A Process Algebraic Approach to Modeling and Performance Evaluation of Blockchain Fraud Reversal Protocols

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Agenda

Context

- Public and private blockchains (digital Euro)
- Fraudulent transactions and smart contracts
- Fraud Reversal Protocols (FRP)
- Can we reverse a (possibly) fraudulent transaction?
- How to model and evaluate the performance of FRPs?

Relevant Questions

- Is there a **optimal time window** to ask for a refund?
- Is there a **cost** associated with the reversal?
- Is there a **trade-off** between the *productivity* of the blockchain and *fairness*, i.e. the possibility of reversing a transaction?

Components

Four main components:

- *Block states*: blockchain evolution throughout the relevant time window
- *User*: a user of the blockchain that is involved in a potentially fraudulent transaction
- *Judges*: a set of judges that can be involved in the transaction reversal process, deciding whether on the reversal (refund) request
- *Hacker*: a malicious user that tries to exploit the blockchain by creating fraudulent transactions

PEPA Specification - Block States

- $\boldsymbol{B_1} = (c_1, \gamma).B_2$
- $\boldsymbol{B_i} = (req, \top).W_i + (c_i, \gamma).B_{i+1}$ for $i = 2, \ldots, n-1$
- $\boldsymbol{B_n} = (req, \top).W_n + (c_n, \gamma).B_1$
- $W_i = (ign, \gamma).B_i + (reset, \delta).B_1$ for $i = 2, \ldots, n$

PEPA Specification - User, Hacker, Judges

- $\boldsymbol{User} = (c_1, \top).Victim + (c_n, \top).User$
- $Victim = (req, r).Req + (c_n, \top).User$
- $Req =$
  $(ign, \top).User + (refund, \top).Granted + (punish, \top).Resume$
- $Granted =$
  $(full, \top).Resume + (partial, \top).Resume + (none, \top).Resume$
- $Resume = (reset, \top).User$
- $\boldsymbol{Hacker} = (full, p\sigma).Hacker + (partial, (1-p)t\sigma).Hacker + (none, (1-p)(1-t)\sigma).Hacker$
- $\boldsymbol{Judges} = (ign, vs).Judges + (refund, (1-v)ws).Judges + (punish, (1-v)(1-w)s).Judges$

PEPA Specification - System Cooperation Equation

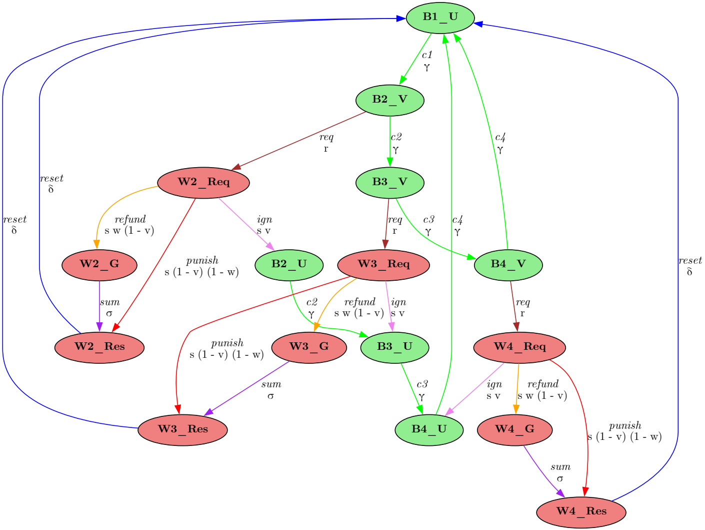
Denoting with

- $L_1 = \{c_1, c_n, reset, ign, req\}$
- $L_2 = \{ign, refund, punish\}$
- $L_3 = \{full, partial, none\}$

the system cooperation equation is:

$$\boldsymbol{B_1} \underset{\boldsymbol{L_1}}{\bowtie} \left(\boldsymbol{User} \underset{\boldsymbol{L_3}}{\bowtie} \boldsymbol{Hacker} \right) \underset{\boldsymbol{L_2}}{\bowtie} \boldsymbol{Judges}$$

Derivation Graph - $n = 4$



Utilization and Refund Probability

The network is *productive* when new blocks are created, i.e., when the counter is not in a waiting state. If $\mathcal{U} = \{User, Victim\}$ and $B_i^* = B_i \underset{L_1}{\bowtie} (* \underset{L_3}{\bowtie} Hacker) \underset{L_2}{\bowtie} Judges$

$$\boldsymbol{U_n} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \sum_{*\in\mathcal{U}} \pi_{\boldsymbol{B_i^*}}$$

