Aggregate Programming
Part 2: Resilient Programs

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Aggregate Programming Stack

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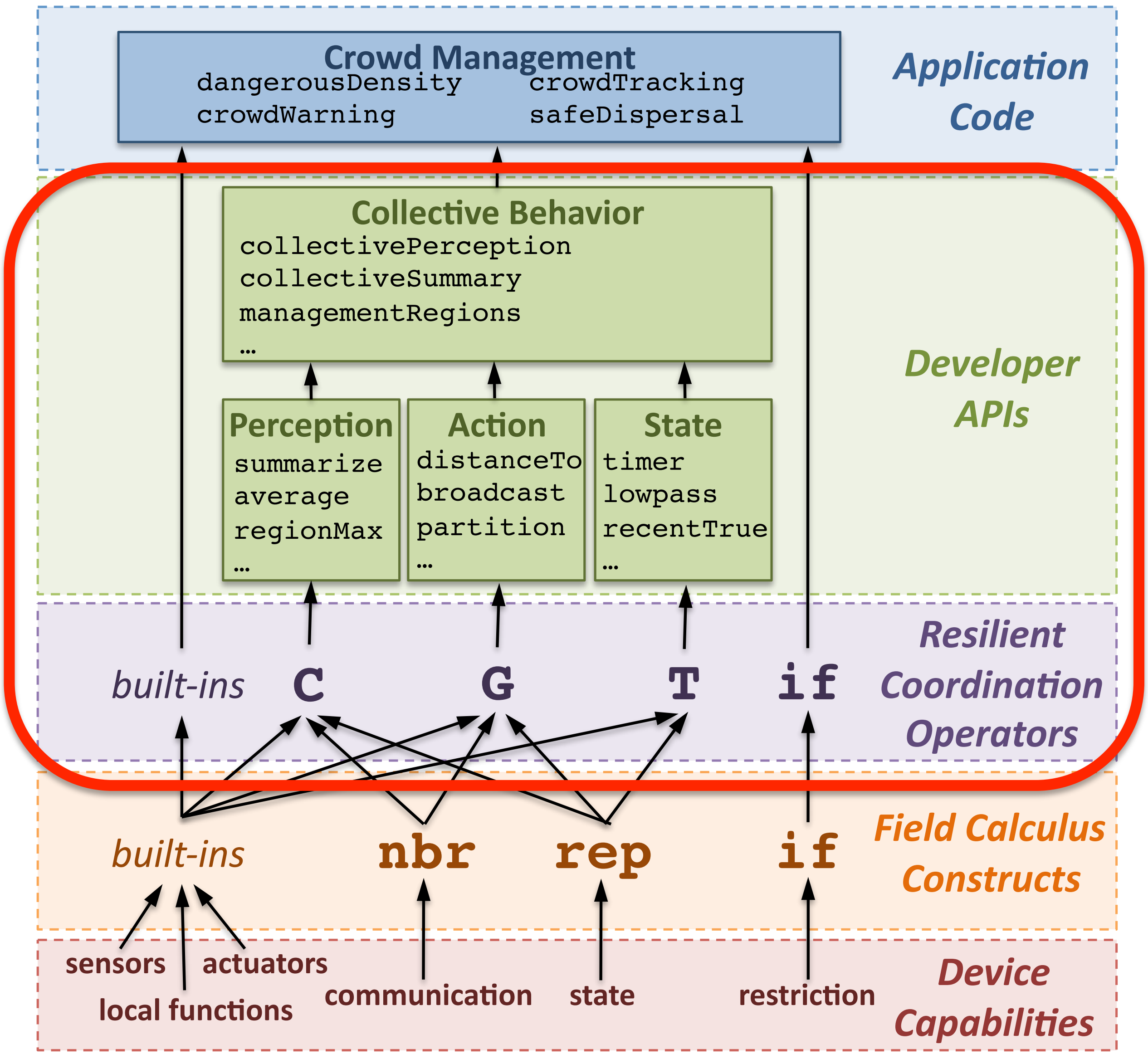

