An Entropy-Based Approach to Evaluating the Economic Efficiency of Cryptocurrencies

Noninterference and Reversibility Analysis in Private Blockchains (NiRvAna) – Final Meeting
May 29-31, 2025, Urbino (PU), Italy

1506 UNIVERSITÀ DEGLI STUDI DI URBINO CARLO BO

Vincenzo Di Perna
(PhD Student – UniUrb)
vincenzo.diperna@unicam.it

1506 UNIVERSITÀ DEGLI STUDI DI URBINO CARLO BO

Michele Foderaro
(PhD Student – UniUrb)
michele.foderaro@unicam.it

Francesco Fabris
(Professor - UniTs)
ffabris@units.it

1506 UNIVERSITÀ DEGLI STUDI DI URBINO CARLO BO

Marco Bernardo
(Professor - UniUrb)
marco.bernardo@uniurb.it

With the support of the PRIN NiRvAna project – Bando 2020 Prot. 20202FCJMH

CONTEXT

Blockchain as a Disruptive Economic Technology

- **Blockchain** enables secure, transparent, and immutable transactions between untrusted nodes without central authorities, acting **as a distributed ledger**.

- **Cryptocurrencies** like Bitcoin (BTC) and Ethereum (ETH) and others build decentralized economies on top of blockchain networks.

- As these systems evolve, assessing their **economic efficiency** — beyond technical metrics such as TPS or energy consumption — becomes increasingly important.

PROBLEMS

Lack of a Conceptual and Methodologically Coherent Tool

- **No unified index** or widely accepted definition exists for economic efficiency in crypto ecosystems.
- **Traditional global metrics** (e.g., GDP, Gini coefficient, Pareto efficiency) do not translate well to decentralized systems and mesaure only one aspect (e.g. wealth imbalance or others).
- Aspects like **supply concentration, user engagement, and asset dormancy** are often overlooked.
- There's a **lack of a solid theoretical framework** connecting on-chain data to economic behavior.

OUR CONTRIBUTION

An Entropy-Based Economic Efficiency Index

- An **axiomatic and theoretically robust framework** that withstands critical scrutiny.
- It measures the **global systemic balance of economic qualities**: a high entropy score reflects a more evenly distributed and functionally engaged qualities of the network.
- **Grounded in information theory**: Shannon entropy — widely applied across disciplines — uniquely captures systemic balance through its **branching and weighting properties**.
- Allows **flexibility in attributes and weights selection** (via Beliș–Guiașu weighted entropy).
- Aggregates **several** heterogeneous on-chain economic indicators into a **single efficiency score**.

THEORETICAL FRAMEWORK #1

Why Entropy?

- **A single index** summarizing balance across multiple economic dimensions.
- Entropy, initially developed as an information measure, can be used to **reward well and harmoniously developed blockchains** with a balanced configuration of economic parameters:

$$\mathcal{H}(P) = -\sum_{i=1}^{k} p_i \log_2 p_i$$

- Entropy, even when interpreted as a measure of good economic balance and efficiency of a cryptocurrency, **is consistent with the original solid theoretical and axiomatic framework**.

THEORETICAL FRAMEWORK #2

Why Entropy?

- Entropy is the unique information measure that satisfies the **branching property axiom**

$$\mathcal{H}_k(p_1, p_2, \ldots, p_k) = \mathcal{H}_{k-1}(p_1 + p_2, p_3, \ldots, p_k) + (p_1 + p_2)\mathcal{H}_2\left(\frac{p_1}{p_1 + p_2}, \frac{p_2}{p_1 + p_2}\right)$$

- Entropy is the unique information measure **whose components can be weighted** without losing its axiomatic properties

$$H(P; \mathcal{W}) = -\sum_{i=1}^{k} w_i p_i \log_2 p_i$$

STEPS #1

The Approach

- **Interpret the phenomenon** of the economic efficiency.
- **Select and model** the data allowing the generation of the dataset to be processed.
- **Process and refine** the data by using normalization as well as outlier removal.
- **Compute the entropy** of the crypto assets.

