Alternative Characterizations of Hereditary History-Preserving Bisimilarity via Backward Ready Multisets

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Truly Concurrent Bisimilarities

- Event structures (Winskel 1980): truly concurrent models based on the notions of *causality*, *conflict*, and *concurrency* among events.
- Configuration structures (Van Glabbeek & Plotkin 1995): finite sets of non-conflicting events that are downward-closed with respect to causality + transitions among them (akin to LTSs).

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- Truly concurrent bisimilarities: *step* bisimilarity, *pomset* bisimilarity, *causal* bisimilarity, *location* bisimilarity, *distributed* bisimilarity, ...
- History-preserving bisimilarity (Rabinovich & Trakhtenbrot 1988): set of *triples* instead of pairs of states, where the third component is a bijection between the events occurred so far in the two structures that preserves *labeling* and *causality* among these events.
- Hereditary history-preserving bisimilarity (Bednarczyk 1991): forward direction and *backward* direction in the bisimulation game.

- HPB and HHPB are extremely discriminating, but respectively represent the coarsest and the finest behavioral equivalences:
  - Capable of respecting causality, branching, and their interplay while abstracting from choices between identical alternatives.
  - Preserved under action refinement (substituting processes for actions).
- Logical characterizations have been provided for both equivalences (Phillips & Ulidowski 2014, Baldan & Crafa 2014).

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- Causal bisimilarity is a set of pairs, not of triples.
- HHPB has an axiomatization over forward-only processes (Fröschle & Lasota 2005).

How To Characterize HHPB?

- The *first* alternative characterization (Bednarczyk 1991) is given by back-and-forth bisimilarity, which is binary and hence does not stepwise build any labeling- and causality-preserving bijection.
- Not to be confused (De Nicola & Montanari & Vaandrager 1990), which coincides with interleaving bisimilarity (backtracking).

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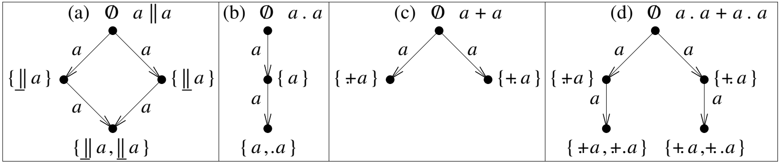
- Not to be confused (De Nicola & Montanari & Vaandrager 1990), which coincides with interleaving bisimilarity (backtracking).
- The characterization result holds in the absence of autoconcurrency.
- Configuration structures (a) and (b) are back-and-forth bisimilar but not HHPB:



- The *second* alternative characterization (Phillips & Ulidowski 2007) relies on a generalization of forward-reverse bisimilarity (CCSK), which is binary, to configuration graphs of prime event structures.
- Forward transition relation and backward transition relation.
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- The characterization result holds in the absence of repeated, identically labeled events along forward computations, which implies the absence of autoconcurrency (and autocausation).
- The absence of equidepth autoconcurrency turns out to be enough on stable configuration structures (Phillips & Ulidowski 2012).
- Absence of identically labeled events occurring at the same depth within a configuration.
- The depth of an event is defined as the length of the longest causal chain of events up to and including the considered event.

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- Subject to no restrictions, but a third component comes into play.
- The idea is to import HHPB in RCCS by encoding memories, i.e., the past behavior, as identified configuration structures.
- Stable configuration structures enriched with unique event identifiers, used in transition labels and exploited when undoing synchronizations.
- The characterizing equivalence, called back-and-forth bisimilarity and defined over RCCS processes, relies on ternary bisimulation relations.
- The third component is a bijection from the set of identifiers of the actions executed so far in the first process to the set of identifiers of the actions executed so far in the second process.

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- CCSK transition labels such as $a[i]$ and $a[j]$ are deemed to be different if the two keys i and j are different.
- Thus $a \parallel a$ and $a \,.\, a$ are told apart by FRB in CCSK because $a \parallel a$ evolves to $a[i] \parallel a[j]$, which can undo $a[i]$ and $a[j]$ in any order, while $a \,.\, a$ evolves to $a[i] \,.\, a[j]$, from which only $a[j]$ can be undone, hence undoing $a[i]$ cannot be matched by undoing $a[j]$.

