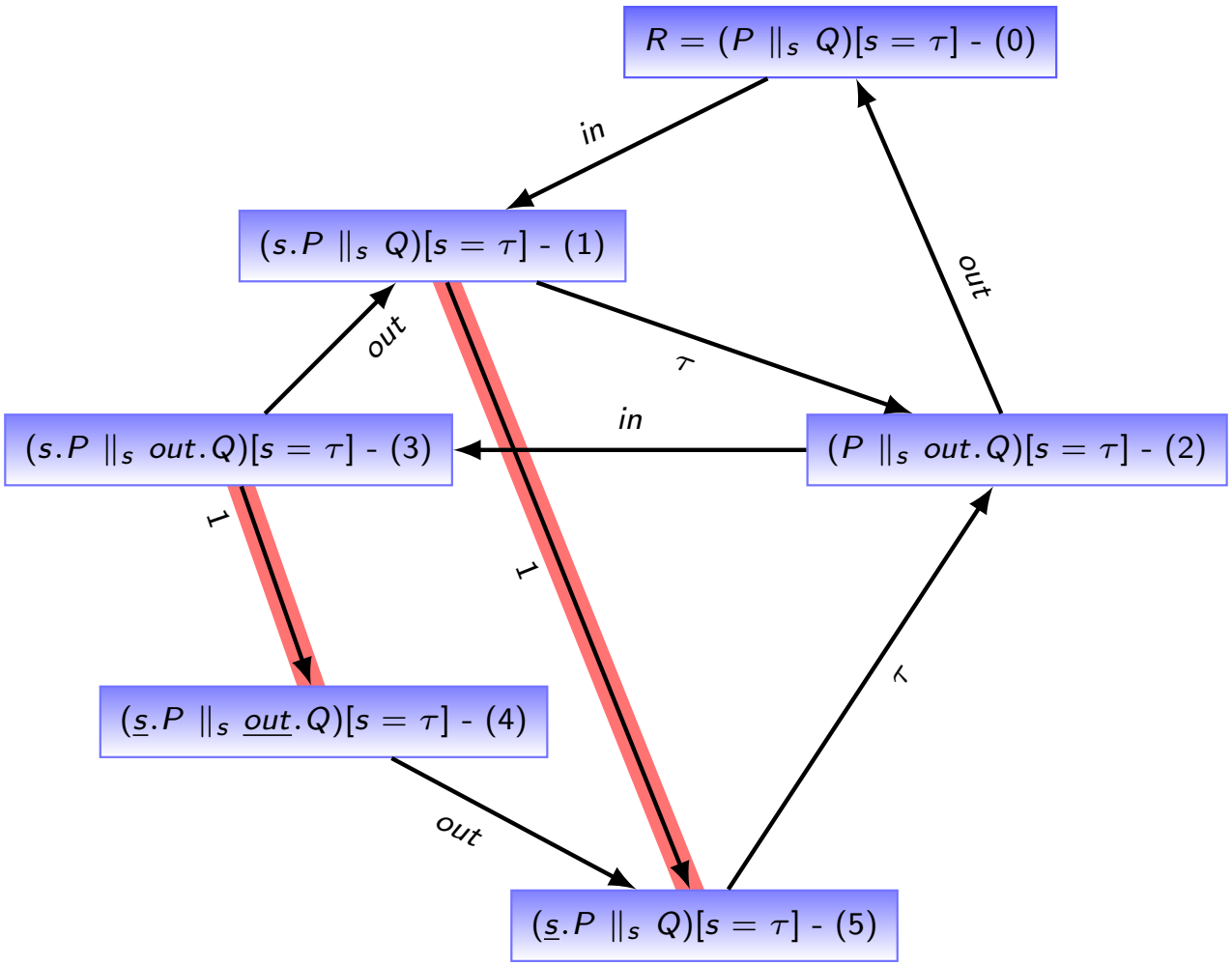

We use the created paths matrix to generate a new graph

For every finite distance:

- We add an edge between the source and the target of a following FTS (if available)
- The edge weight is equal to the corresponding matrix path value

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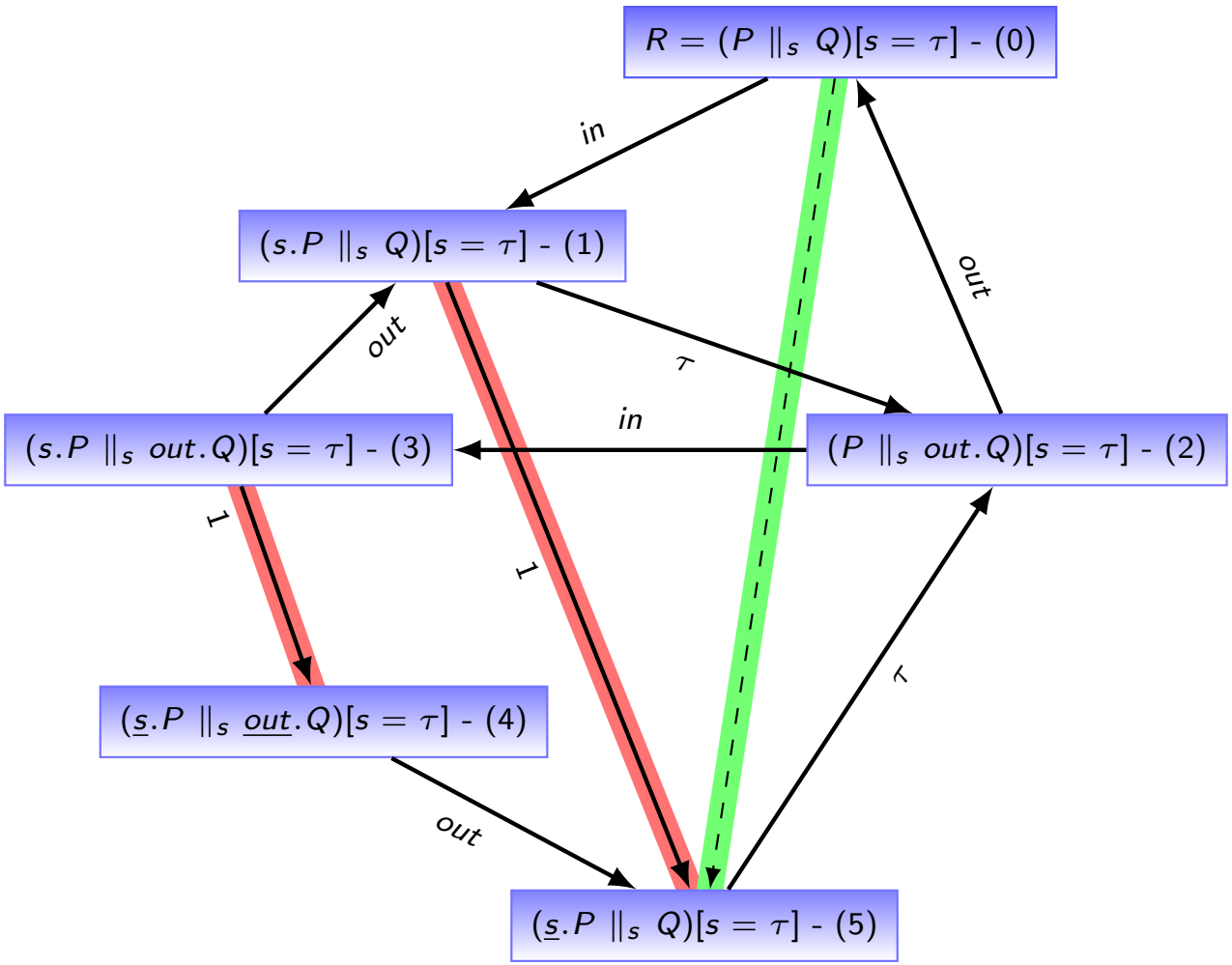