STEPS #2

From Normalization to Entropy-Based Efficiency

- **Step 1 – Normalization**

  Given a set of values q_i, we normalize each to obtain $r_i \in [0,1]$:

$$R = \{r_1, r_2, \ldots, r_k\}$$

- **Step 2 – Define Inefficiency**

  To represent inefficiency, we introduce a function $\mathcal{I}(r_i)$ such that:

$$\mathcal{I}(1) = 0 \quad , \quad \mathcal{I}(r_i \to 0) \to +\infty$$

We choose the analytic function:

$$\mathcal{I}(r_i) = -\log_2 r_i$$

STEPS #3

From Normalization to Entropy-Based Efficiency

- **Step 3 – Probability Distribution**

  We derive a probability distribution P from the normalized values:

$$p_i = \frac{r_i}{\sum_{j=1}^{k} r_j} \quad \Rightarrow \quad P = \{p_1, \dots, p_k\}$$

- **Step 4 – Shannon Entropy**

  We compute the entropy of the distribution:

$$\mathcal{H}(P) = -\sum_{i=1}^{k} p_i \log_2 p_i \quad \in [0, \log_2 k]$$

Normalizing with base k:

$$\mathcal{H}(P) \in [0, 1]$$

PHENOMENON DEFINITION #1

What Are We Measuring?

- Economic efficiency in crypto is not just about value or speed. It includes **user engagement**, **wealth distribution**, **assets activity** and possibly others.
- Build an index that reflects the health and balance of a crypto economy — not just its size.
- **Primitive economic qualities** (**PQ**): Direct on-chain metrics (e.g., number of transfers, active addresses,...).
- **Derived economic qualities** (**DQ**): Combinations of primitives (e.g., participation rate, turnover ratio,...).

PHENOMENON DEFINITION #2

An Example of Possible Primitive Qualities

Economic Attribute	Acronym	Definition	Formula	Unit	Interval
PQ1 - Transferred Value	T_{value}	The total Ntv value exchanged within the system over a given period, excluding issuance account transfers, which record asset creation (*e.g.*, Bitcoin coinbase transactions).	$T_{\text{value}} = \sum_{t \in T} \text{amount}(t)$	Ntv.	$[0, +\infty)$
PQ2 - Transfer Count	T_{count}	The total number of transfers t executed within the network during a given period.	$T_{\text{count}} = \|T\|$	Count	$[0, +\infty)$
PQ3 - Number of Funded Addresses	NA_{funded}	The number of addresses continuosly holding at least 1 USD during a given period.	$NA_{\text{funded}} = \|A_{\text{funded}}\|$	Count	$[0, +\infty)$
PQ4 - Number of Active Addresses Sent	NA_{active}	The total number of unique addresses, counted only once, that are involved in sending transfers over a given period.	$NA_{\text{active}} = \|A_{\text{active}}\|$	Count	$[0, +\infty)$
PQ5 - Active Supply	S_{active}	The amount of coins/tokens that have been moved at least once within a given time period, excluding double counting of the same units being recycled.		Ntv.	$[0, +\infty)$
PQ6 - Daily Digital Asset to USD Price Rate	USD_{price}	The price in USD per native unit of the coin or token at the close of the day.		USD	$(0, +\infty)$
PQ7 - Market Capitalization	Cap	The total value of a cryptocurrency in USD, calculated by multiplying its current price by the total circulating supply of coins.	$Cap = S \cdot USD_{\text{price}}$	USD	$(0, +\infty)$