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- Identified RCCS transition labels like $i{:}a$ and $j{:}a$ are viewed as compatible even if i and j are different.
- But $a \parallel a$ and $a \,.\, a$ are distinguished by back-and-forth bisimilarity because, although undoing $i{:}a$ can be matched by undoing $j{:}a$, it is not possible to establish a bijection from a distributed memory containing $i{:}a$ in a location and $j{:}a$ in another location ($a \parallel a$) to a centralized memory containing $j{:}a$ on top of $i{:}a$ ($a \,.\, a$).

Other Ways of Characterizing HHPB via FRB?

- To what extent do we need a systematic identification of events?
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- Multiset of actions that label incoming transitions.

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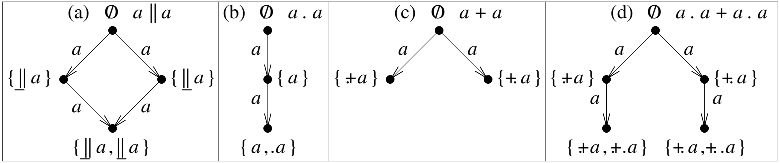
- What is the minimal information to add to the FRB game?
- Backward ready multisets capture the degree of concurrency.
- Multiset of actions that label incoming transitions.
- Counting events instead of identifying them.
- Configuration structures (a) and (b) are neither HHPB nor FRB:brm:



- A labeled configuration structure is a tuple $C = (\mathcal{E}, \mathcal{C}, \mathcal{A}, l)$ where:
  - $\mathcal{E}$ is a set of events.
  - $\mathcal{C} \subseteq \mathscr{P}_{\mathrm{fin}}(\mathcal{E})$ is a set of configurations.
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- A configuration structure C is stable iff it is:
  - Rooted: $\emptyset \in \mathcal{C}$.
  - Connected: $\forall X \in \mathcal{C} \setminus \{\emptyset\}. \exists e \in X. X \setminus \{e\} \in \mathcal{C}$.
  - Closed under bounded unions and intersections: $\forall X, Y, Z \in \mathcal{C}. X \cup Y \subseteq Z \implies X \cup Y \in \mathcal{C} \wedge X \cap Y \in \mathcal{C}$.

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- The causality relation over $X \in \mathcal{C}$ is defined by $e_1 \leq_X e_2$ iff $e_2 \in Y \implies e_1 \in Y$ for all configurations $Y \subseteq X$.
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- The concurrency relation over X is $co_X = (X \times X) \setminus (\leq_X \cup \geq_X)$.
- $X \xrightarrow{a}_C X'$ for $X, X' \in \mathcal{C}$ iff $X \subseteq X' \wedge X' \setminus X = \{e\} \wedge \ell(e) = a$.

- Two stable configuration structures $C_k = (\mathcal{E}_k, \mathcal{C}_k, \mathcal{A}, \ell_k)$, $k \in \{1, 2\}$, are hereditary history-preserving bisimilar, written $C_1 \sim_{\mathrm{HHPB}} C_2$, iff there exists a relation $\mathcal{B} \subseteq \mathcal{C}_1 \times \mathcal{C}_2 \times \mathscr{P}(\mathcal{E}_1 \times \mathcal{E}_2)$ such that:
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        - $\mathit{brm}(X_1) = \mathit{brm}(X_2)$.
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- $\mathit{brm}(X) = \{\!| a \in \mathcal{A} \mid X' \xrightarrow{a}_{C} X |\!\}$ for $X \in \mathcal{C}$.
- $\sim_{\mathrm{FRB:brm}}$ and $\sim_{\mathrm{HHPB}}$ coincide over stable configuration structures even in the presence of autoconcurrency, if all conflicts are local (i.e., in each maximal set of conflicting events, all events are caused by the same event).
- We also have an operational characterizazion based on a variant of PRPC – Proved Reversible Process Calculus (events given by proof terms).