Example of a complex service

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```
(def evacuate (zone coordinator alert)
  (let ((alerted
         (if zone
          (broadcast coordinator
           (collect-region
            (distance-to commander)
            alert))
          0))))
    (* alerted
     (follow-gradient
      (distance-to (not zone))))))
```

Self-stabilization is hard to get right

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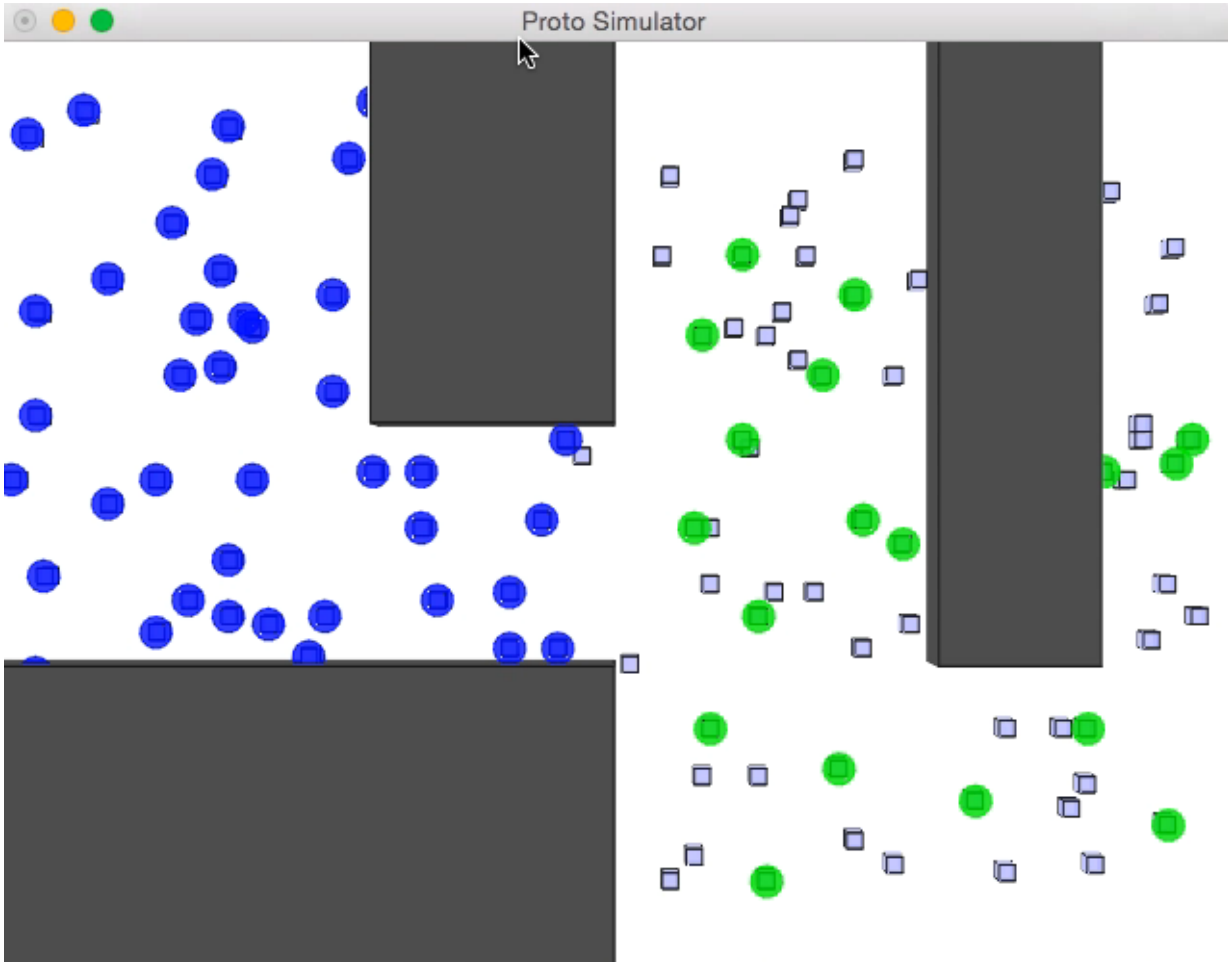

Naïve geometry: when stationary, fine…

Self-stabilization is hard to get right

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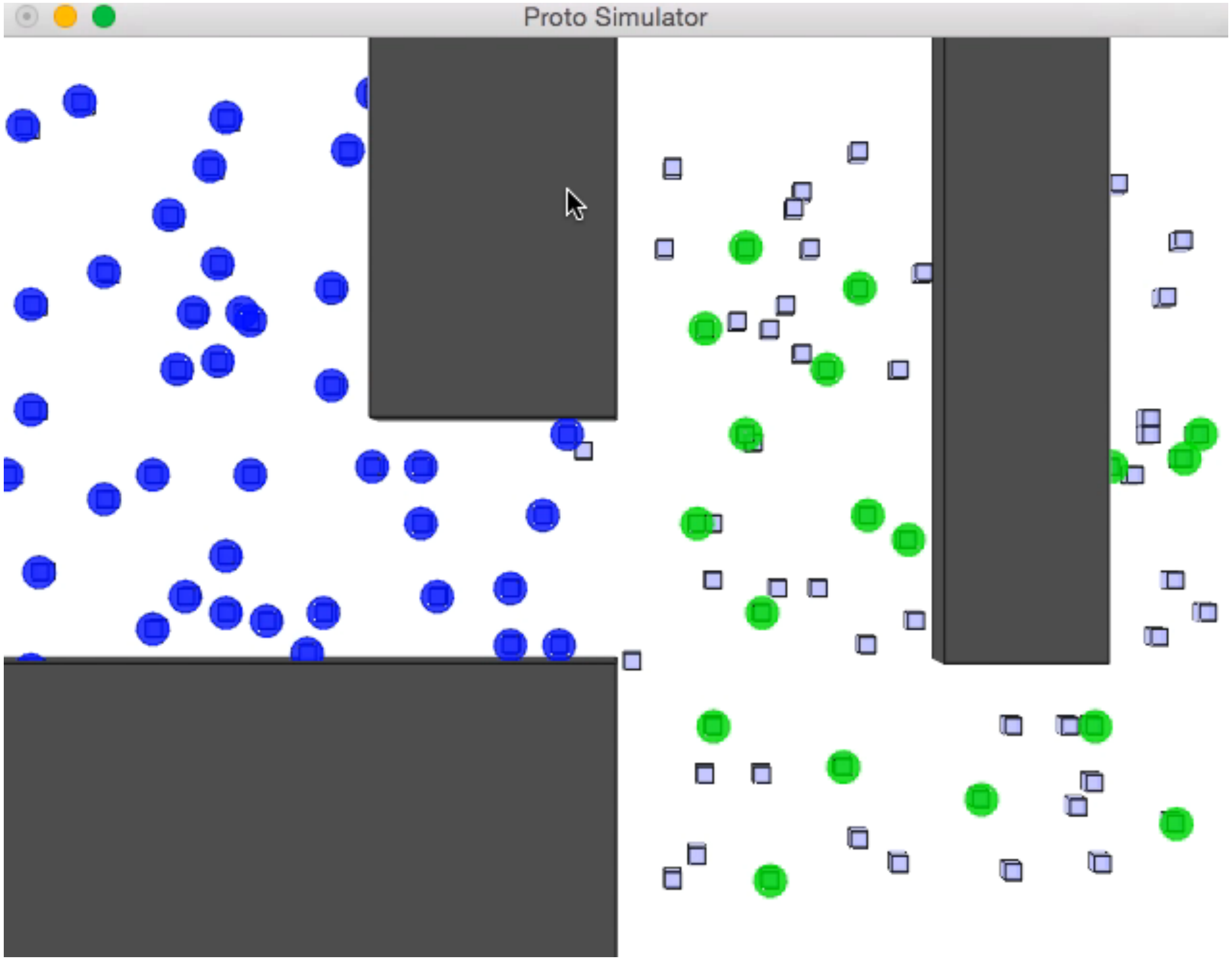