The *refund probability* is the probability of being in a state where the user is granted a refund. If $B_i^G = B_i \underset{L_1}{\bowtie} (Granted \underset{L_3}{\bowtie} Hacker) \underset{L_2}{\bowtie} Judges$

$$\boldsymbol{R_n} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \pi_{\boldsymbol{B_i^G}}$$

Refund Cost

Accepting a refund request has a cost, which is directly proportional to the length of the time window between the fraudolent transaction and the refund request:

$$C_n = \frac{\alpha \sum_{i=1}^{n} i^e \pi_{B_i^G}}{\sum_{i=1}^{n} \pi_{B_i^G}}$$

One possible choice for α is the average number of transactions per block - this value can be estimated from empirical blockchain data or set according to the specific scenario being modeled.

Time Window Optimization

Optimal scenario:

- *maximum* utilization: blockain is as productvie as possible
- *maximum* refund probability: fairness condition in which fraudolent transaction get *reversed*
- *minimal* refund cost: minimal disruption to the blockain

We define the *Blockchain Efficiency-Fairness Index (BEFI)* as

$$\boldsymbol{BEFI_n = \frac{U_n R_n}{C_n}}$$

U_n and R_n have *opposite* trends $\rightarrow$ $BEFI_n$ captures the **trade-off** between these two indices and the cost of the refund mechanism.

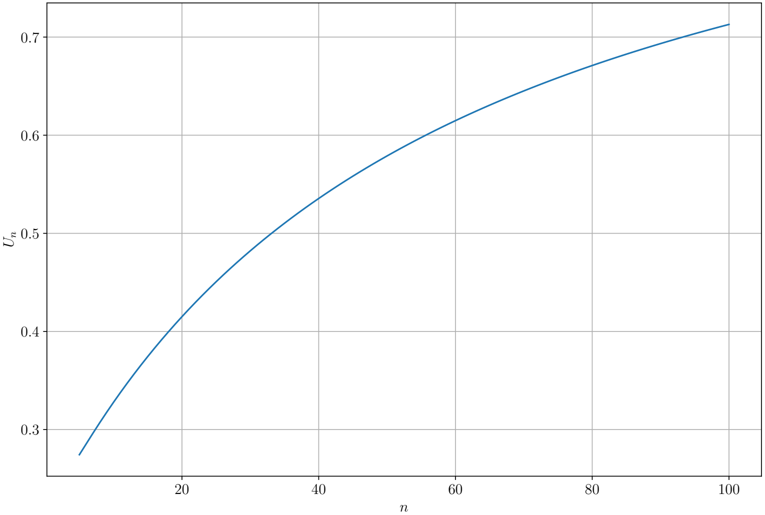
Experimental Setting

Scenario: introduce refund mechanism in Ethereum blockchain

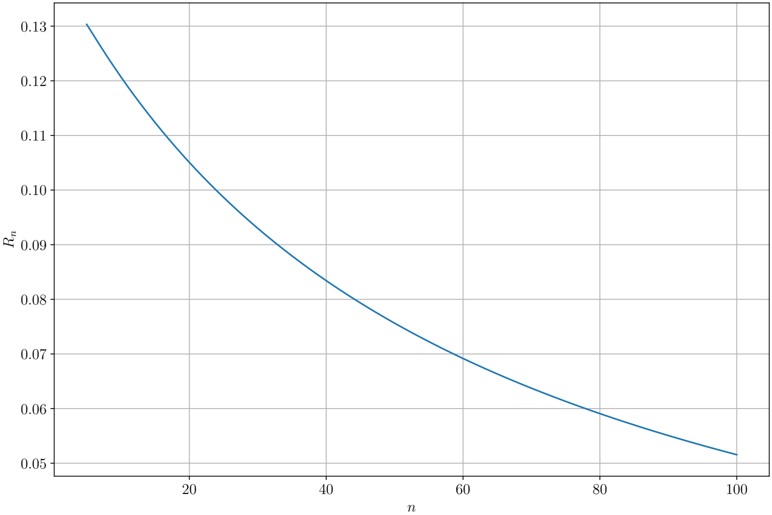
Parameters:

- block production rate $\gamma = \frac{1}{12s}$
- refund request rate $r \geq \gamma$
- request assessment rate (can lead to refund, punishment, dismissal with no processing) $s = \gamma$
- granted refund rate $\sigma = \gamma$ (need to write a block where the transaction is reversed)
- system reset rate $\beta = m\gamma$ for $1 < m < n$ (need to re-create all blocks after the fraudolent transaction)