Logical Characterizations

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- We focus on event identifier logic $\mathcal{L}_{\mathrm{EI}}$:

$$\phi \ ::= \ \mathsf{true} \mid \neg\phi \mid \phi \wedge \phi \mid \langle x : a \rangle\!\rangle \phi \mid (x : a)\phi \mid \langle\!\langle x \rangle \phi$$

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- $\langle x : a \rangle\!\rangle\, \_$ and $(x : a)\, \_$ act as binders for identifiers.
- For an *environment* $\rho : \mathcal{I} \rightharpoonup \mathcal{E}$ that maps identifiers to events, the semantics is:

$$\begin{array}{lllll} X & \models_\rho & \langle x : a \rangle\!\rangle \phi & \text{iff} & \exists X \xrightarrow{l(e)}_C X' \text{ s.t. } l(e) = a \text{ and } X' \models_{\rho[x \mapsto e]} \phi \\ X & \models_\rho & (x : a)\phi & \text{iff} & \exists e \in X \text{ s.t. } l(e) = a \text{ and } X \models_{\rho[x \mapsto e]} \phi \\ X & \models_\rho & \langle\!\langle x \rangle \phi & \text{iff} & \exists X' \xrightarrow{l(e)}_C X \text{ s.t. } \rho(x) = e \text{ and } X' \models_\rho \phi \end{array}$$

- Backward ready multiset logic $\mathcal{L}_{\mathrm{BRM}}$ characterizes $\sim_{\mathrm{FRB:brm}}$.
- It is interpreted over a variant of PRPC:

$$
\begin{aligned}
P &::= \underline{0} \mid a\,.\,P \mid a^{\dagger\xi}.\,P \mid P + P \mid P \parallel_L P \\
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- Process semantics is based on *proof terms*:

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- Syntax of formulas:

$$\phi \;::=\; \mathsf{true} \mid M \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi \mid \langle a^{\dagger} \rangle \phi$$

- Semantics of formulas:

$$\begin{array}{lcll} P & \models & M & \text{iff} \quad \mathit{brm}(P) = M \\ P & \models & \langle a \rangle \phi & \text{iff} \quad \exists P \xrightarrow{\theta} P' \text{ s.t. } \mathit{act}(\theta) = a \text{ and } P' \models \phi \\ P & \models & \langle a^{\dagger} \rangle \phi & \text{iff} \quad \exists P' \xrightarrow{\theta} P \text{ s.t. } \mathit{act}(\theta) = a \text{ and } P' \models \phi \end{array}$$

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- We can reinterpret $\mathcal{L}_{\mathrm{EI}}$ over PRPC (with ρ mapping identifiers to proof terms).
- Main challenge: how to treat $(x : a)_{-}$?

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- We can reinterpret $\mathcal{L}_{\mathrm{EI}}$ over PRPC (with ρ mapping identifiers to proof terms).
- Main challenge: how to treat $(x : a)_{\_}$?
- Reinterpretation:

$$
\begin{array}{lll}
P \models_\rho \mathsf{true} & & \\
P \models_\rho \neg\phi & \text{iff} & P \not\models_\rho \phi \\
P \models_\rho \phi_1 \wedge \phi_2 & \text{iff} & P \models_\rho \phi_1 \text{ and } P \models_\rho \phi_2 \\
P \models_\rho \langle x : a\rangle\rangle\phi & \text{iff} & \exists P \xrightarrow{\theta} P' \text{ s.t. } \mathit{act}(\theta) = a \text{ and } P' \models_{\rho[x \mapsto \theta]} \phi \\
P \models_\rho (x : a)\phi & \text{iff} & \exists a^\dagger.\, P' \in \mathit{sp}(P) \text{ s.t. } P \models_{\rho[x \mapsto \mathit{apt}(a^\dagger.\, P', P)]} \phi \\
P \models_\rho \langle\langle x\rangle\phi & \text{iff} & \exists P' \xrightarrow{\theta} P \text{ s.t. } \rho(x) = \theta \text{ and } P' \models_\rho \phi
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- Two processes are $\sim_{\mathrm{FRB:brm}}$-equivalent iff they satisfy the same formulas of $\mathcal{L}_{\mathrm{EI}}$ (for all permissible environments ρ).

- We can reinterpret $\mathcal{L}_{\mathrm{BRM}}$ over stable configuration structures.
- $[\![P]\!]$ is the stable configuration structure associated with process P, whose events are formalized as proof terms.
- Reinterpretation:

$$
\begin{array}{lcll}
[\![P]\!] & \models & \mathsf{true} & \\
[\![P]\!] & \models & M & \text{iff } \{\!| a \in \mathcal{A} \mid X_{P'} \xrightarrow{a}_{c_{P'}} X_P |\!\} = M \\
[\![P]\!] & \models & \neg\phi & \text{iff } [\![P]\!] \not\models \phi \\
[\![P]\!] & \models & \phi_1 \wedge \phi_2 & \text{iff } [\![P]\!] \models \phi_1 \text{ and } [\![P]\!] \models \phi_2 \\
[\![P]\!] & \models & \langle a \rangle \phi & \text{iff } \exists X_P \xrightarrow{a}_{c_P} X_{P'} \text{ s.t. } [\![P']\!] \models \phi \\
[\![P]\!] & \models & \langle a^{\dagger} \rangle \phi & \text{iff } \exists X_{P'} \xrightarrow{a}_{c_{P'}} X_P \text{ s.t. } [\![P']\!] \models \phi
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\end{array}
$$