PHENOMENON DEFINITION #3

An Example of Possible Derived Qualities

DQ1 - Participation	$A_{\text{participation}}$	The proportion of active addresses relative to the total addresses holding at least 1 USD. A higher ratio suggests a more engaged user base and a healthy level of participation within the cryptocurrency ecosystem.	$A_{\text{participation}} = NA_{\text{active}} / NA_{\text{funded}}$	Count	$[0,1]$
DQ2 - Mean Transfer Size	MTS	The mean size of a transfer, measured in USD. It is calculated by dividing the total value transferred by the number of transfers between distinct addresses during a given period.	$MTS = T_{\text{value}} / T_{\text{count}}$	Ntv.	$[0,+\infty)$
DQ3 - Mean Transfers per Active Address	MTA_{active}	The mean number of transfers per active address. It is calculated by dividing the total number of transfers by the number of active addresses during a given period. This metric provides insights into the intensity of usage per user.	$MTA_{\text{active}} = T_{\text{count}} / NA_{\text{active}}$	Count	$[0,+\infty)$
DQ4 - Active Supply Turnover Rate	TR	The ratio between the total value transferred and the active supply over a given period. A higher TR indicates more frequent economic activity and greater liquidity.	$TR = T_{\text{value}} / S_{\text{active}}$	Count	$[0,+\infty)$
DQ5 - Active Supply Ratio	ASR	The proportion of the current supply that is actively participating in transactions. A higher ASR signifies that a larger portion of the available cryptocurrency is being used rather than held passively.	$ASR = S_{\text{active}} / S_{\text{current}}$	Count	$[0,1]$
DQ6 - Wealth Distribution	WD	The degree to which wealth is distributed across the network's participants.	$WD = \frac{\sum_{k \in I} \sum_{m \in I} NA_{\text{funded}}(k) \cdot NA_{\text{funded}}(m) \lvert \overline{balance}(k) - \overline{balance}(m) \rvert}{2 \cdot NA_{\text{funded}} \cdot \sum_{j \in I} S_{\text{current}}(j)}$	Count	$[0,1]$
DQ7 - Mean Transfer per Market Cap	$MTMC$	The mean size of a transfer over a specific period relative to the cryptocurrency's market capitalization.	$MTMC = (T_{\text{value}} \cdot USD_{\text{price}}) / Cap$	USD	$[0,+\infty)$

DATA MODELING

Criteria for On-Chain Metric Selection

- Sourced from Coin Metrics for reliable, high-quality on-chain data, and mapped to the primitive and derived crypto-economic qualities.

PQ	Coin Metrics Parameter	Description	Unit
PQ1	Xfer'd Val	The sum of Ntv. transferred (*i.e.*, the aggregate "size" of all transfers) that interval.	Ntv.
PQ2	Xfer Cnt	The sum count of transfers that day, including all user-initiated actions recorded on the chain, excluding protocol-mandated changes like coinbase transactions or new issuance.	Count
PQ3	Addresses Count with Balance $\geq$ 1 USD	The total count of unique addresses holding at least 1 USD by the end of the day.	Count
PQ4	Active Addresses (Sent)	The total count of unique sending addresses active in the network that day, excluding duplicates from previous activity.	Count

PQ5	1-Day Active Supply	The sum of unique native units that transacted at least once in the trailing X time to that interval. Native units that transacted more than once are only counted once.	Ntv.
PQ6	USD Denominated Closing Price	The price of the asset denominated in U.S. Dollars.	USD
PQ7	Market Cap	The sum USD value of the current supply. Also referred to as network value or market capitalization.	USD
-	Current Supply	The sum of all native units ever created and currently visible on the ledger (*i.e.*, issued) as of that day.	Ntv.
-	Val in Addrs w/ Bal $\geq$ X Ntv	The total of native units held in addresses with a balance of X Ntv or more at day's end, excluding non-native tokens.	Ntv.

DATA PROCESSING

- **Min–max normalization**: all metrics rescaled to [0,1] for comparability of each asset across its full lifetime.

- **Native unit values** (e.g., BTC, ETH) **converted to USD** for uniformity.

- Wealth Distribution (DQ6) inverted
  → high concentration = low efficiency.

- Temporal alignment across all series using the earliest common timestamp.

EVALUATIONS

- Apply the entropy-based index to six major cryptocurrencies: **Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), USD Coin (USDC), Dogecoin (DOGE), and Cardano (ADA)**.

- Evaluation using **two sets** of derived quality attributes:

  - **Set1**: Basic financial activity indicators (e.g., market cap, participation).
  - **Set2**: Structural and behavioral indicators (e.g., wealth distribution, turnover, transfer size).

- On-chain data sourced from **Coin Metrics**, using standardized normalization and conversion processes.