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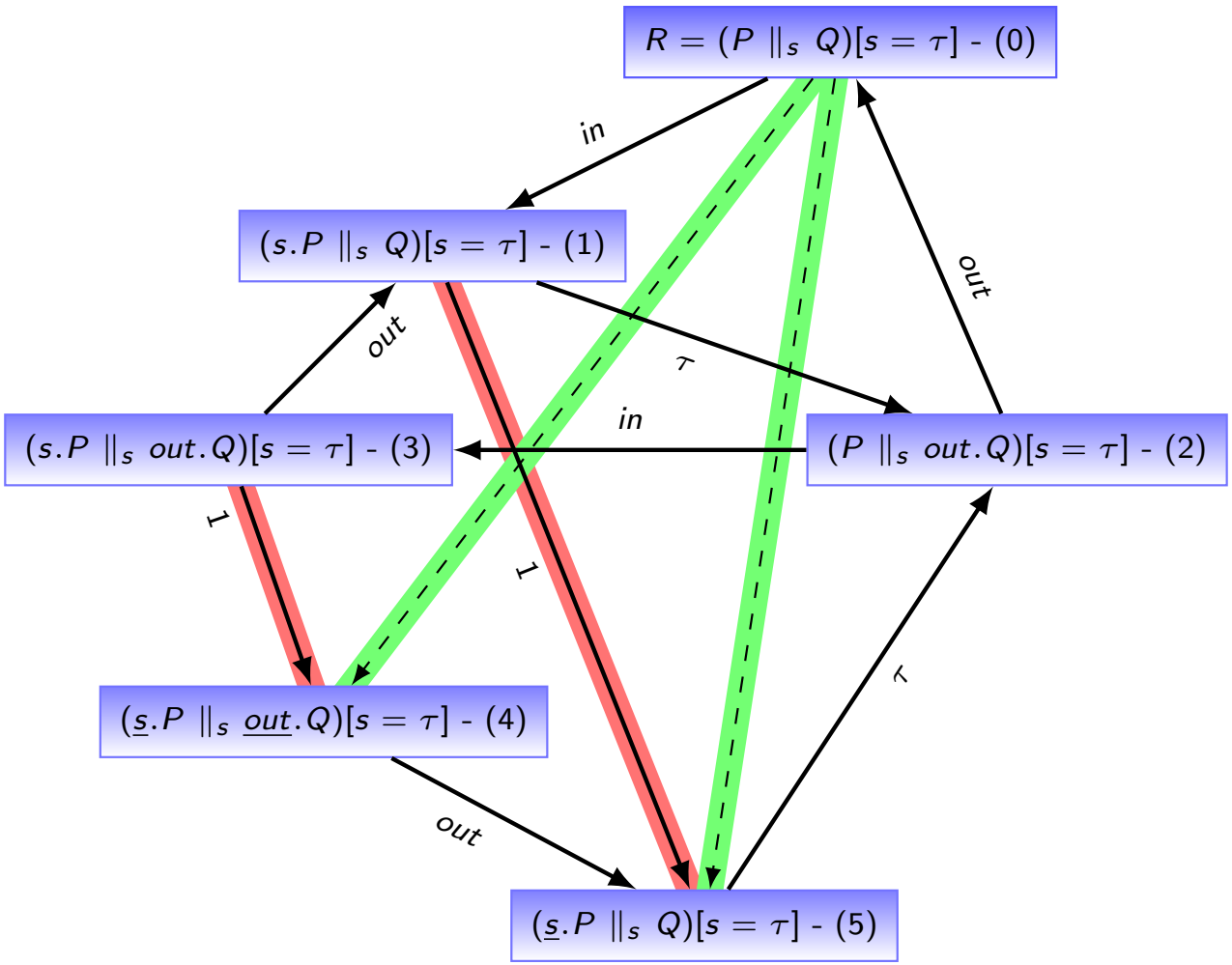

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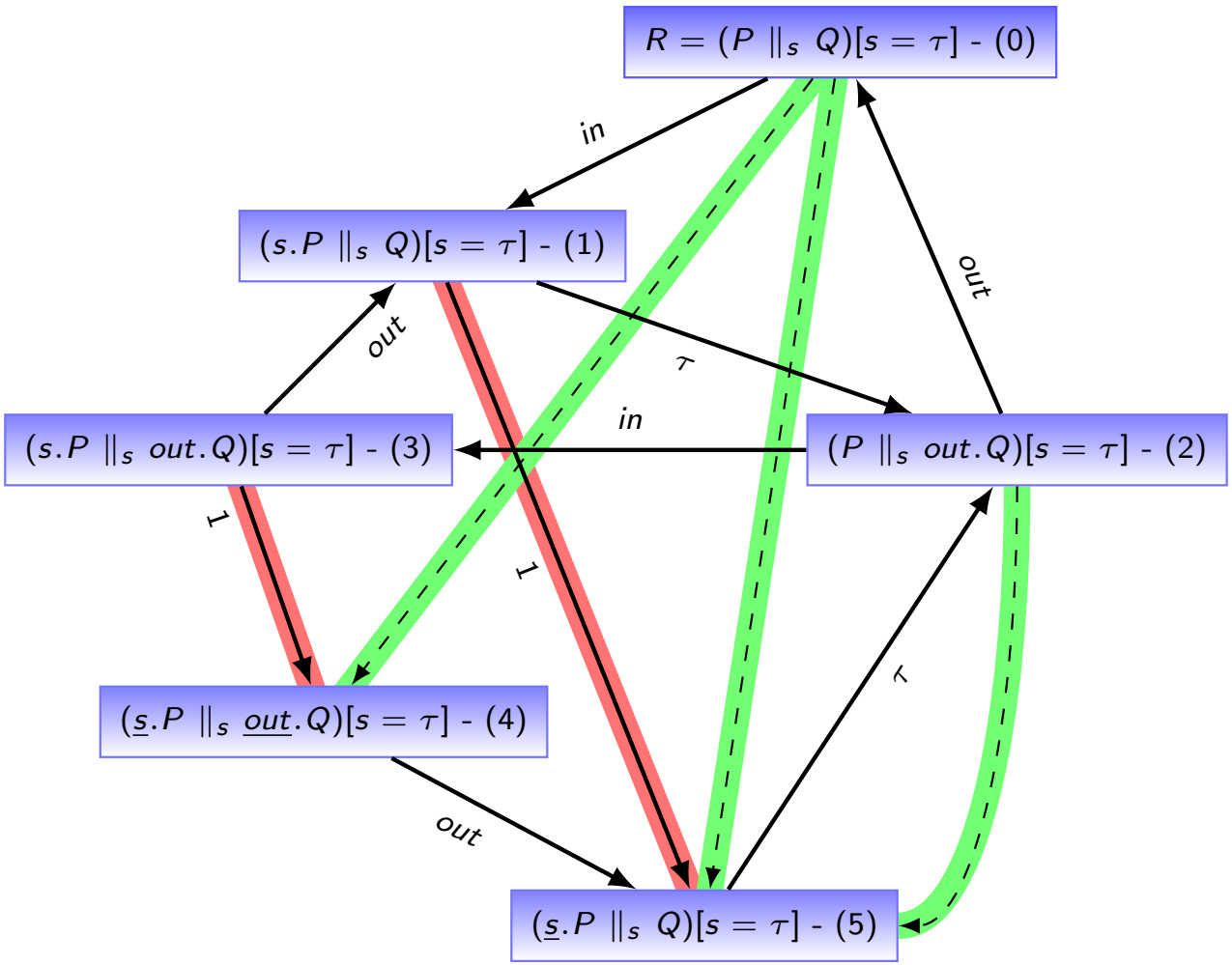