… but doesn't correct properly for change.

Self-Stabilizing Building Blocks

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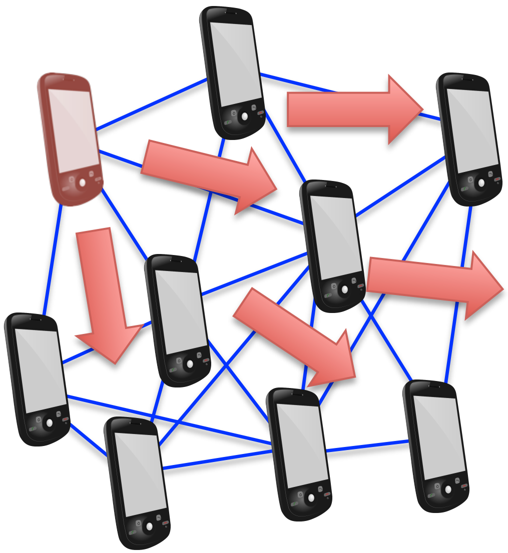
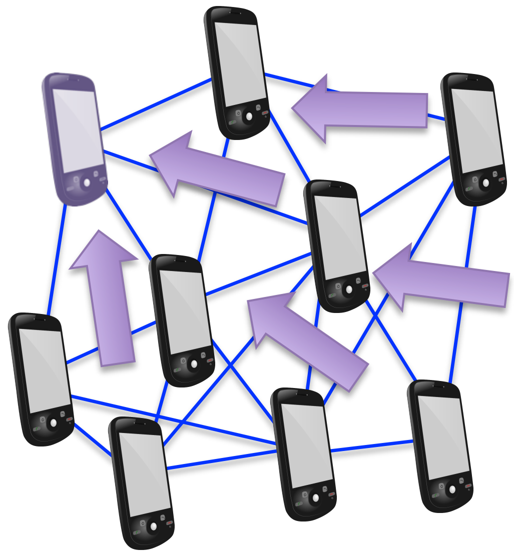
G



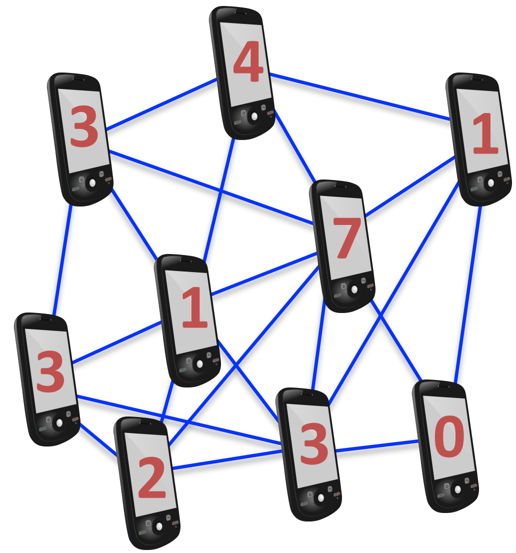
Information spreading

C



Information collection

T



Short-term memory

Resilience by construction: all programs from these building blocks are also self-stabilizing!

Building Block: G

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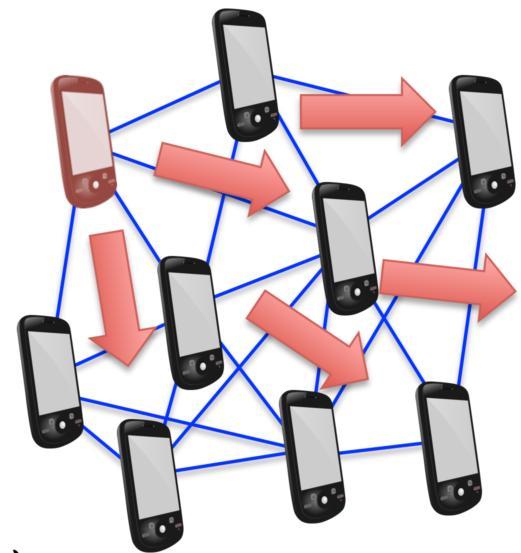
Information spreading



Field Calculus Implementation:

```
(def G (source initial metric accumulate)
 (2nd
  (rep distance-value
   (tuple infinity initial)
   (mux source (tuple 0 initial)
    (min-hood
      (tuple
       (+ (1st (nbr distance-value)) (metric))
       (accumulate (2nd (nbr distance-value)))))))))
```

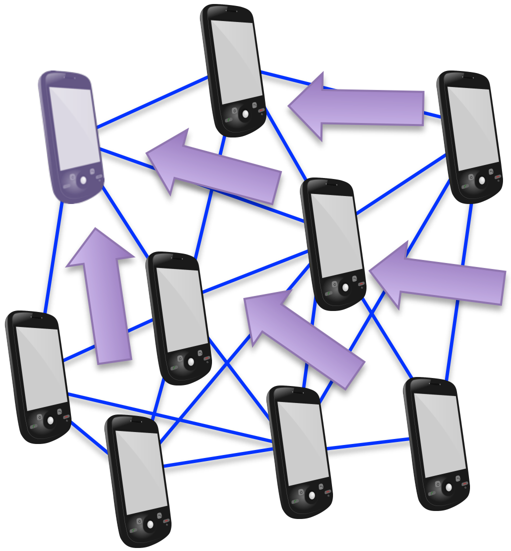
Library Examples:

```
(def distance-to (source)
 (G source 0 nbr-range (fun (v) (+ v (nbr-range)))))
```

```
(def broadcast (source value)
 (G source value nbr-range identity))
```

Building Block: C

Information collection



Field Calculus Implementation:

```
(def C (potential accumulate local null)
 (rep v local
  (accumulate local
   (accumulate-hood accumulate
    (mux (= (nbr (find-parent potential)) (uid))
     (nbr v) null)))))
(def find-parent (potential)
 (mux (< (1st (min-hood (nbr potential))) potential)
   (2nd (min-hood (nbr (tuple potential (uid)))))
   NaN))
```

Library Examples:

```
(def summarize (sink accumulate local null)
 (broadcast sink
  (C (distance-to sink) accumulate local null)))

(def average (sink value)
  (/ (summarize sink + value 0)
     (summarize sink + 1 0)))
```

Building Block: T

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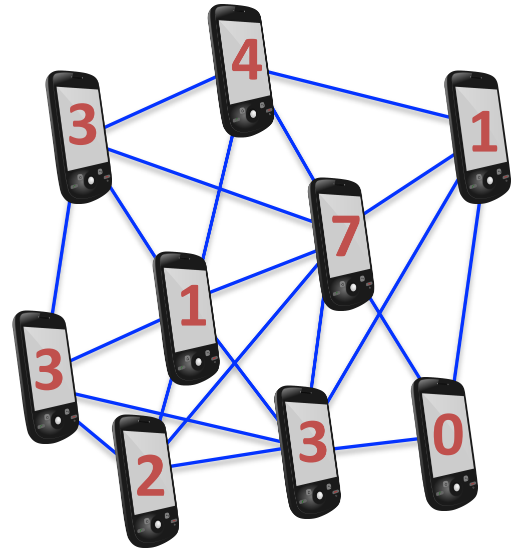
Time-summarization of information

Field Calculus Implementation:

```
(def T (initial decay)
 (rep v initial
  (min initial
   (max 0 (decay v)))))
```



Library Examples:

```
(def timer (length)
 (T length (fun (t) (- t (dt)))))
```

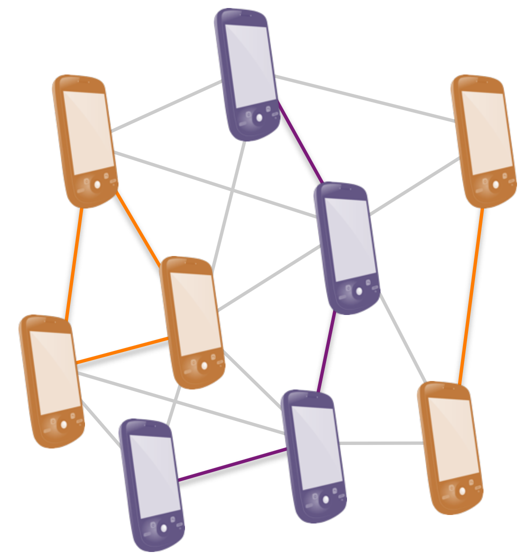
```
(def limited-memory (value timeout)
 (2nd (T (tuple timeout value)
        (fun (t) (tuple (- (1st t) (dt)) (2nd t))))))
```

Building Block: `if`

Restrict scope to subspaces

Field Calculus Implementation:

```
(if test
    true-expression
    false-expression)
```



Library Examples:

```
(def distance-avoiding-obstacles (source obstacles)
  (if obstacles
      infinity
      (distance-to source)))


(def recent-event (event timeout)
  (if event true (> (timer timeout) 0)))
```

All combinations are self-stabilizing!

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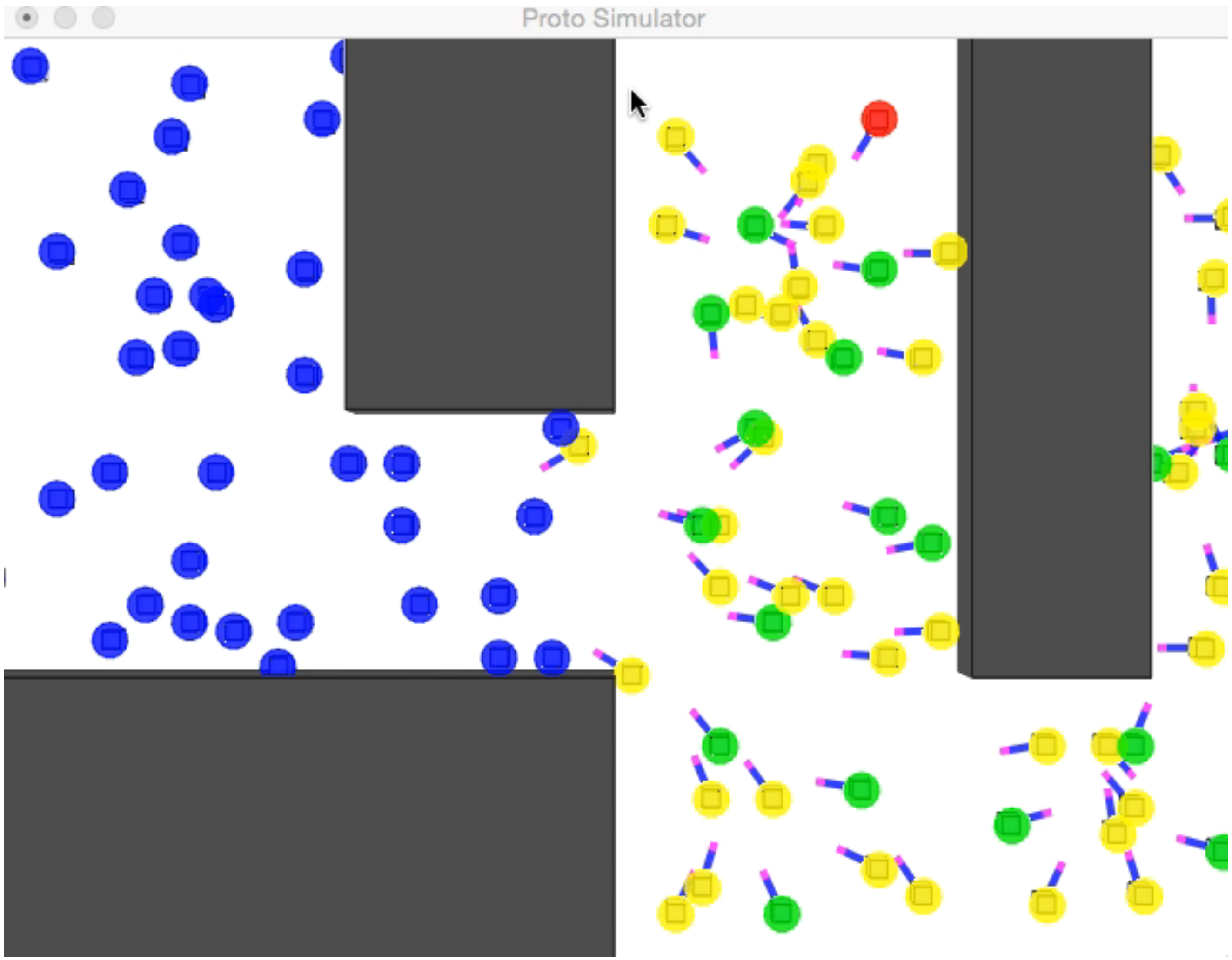


Now program rapidly converges following changes

Applying building blocks:

Example API algorithms from building blocks:

distance-to (source)
broadcast (source value)
summarize (sink accumulate local null)
integral (sink value)
timer (length)
random-voronoi (grain metric)
broadcast-region (region source value)
distance-avoiding-obstacles (source obstacles)
max-likelihood (source p)
path-forecast (source obstacle)
average (sink value)
region-max (sink value)
limited-memory (value timeout)
group-size (region)
recent-event (event timeout)

Since based on these building blocks, all programs built this way are self-stabilizing!

Complex Example: Crowd Management

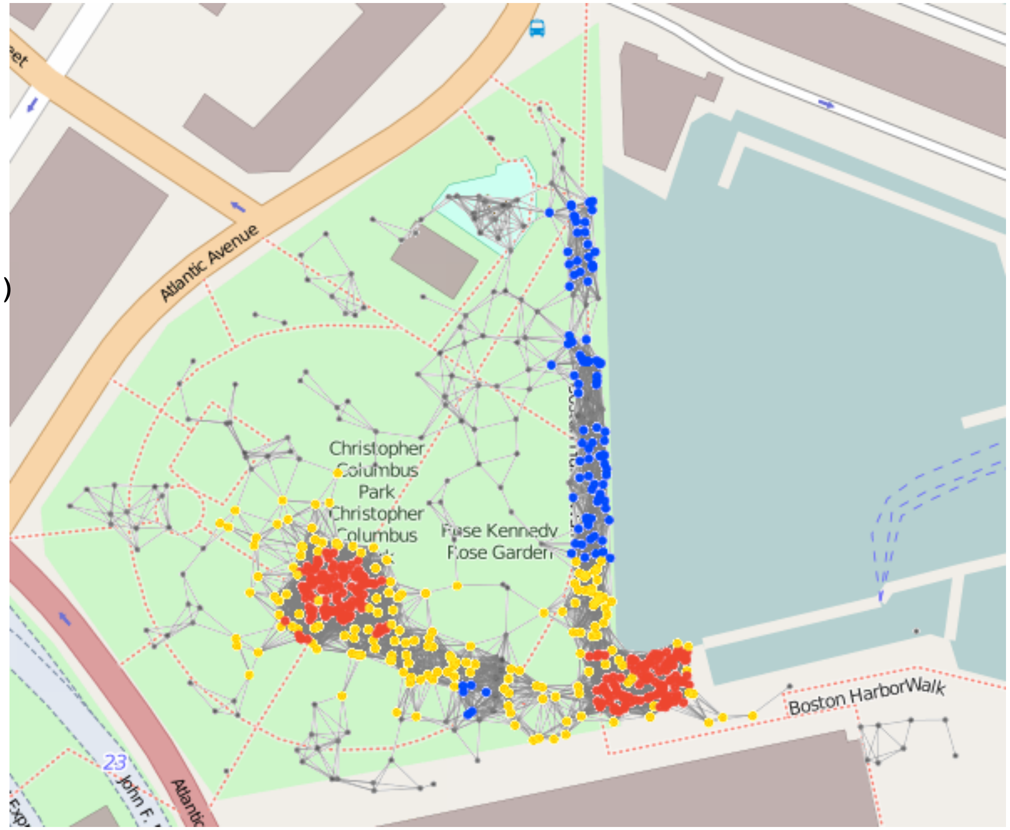