CRYPTOCURRENCIES

- ***Assets*** *with* ***high market cap***, **diverse economic functions**, and sufficient on-chain data availability on Coin Metrics:

- Bitcoin (BTC)

- Ethereum (ETH)

- Ripple (XRP)

- USD Coin (USDC)

- Dogecoin (DOGE)

- Cardano (ADA)

Asset	Category	Coin Metrics Capitalization	Max Supply	Avg. Network Size	Underlying Blockchain	Consensus Mechanism
Bitcoin (BTC) [2]	Payment	$1,653.3 billion	21 million BTC	21.886 [69]	Bitcoin	PoW
Ethereum (ETH) [3]	Smart Contract	$217.6 billion	No fixed supply	13.347 [70]	Ethereum	PoS
Ripple (XRP) [4]	Payment	$214 billion	100 billion XRP	150+ [71]	XRP Ledger	Consensus Protocol
USD Coin (USDC) [26]	Stablecoin	$40 billion	No fixed supply	Depends on the blockchain	Operates on multiple blockchains	Depends on the blockchain
Dogecoin (DOGE) [27]	Memecoin/Payment	$25.1 billion	No fixed supply	315 [72]	Dogecoin	PoW
Cardano (ADA) [28]	Payment/Smart Contract	$23.3 billion	45 billion ADA	N.A.	Cardano	PoS