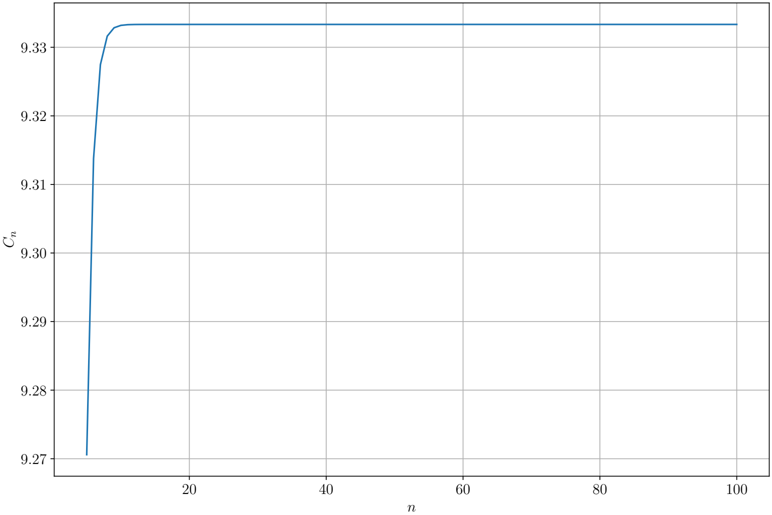
Utilization



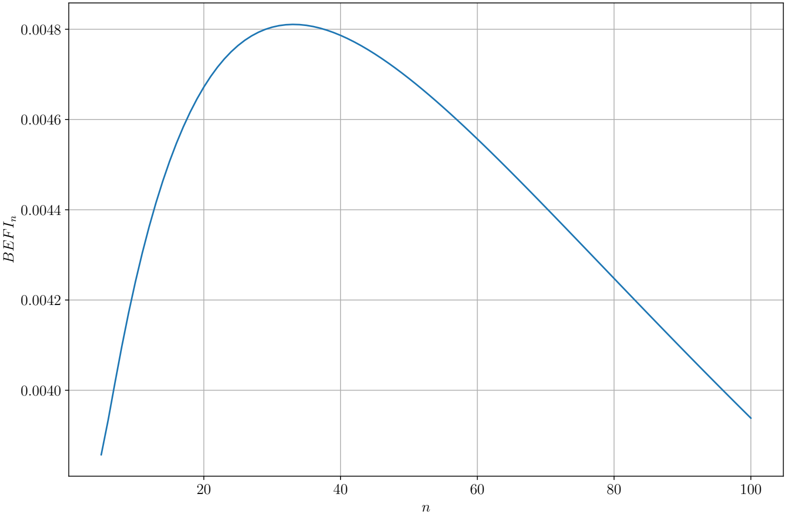
Refund Probability



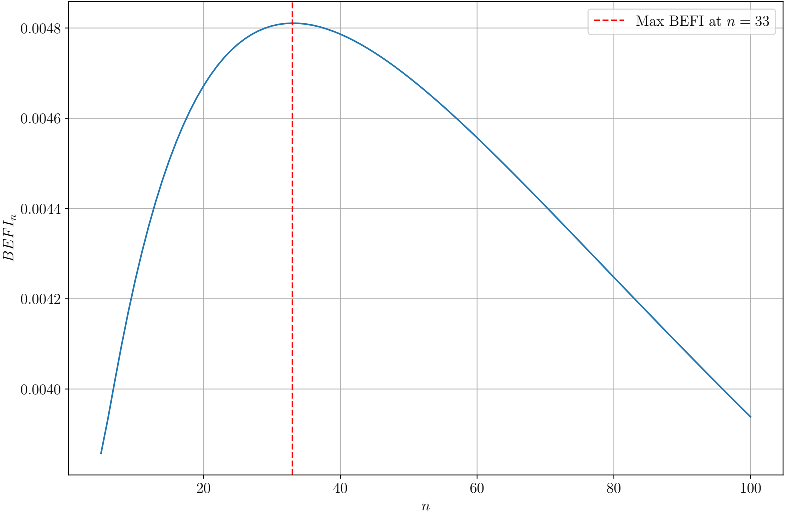
Total Refund Cost



BEFI



Trade-off Analysis



Summary

Assumptions:

- Simplified user behavior (e.g., single user, single hacker)
- Possibility to establish a pool of *Judges*.

Strengths:

- Agnostic model that can be applied to different blockchain scenarios (potentially even private blockchain)
- Model takes into account both productivity of the chain and the push for fairness and fraud remedy
- We acknlowledge the possibility that a refund request may come in too late (hacker has already spent all/part of the hacked sum)

Future Work

- Investigate protocol applicability in concrete scenarios, such as private blockchains
- Analyse empirical data to understand transaction dependence and estimate spending speed
- Extend the model to capture fraudolent spending speed more precisely