```
(def crowd-tracking (p)
 ;; Consider only Fruin LoS E or F within last minute
 (if (recently-true (> (density-est p) 1.08) 60)
   ;; Break into randomized "cells" and estimate danger of each
   (+ 1 (dangerous-density (sparse-partition 30) p))
   0))

(def recently-true (state memory-time)
 ;; Make sure first state is false, not true...
 (rt-sub (not (T 1 1)) state memory-time))
(def rt-sub (started s m)
 (if state 1 (limited-memory s m)))

(def dangerous-density (partition p)
 ;; Only dangerous if above critical density threshold...
 (and
  (> (average partition (density-est p)) 2.17)
  ;; ... and also involving many people.
  (> (summarize partition + (/ 1 p) 0) 300)))
```

```
(def crowd-warning (p range)
  (> (distance-to (= (crowd-tracking p) 2))
   range)

(def safe-navigation (destination p)
  (distance-avoiding-obstacles
   destination (crowd-warning p)))
```



18 lines non-whitespace code

10 library calls (21 ops)

IF: 3 G: 11 C: 4 T: 3

Generalization: Self-Stabilizing Calculus

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$\mathtt{e} ::= \mathtt{x} \mid \mathtt{v} \mid (\mathtt{e}\ \overline{\mathtt{e}}) \mid (\mathtt{rep}\ \mathtt{x}\ \mathtt{w}\ \mathtt{e}) \mid (\mathtt{nbr}\ \mathtt{e}) \mid (\mathtt{if}\ \mathtt{e}\ \mathtt{e}\ \mathtt{e})$		expression