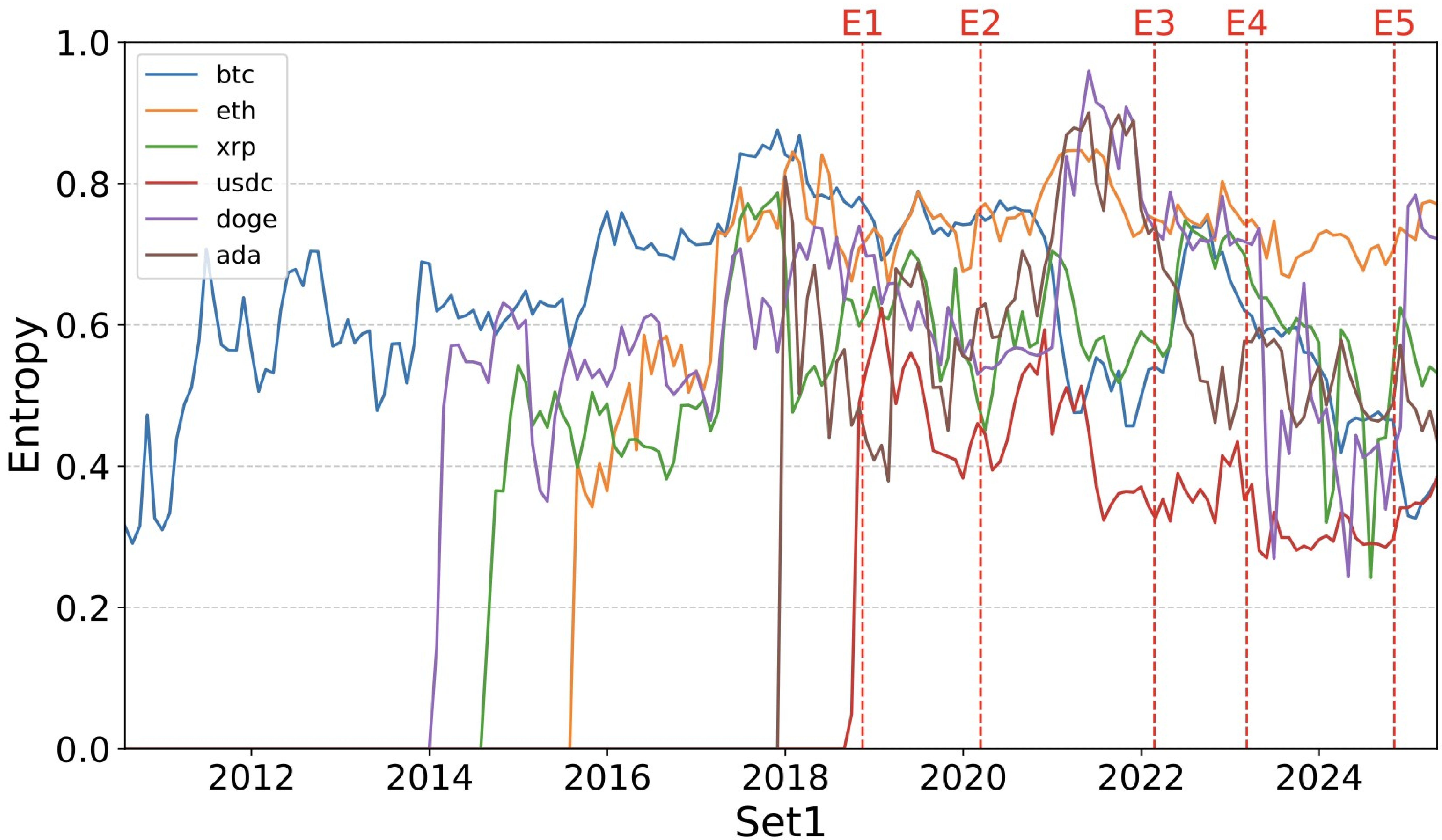
SETS

- Compare systemic balance under different economic lenses.

Set	Qualities	Description
Set1		It combines user activity metrics with market financial indicators.
	DQ1	Participation ($A_{\text{participation}}$)
	DQ3	Mean Transfers per Active Address (MTA_{active})
	DQ7	Mean Transfer per Market Cap ($MTMC$)
	PQ7	Market Capitalization (Cap)
Set2		It comprises six dynamic qualities, with a focus on user engagement, transaction characteristics, and the distribution of wealth within the network.
	DQ1	Participation ($A_{\text{participation}}$)
	DQ2	Mean Transfer Size (MTS)
	DQ3	Mean Transfers per Active Address (MTA_{active})
	DQ4	Active Supply Turnover Rate (TR)
	DQ5	Active Supply Ratio (ASR)
	DQ6	Wealth Distribution (WD)