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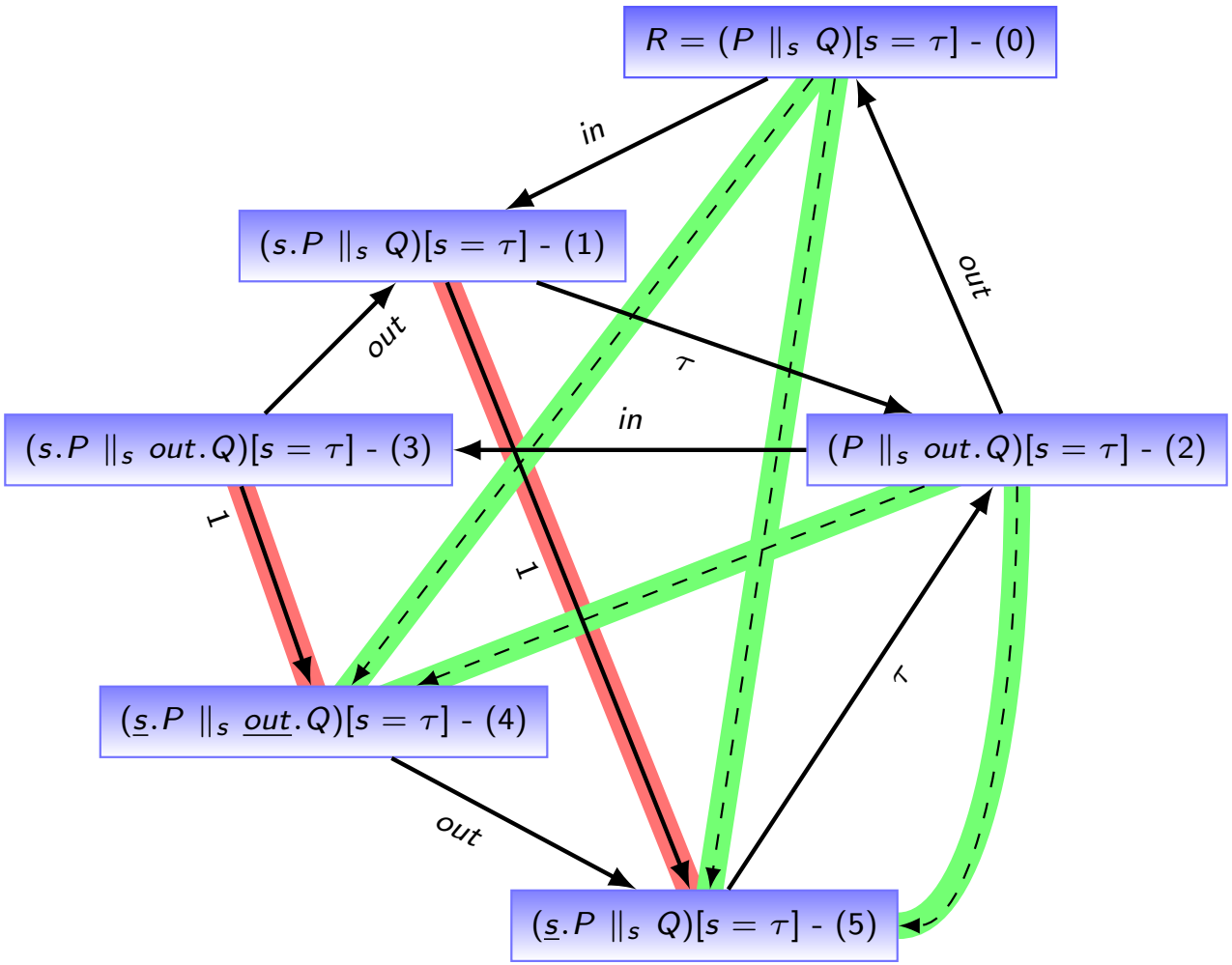