$\mathtt{v} ::= \ell \mid \phi$		value
$\ell ::= \mathtt{b} \mid \mathtt{n} \mid \langle \ell, \ell \rangle \mid \mathtt{o} \mid \mathtt{f} \mid (\mathtt{fun}\ (\overline{\mathtt{x}})\ \mathtt{e})$		local value
$\mathtt{w} ::= \mathtt{x} \mid \ell$		variable or local value
$\mathtt{F} ::= (\mathtt{def}\ \mathtt{f}(\overline{\mathtt{x}})\ \mathtt{e})$		user-defined function
$\mathtt{P} ::= \overline{\mathtt{F}}\ \mathtt{e}$		program

Restrict field calculus by replacing **e** *with* **s**:

$$\mathtt{s} ::= \ell \mid \mathtt{x} \mid (\mathtt{s}\ \overline{\mathtt{s}}) \mid (\mathtt{nbr}\ \mathtt{s}) \mid (\mathtt{if}\ \mathtt{s}\ \mathtt{s}\ \mathtt{s})$$

$$\mid\ \mathbf{T}\ (\mathtt{rep}\ \mathtt{x}\ \mathtt{w}\ (\pi^{\mathsf{MB}}\ \mathtt{x}\ \overline{\mathtt{s}})) \qquad \mathtt{x} \notin \mathbf{FV}(\overline{\mathtt{s}})$$

$$\mid\ \mathbf{C}\ (\mathtt{rep}\ \mathtt{x}\ \mathtt{w}\ (\pi^{\mathsf{F}}\ \mathtt{s}^{\mathsf{A}}\ (\mathtt{nbr}\ (\mathtt{s}\ \mathtt{x}))\ \overline{\mathtt{s}})) \qquad \mathtt{x} \notin \mathbf{FV}(\mathtt{s}, \overline{\mathtt{s}}, \mathtt{s}^{\mathsf{A}})$$