ILLUSTRATIVE EXAMPLE #1

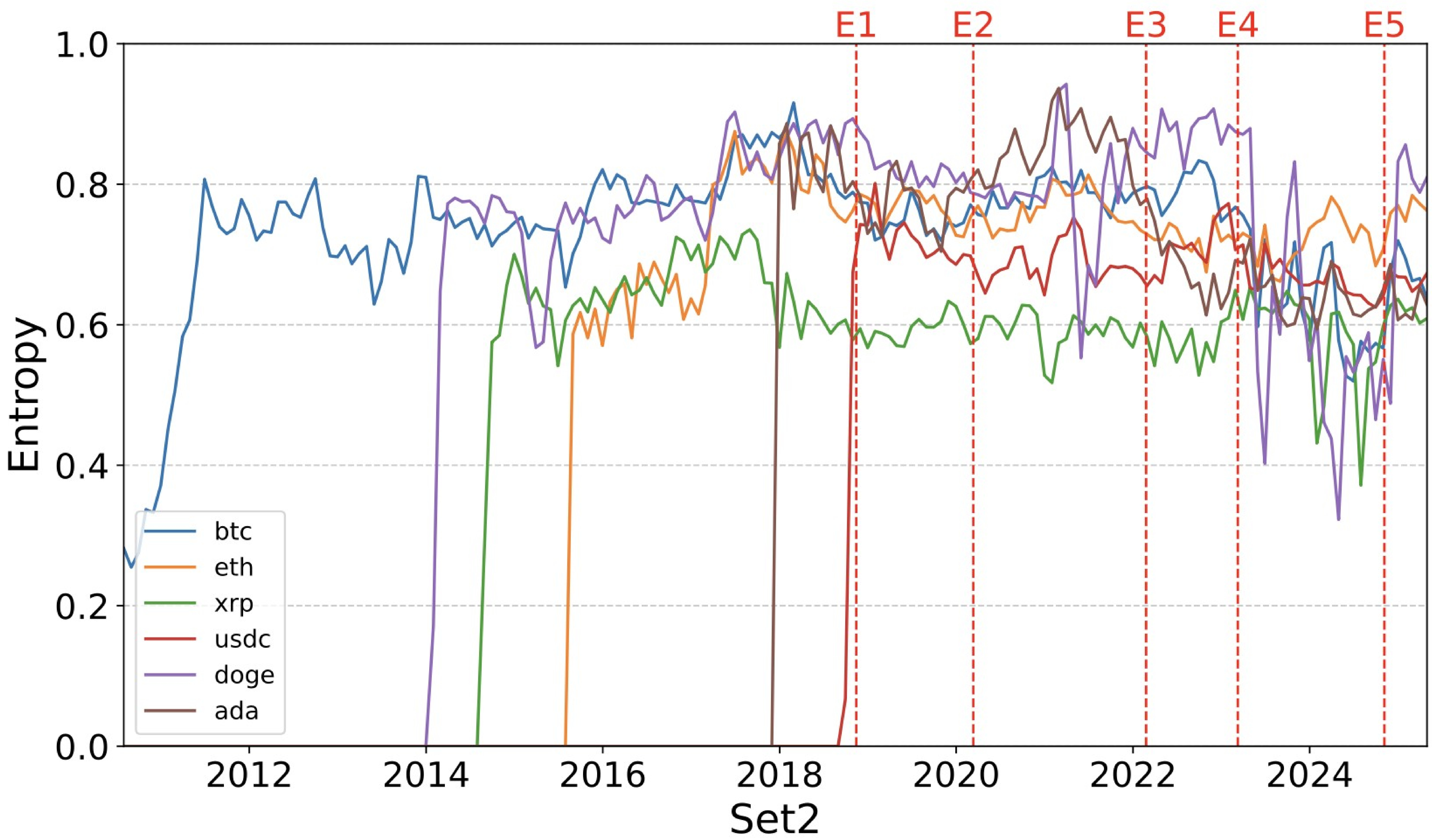
Entropy evolution over time: Set 1



(a) Entropy variations for ***Set1***

ILLUSTRATIVE EXAMPLE #2

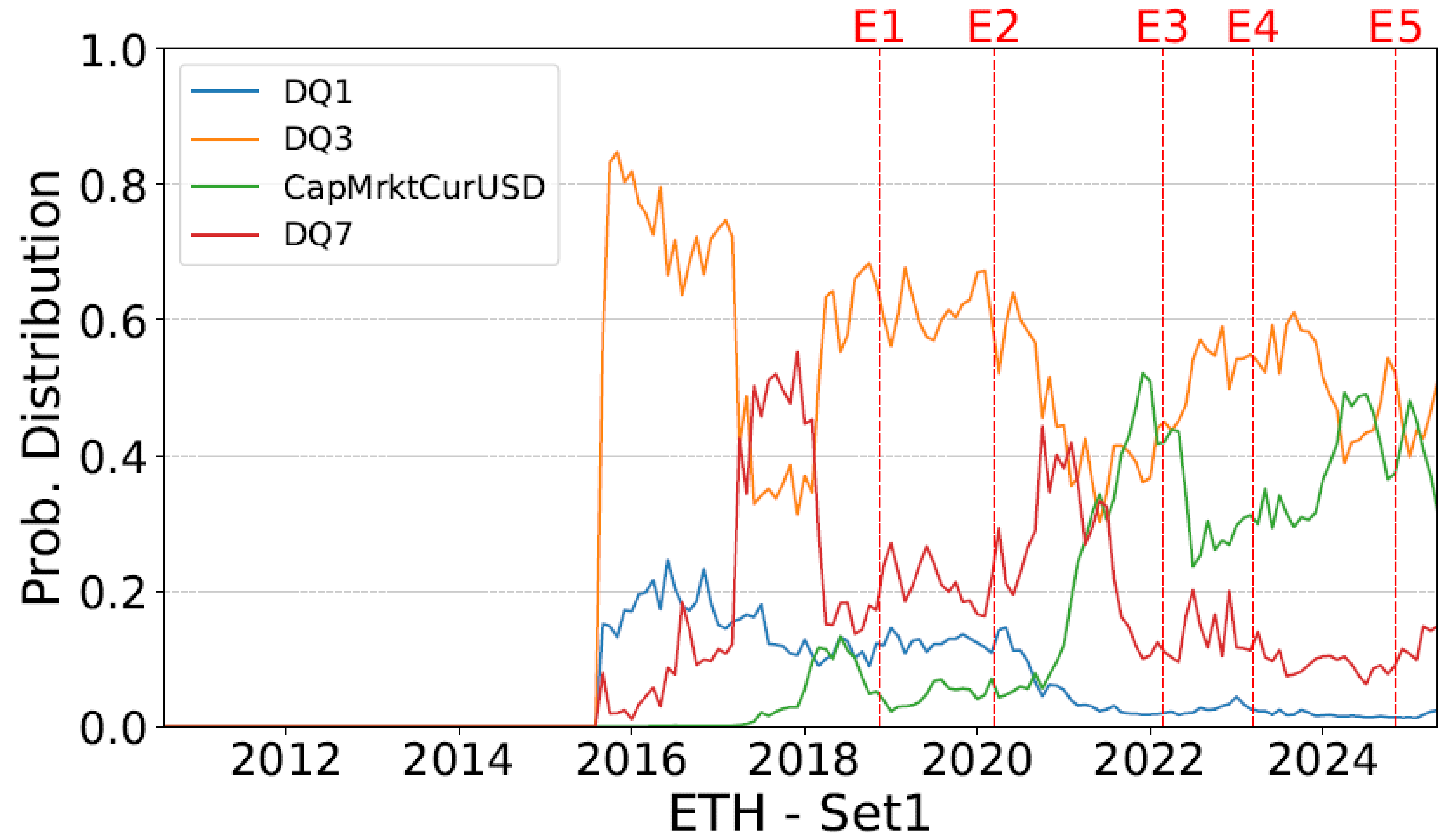
Entropy evolution over time : Set 2

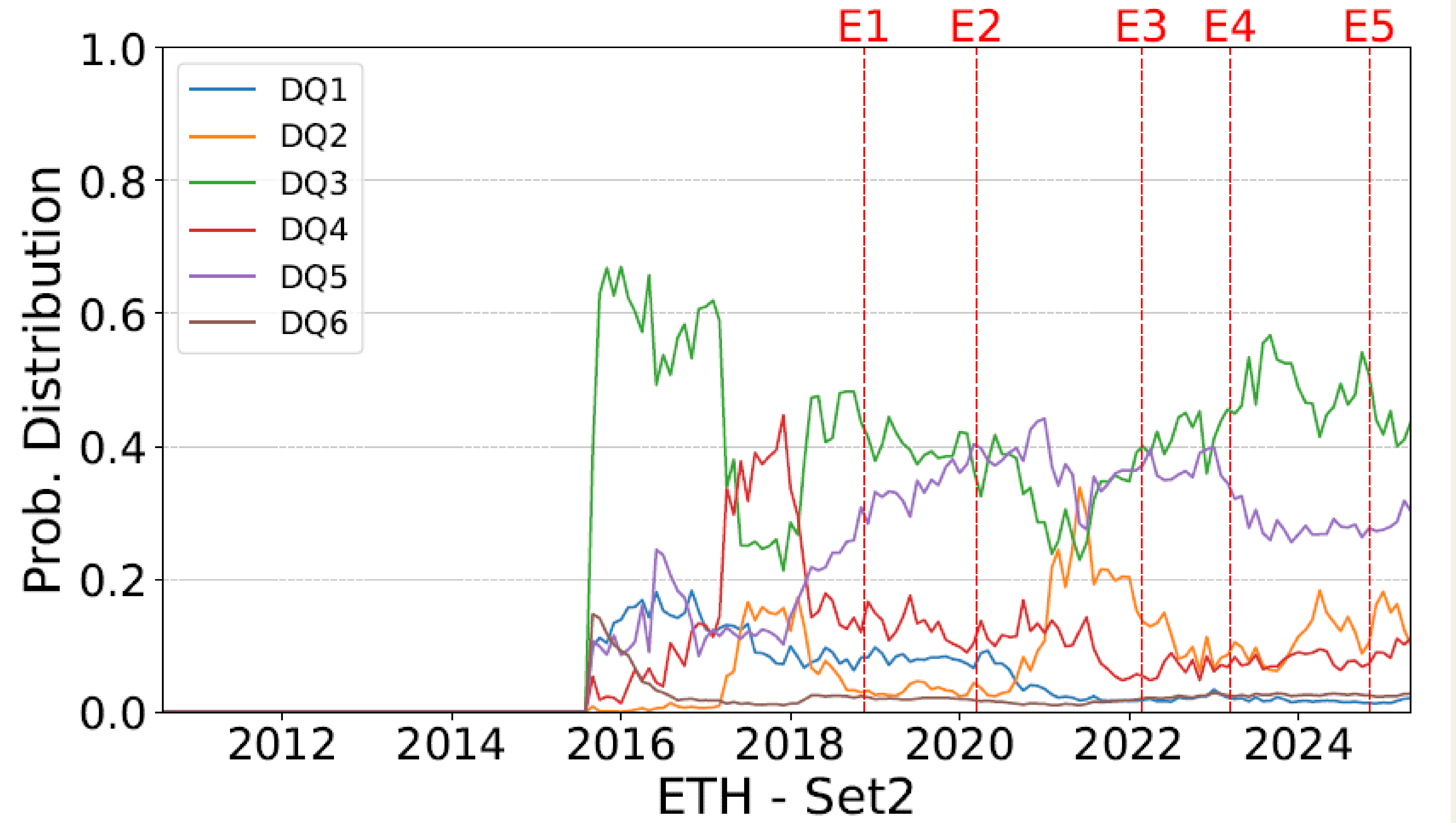


(b) Entropy variations for ***Set2***

ILLUSTRATIVE EXAMPLES: PROBABILITIES

Probability distributions of parameters for ETH





DQ1	Participation ($A_{\text{participation}}$)
DQ3	Mean Transfers per Active Address (MTA_{active})
DQ7	Mean Transfer per Market Cap ($MTMC$)
PQ7	Market Capitalization (Cap)

DQ1	Participation ($A_{\text{participation}}$)
DQ2	Mean Transfer Size (MTS)
DQ3	Mean Transfers per Active Address (MTA_{active})
DQ4	Active Supply Turnover Rate (TR)
DQ5	Active Supply Ratio (ASR)
DQ6	Wealth Distribution (WD)

ILLUSTRATIVE EXAMPLES: FINDINGS

- **Shannon entropy is suitable to measure the good global balance of economic parameters**:

  high entropy signals a good balance across all the qualities describing economic efficiency; low entropy indicates the dominance of some parameters and/or stagnation in others.