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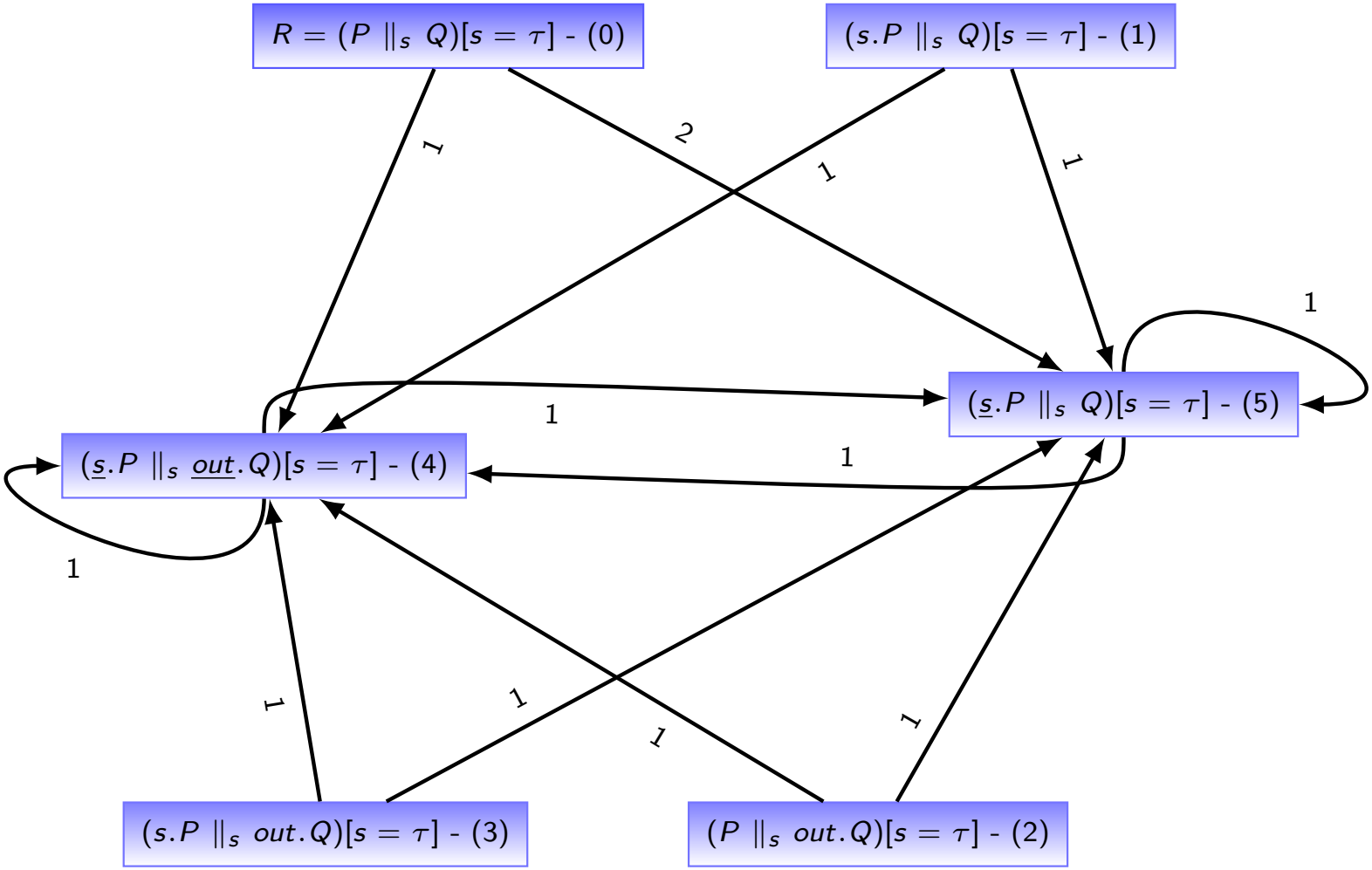
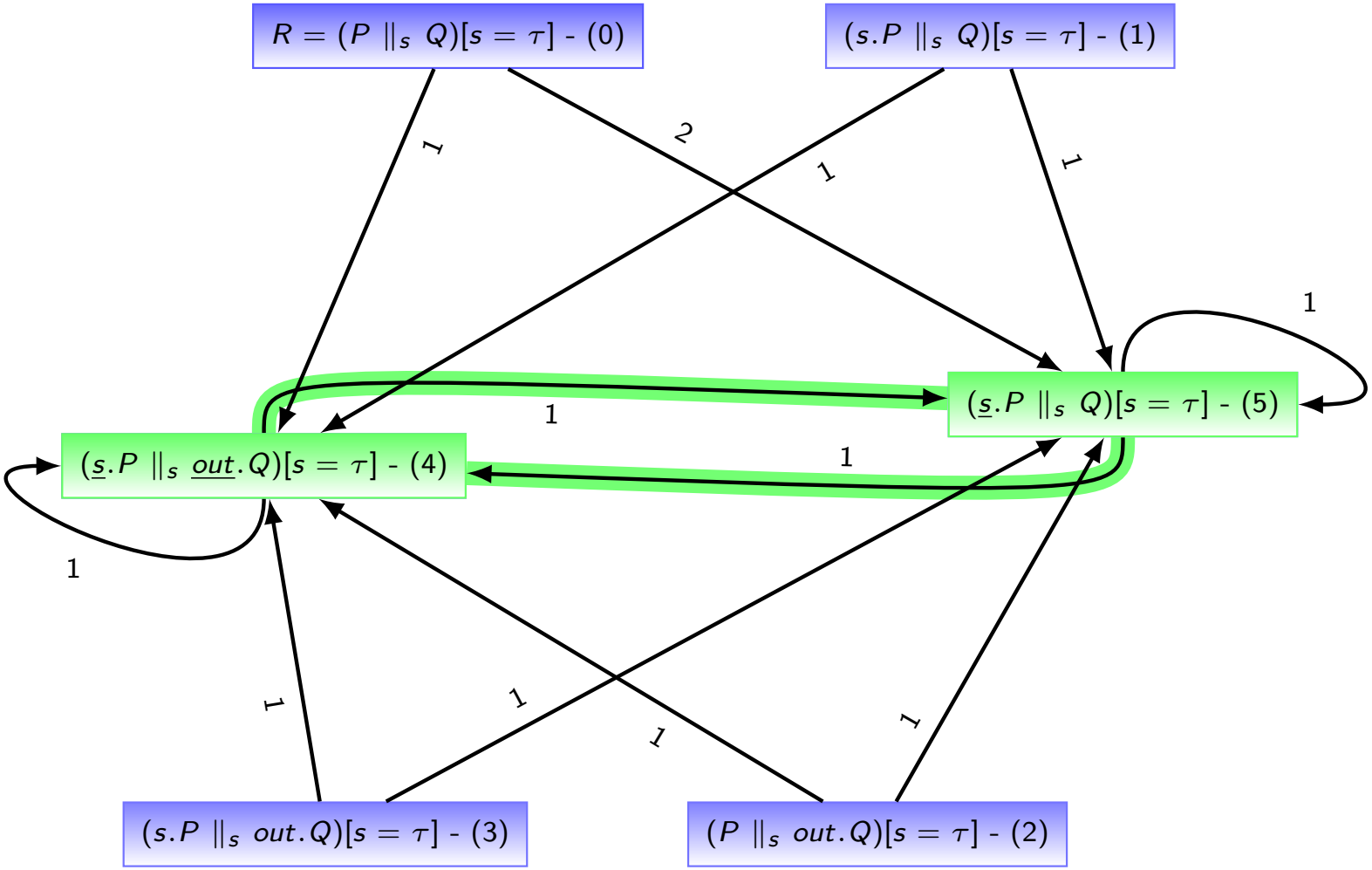
PIPE - Bad cycle(s) calculation graph



On the obtained graph we apply the **Karp algorithm** for finding the minimum mean cycle