- Two stable configuration structures are $\sim_{\mathrm{HHPB}}$-equivalent iff they satisfy the same formulas of $\mathcal{L}_{\mathrm{BRM}}$.
- P and $[\![P]\!]$ satisfy the same formulas, of both $\mathcal{L}_{\mathrm{BRM}}$ and $\mathcal{L}_{\mathrm{EI}}$.

Translations

- We can translate $\mathcal{L}_{\mathrm{BRM}}$ into $\mathcal{L}_{\mathrm{EI}}$.
- Main challenge: how to translate backward ready multisets?
- Translation $\mathcal{T}_{\mathrm{BE}}$:

$$
\begin{array}{rcll}
\mathcal{T}_{\mathrm{BE}}(\mathsf{true}, A, \varrho_n) & = & \mathsf{true} & \\
\mathcal{T}_{\mathrm{BE}}(M, A, \varrho_n) & = & \bigwedge_{a_i \in \mathit{supp}(M)} \left(\bigwedge_{k=1}^{M(a_i)} \langle\!\langle x_{i,k} \rangle\!\rangle \mathsf{true} \wedge \bigwedge_{h=1}^{\sharp(a_i, \varrho_n) - M(a_i)} \neg \langle\!\langle z_{i,h} \rangle\!\rangle \mathsf{true} \right) & \\
 & & \wedge \bigwedge_{b \in A \setminus \mathit{supp}(M)} \neg (y : b) \langle\!\langle y \rangle\!\rangle \mathsf{true} & \text{with } y \text{ fresh} \\
\mathcal{T}_{\mathrm{BE}}(\neg\phi, A, \varrho_n) & = & \neg \mathcal{T}_{\mathrm{BE}}(\phi, A, \varrho_n) & \\
\mathcal{T}_{\mathrm{BE}}(\phi_1 \wedge \phi_2, A, \varrho_n) & = & \mathcal{T}_{\mathrm{BE}}(\phi_1, A, \varrho_n) \wedge \mathcal{T}_{\mathrm{BE}}(\phi_2, A, \varrho_n) & \\
\mathcal{T}_{\mathrm{BE}}(\langle a \rangle \phi, A, \varrho_n) & = & \langle x : a \rangle\!\rangle \mathcal{T}_{\mathrm{BE}}(\phi, A, \varrho_n \cup \{(n+1, (x, a))\}) & \text{with } x \text{ fresh} \\
\mathcal{T}_{\mathrm{BE}}(\langle a^{\dagger} \rangle \phi, A, \varrho_n) & = & (x : a) \langle\!\langle x \rangle \mathcal{T}_{\mathrm{BE}}(\phi, A, \varrho_n) & \text{with } x \text{ fresh}
\end{array}
$$

- $\mathcal{T}_{\mathrm{BE}}$ preserves satisfiability.

Future Work

- Translating $\mathcal{L}_{\mathrm{EI}}$ into $\mathcal{L}_{\mathrm{BRM}}$:
  - How to translate $(x : a)\phi$?
  - What role is played by backward ready multisets?
- Investigating the relationships between $\mathcal{L}_{\mathrm{BRM}}$ and the logical characterization of $\sim_{\mathrm{HHPB}}$ in (Baldan & Crafa 2014).
- Extension to non-stable configuration structures.
- Decidability and time complexity of $\sim_{\mathrm{FRB:brm}}$:
  - $\sim_{\mathrm{HHPB}}$ is undecidable over finite labeled transition systems extended with an independence relation over transitions (Jurdzinski & Nielsen & Srba 2003).
  - $\sim_{\mathrm{HHPB}}$ is decidable in polynomial time over basic parallel processes (Fröschle & Jančar & Lasota & Sawa 2010).