- **Qualitative behavior of some cryptocurrencies:**

- Roughly speaking,

  Ethereum seems to be the most balanced asset across both attribute sets, showing limited volatility in its entropy over time.

  XRP seems to exhibit low engagement and poor wealth distribution, while DOGE displays high inconsistency across metrics.

ILLUSTRATIVE EXAMPLES: FINDINGS

- Assets behave differently depending on the attribute set (e.g., XRP appears efficient in Set1 but not in Set2), highlighting the **importance of attribute selection** in interpreting economic efficiency.

- Our goal **is limited to offering a solid theoretical framework** based on Shannon entropy, ***not to assessing a (final) ranking of cryptocurrencies***.

CONCLUSIONS

- The proposed entropy-based framework provides a robust, flexible, and **theoretically sound approach** to assess cryptoassets efficiency.
- Enables **subjective analysis** of crypto-economic efficiency by tracking custom parameter distributions over time.
- **Can use the Beliș–Guiașu weighted entropy** to reflect real-world relevance (e.g. weighting entropy with the absolute value r_i of parameters, prioritizing user engagement over market cap,…).

$$H(P;\mathcal{W}) = -\sum_{i=1}^{k} w_i p_i \log_2 p_i$$

FUTURE WORKS

- Select the best set of on-chain metrics and derived qualities so as to obtain a **widely accepted definition** of economic efficiency for a cryptocurrency ecosystem.
- Specialize the entropy-based economic efficiency index to the **several case-uses for cryptocurrencies**, such as digital payments, store of value, decentralized finance, smart-contracts, etc..., by selecting different sets of parameters.
- **Use the Beliș–Guiașu weighted entropy** with $w_i = r_i$, the absolute value of the normalized parameters, so as to jointly reward not only the well-balanced economic structure of the cryptocurrency, but also its absolute value.

Thanks for your attention

References

C. E. Shannon, A mathematical theory of communication, The Bell System Technical Journal 27 (1948) 379–423. doi:10.1002/j.1538-7305.1948.tb01338.x.

Michele Foderaro
(PhD Student – UniUrb)

michele.foderaro@unicam.it