PIPE - Bad cycle(s) calculation graph



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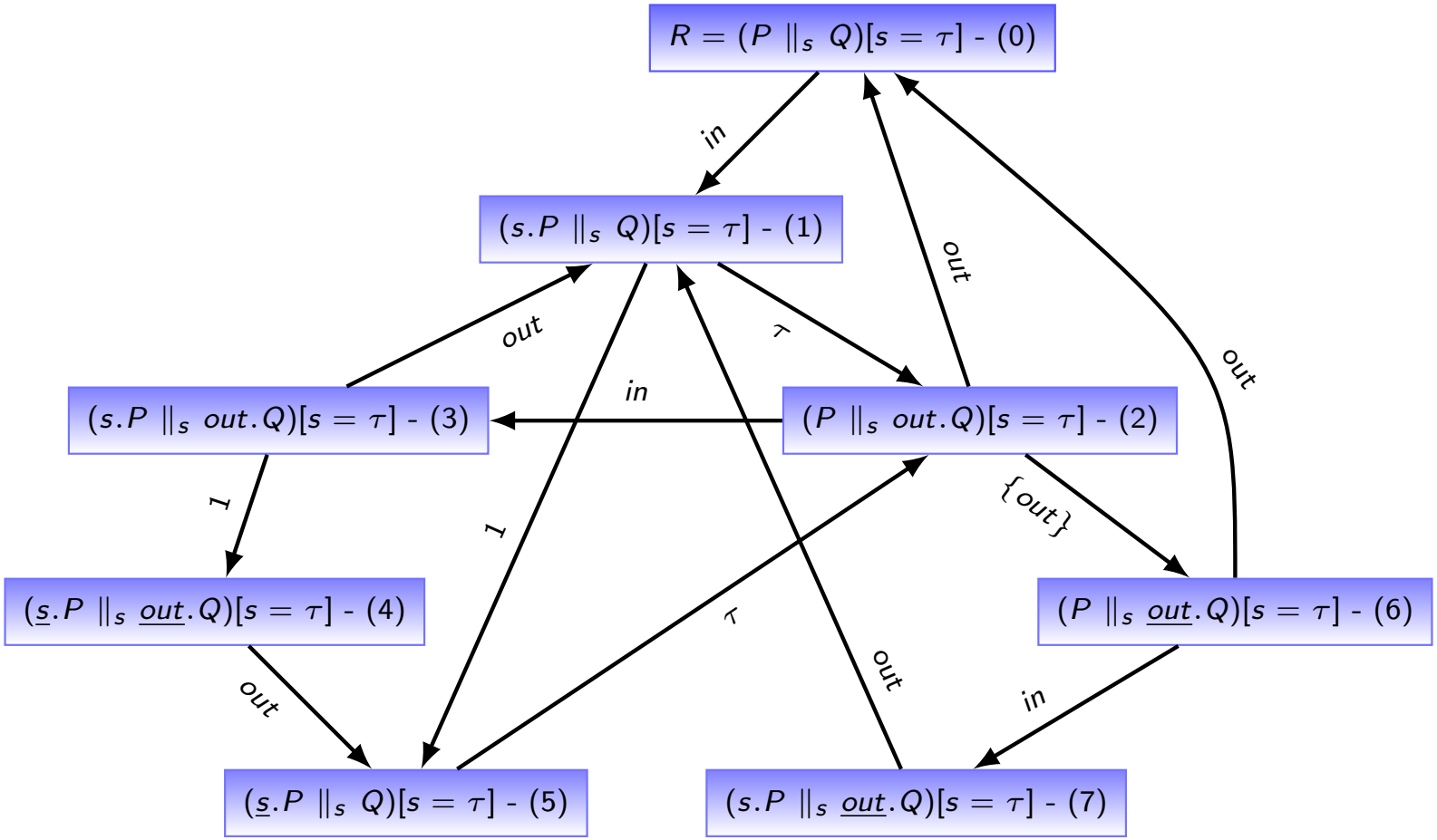
PIPE - Bad cycle(s) example - final result



$$R = (P|s|Q)[s = \tau];$$
$$P = \underline{in}.s.P;$$
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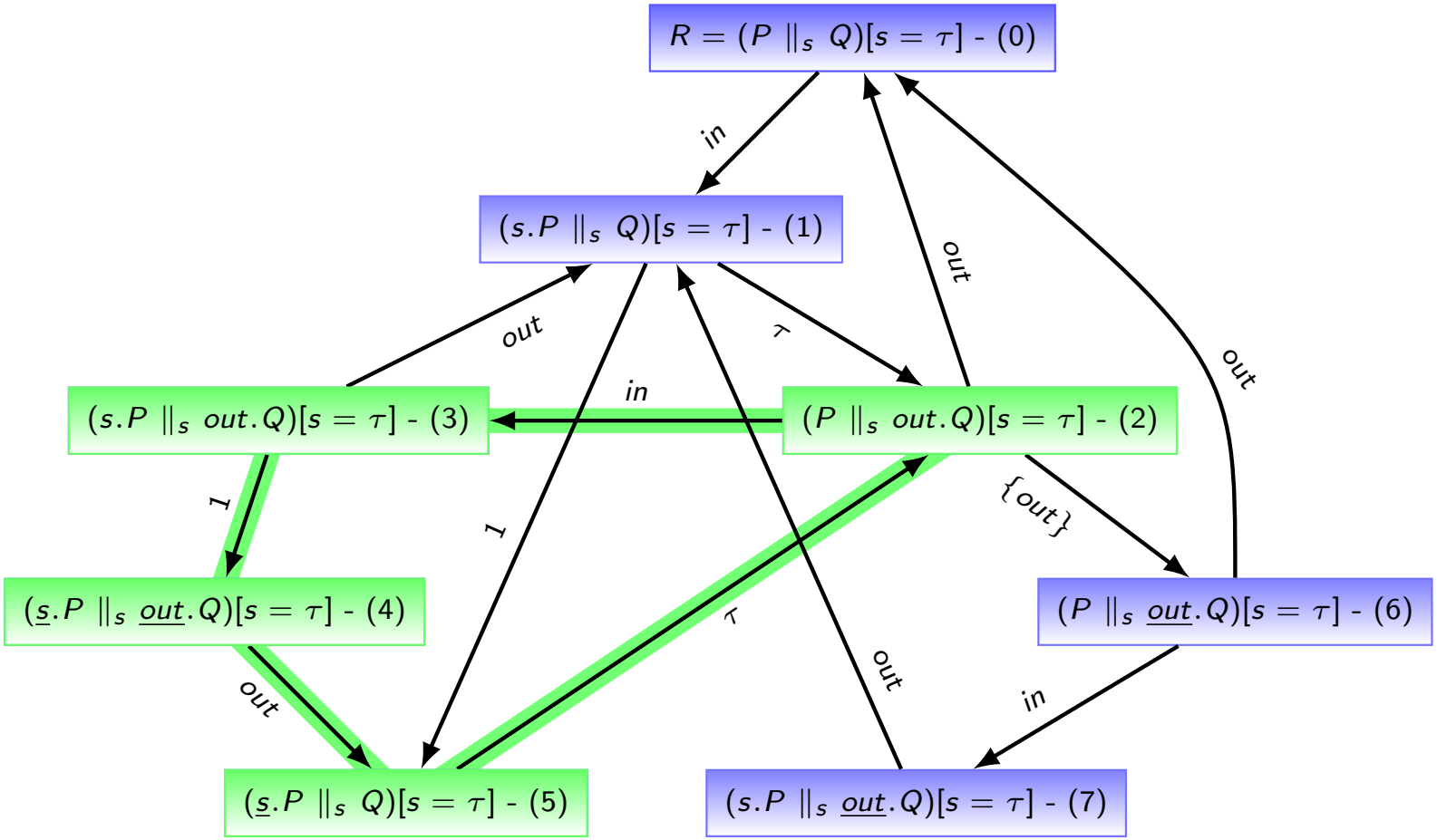

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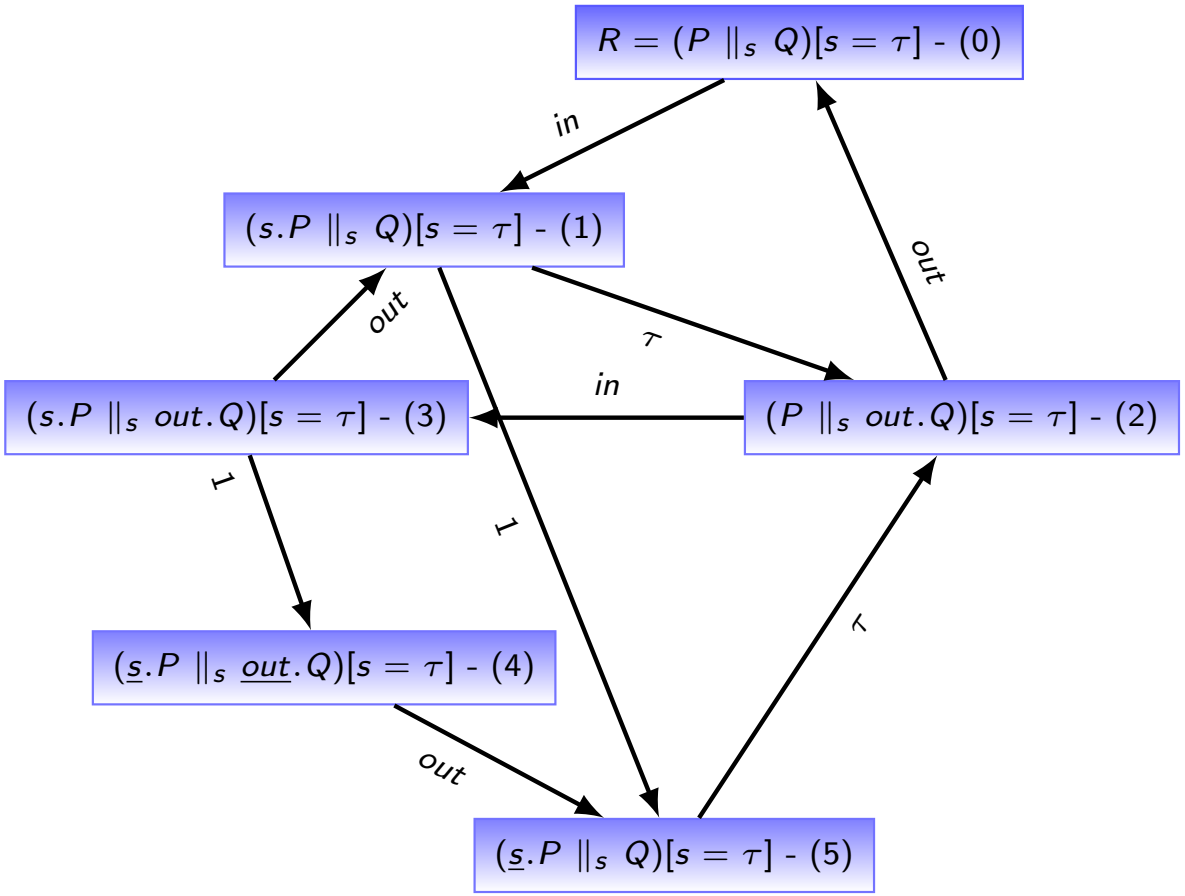
2. We apply *B-F* on each time step (u, v) in G with root v. The weights are:
   - $|N(G)|$ for *in* actions
   - -1 for time steps
   - 0 for *out* and τ actions

PIPE - New Bad cycle(s) example



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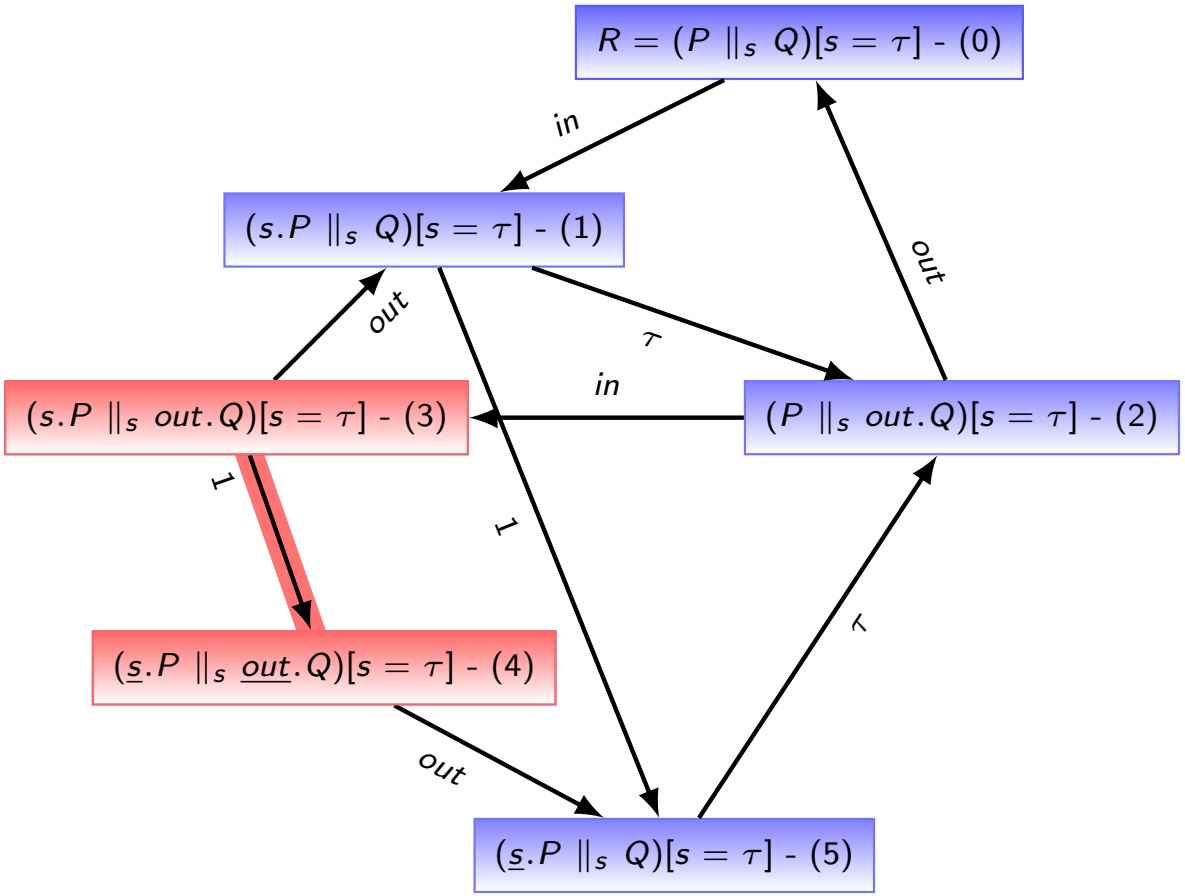

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$4 \rightsquigarrow 3$

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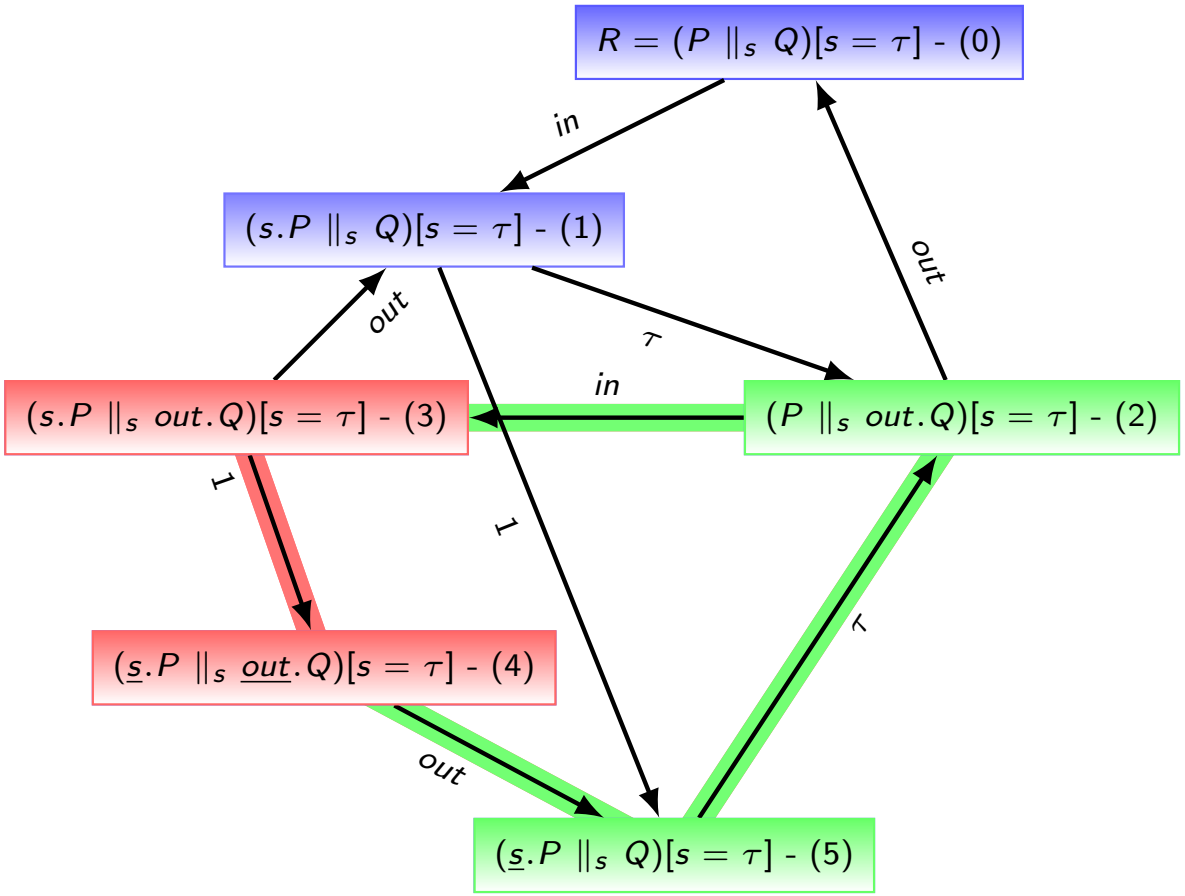

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1 *in* - 0 *ts* (+1 due to $(3, 4)$)
average performance $= 1$

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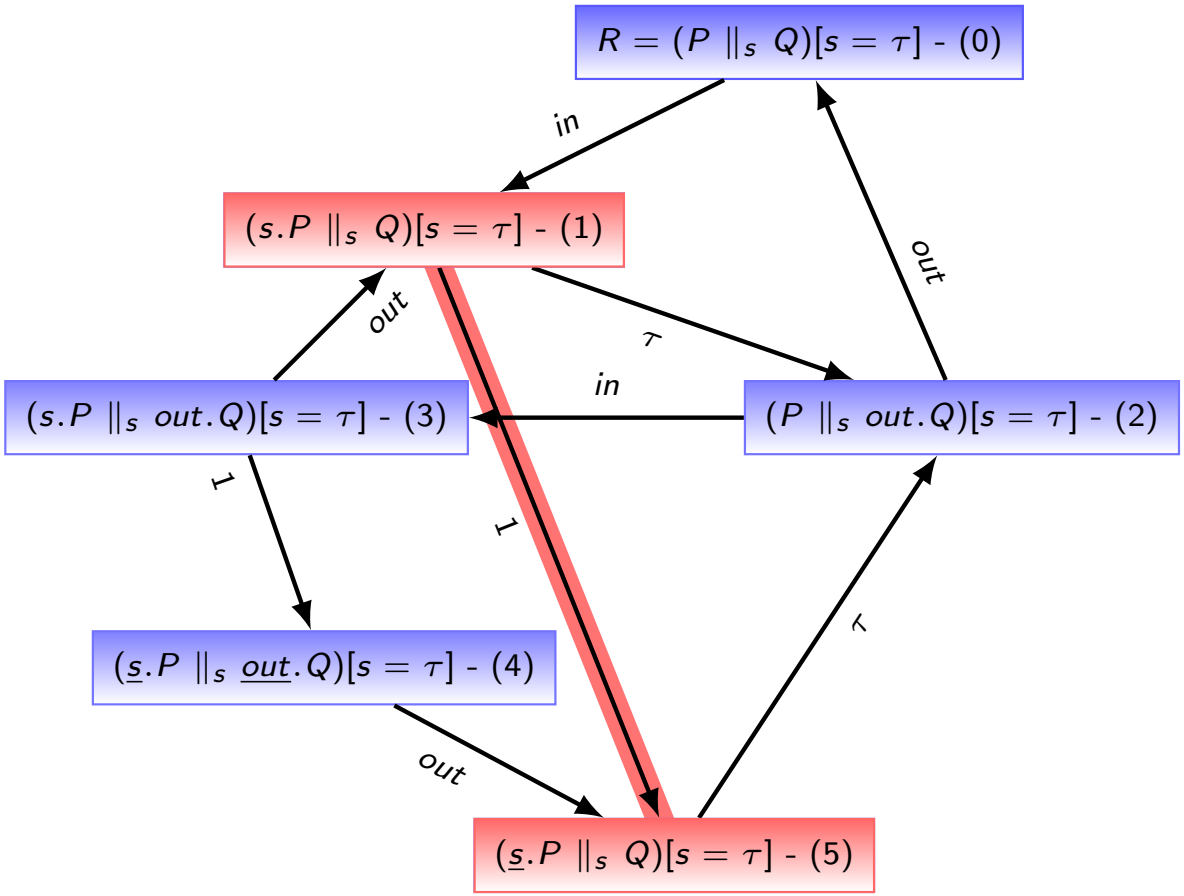

$R = (P|s|Q)[s = \tau];$
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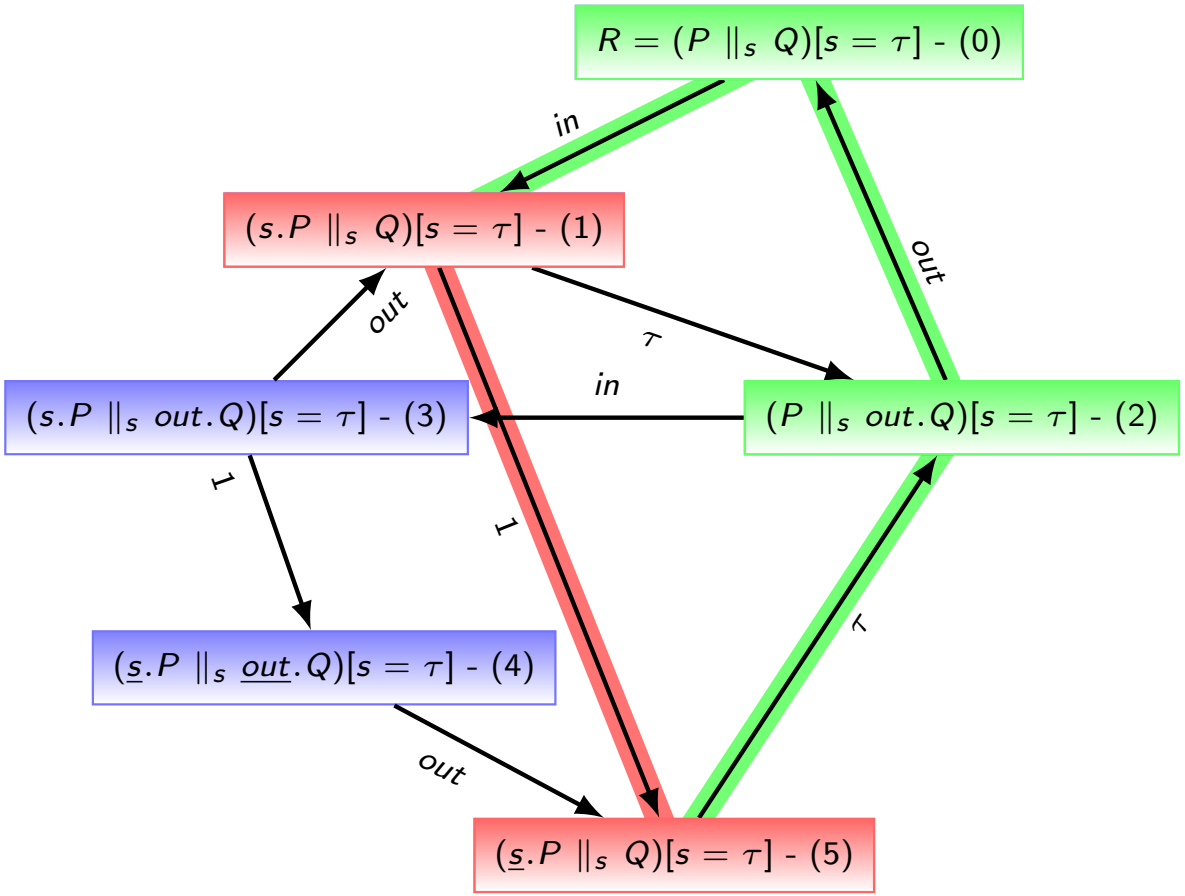
$Q = \underline{s}.out.Q;$

$4 \rightsquigarrow 3$
1 *in* - 0 *ts* ($+1$ due to $(3, 4)$)
average performance $= 1$

$5 \rightsquigarrow 1$

PIPE - New Bad cycle(s) example



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$4 \rightsquigarrow 3$
1 *in* - 0 *ts* (+1 due to $(3, 4)$)
average performance $= 1$

$5 \rightsquigarrow 1$
1 *in* - 0 *ts* (+1 due to $(1, 5)$)
average performance $= 1$

Improving Bad cycles calculation

- Applying the procedure for each time step (u, v) in each SCC, we retain only the paths that give the maximum ratio between time and *in* actions, i.e. the *bad cycle(s)*
- The complexity of the new algorithm is $\mathbf{O(N^2E)}$:
  1. The complexity of *Bellman-Ford* is $O(NE)$
  2. We could have at most N full time steps in $rRTS(P)$
- Even if this complexity value seems not much better than the previous one, experimental results show a drastic improvement in bad cycle(s) calculation

Bad Cycle(s) algorithms performance comparison

	nodes/edges	*Floyd + Karp*	*Bellman − Ford*	*Gain*
$Fifo_3$	9/18	0	0	0%
$Pipe_3$	20/40	32	15	53.2%
$Buff_3$	17/34	281	47	83.2%
$Fifo_7$	17/38	15	0	100%
$Pipe_7$	648/1896	40375	1063	97.3%
$Buff_7$	240/560	5547	1531	72.3%
$Fifo_9$	21/48	16	0	100%
$Pipe_9$	3680/12648	-	34110	100%
$Buff_9$	448/1064	32594	8312	74.4%

Table: Bad cycles detection time (expressed in *ms*)

And...

Thanks!