$$\mid\ \mathbf{G}\ (\mathtt{rep}\ \mathtt{x}\ \mathtt{w}\ (\pi\ (\pi'\ (\mathtt{nbr}\ (\pi''\ \mathtt{x}\ \overline{\mathtt{s}}''))\ \overline{\mathtt{s}}')\ \overline{\mathtt{s}})) \qquad \pi' \circ \pi = \pi^{\mathsf{MD}}, \pi'' \circ \pi' = \pi^{\mathsf{MBP}}, \mathtt{x} \notin \mathbf{FV}(\overline{\mathtt{s}}, \overline{\mathtt{s}}', \overline{\mathtt{s}}'')$$

Self-Stabilization → Substitution

Self-Stabilising Calculus	⊂	**(Higher-Order) Field Calculus**
⋓		⋓
Building Block Operator	≃	*Coordination Mechanism*

Given functions λ,λ' with same type, λ is substitutable for λ' iff for any self-stabilising list of expressions e, (λ e) always self-stabilises to the same value as (λ' e).

Optimization of Dynamics

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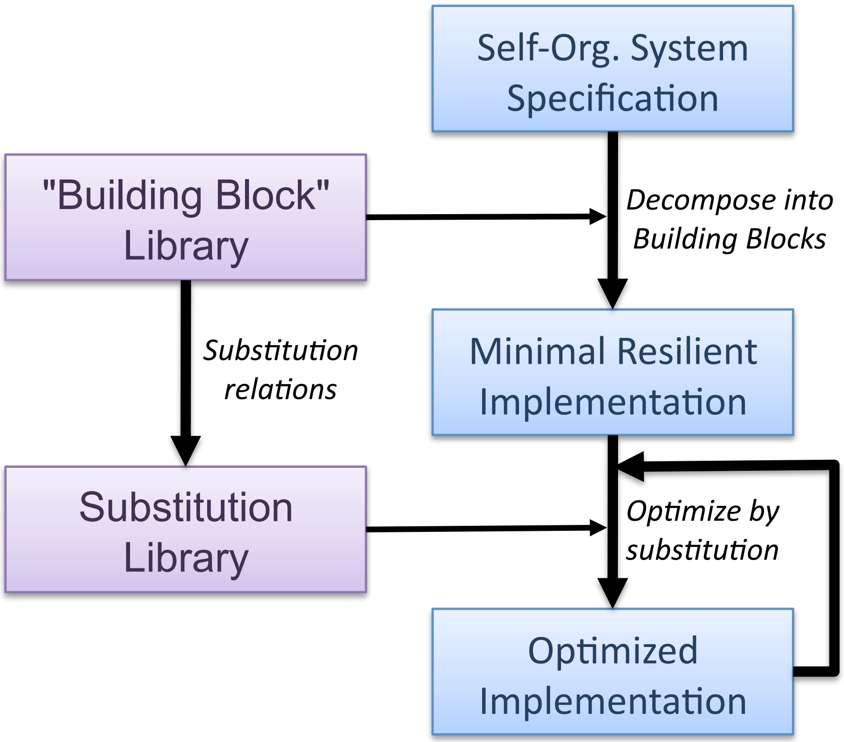

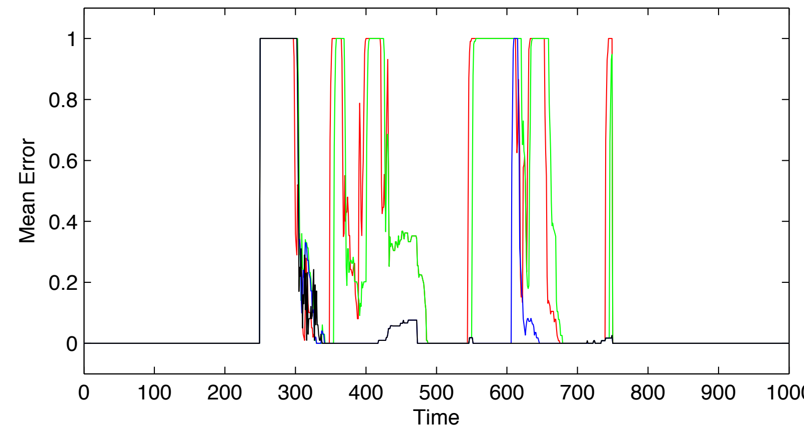

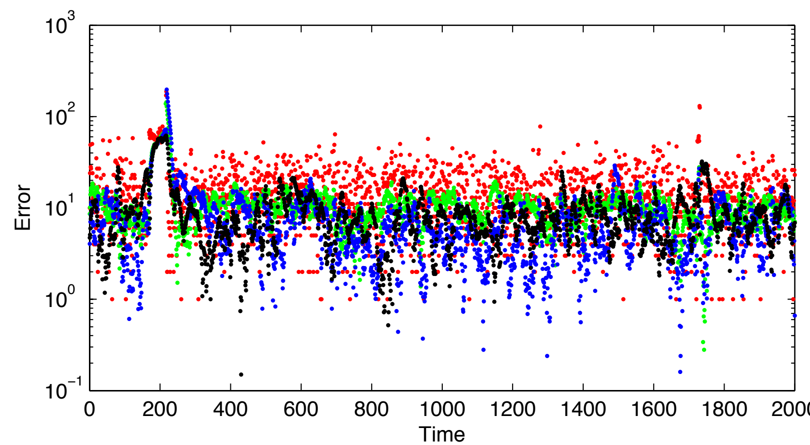

[Viroli, Beal, Damiani & Pianini, SASO ‘15]

Optimization Example: Crowd Alert

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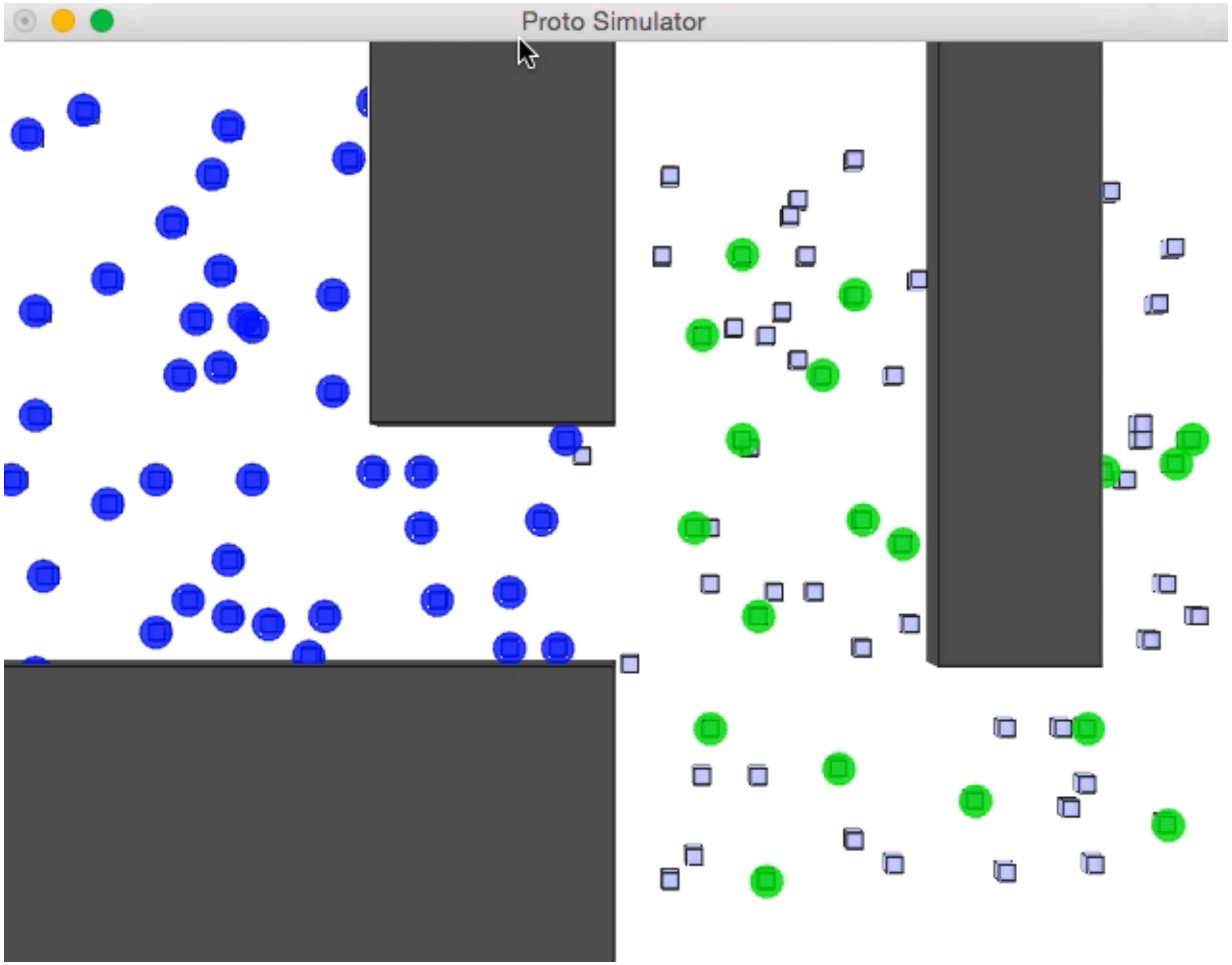
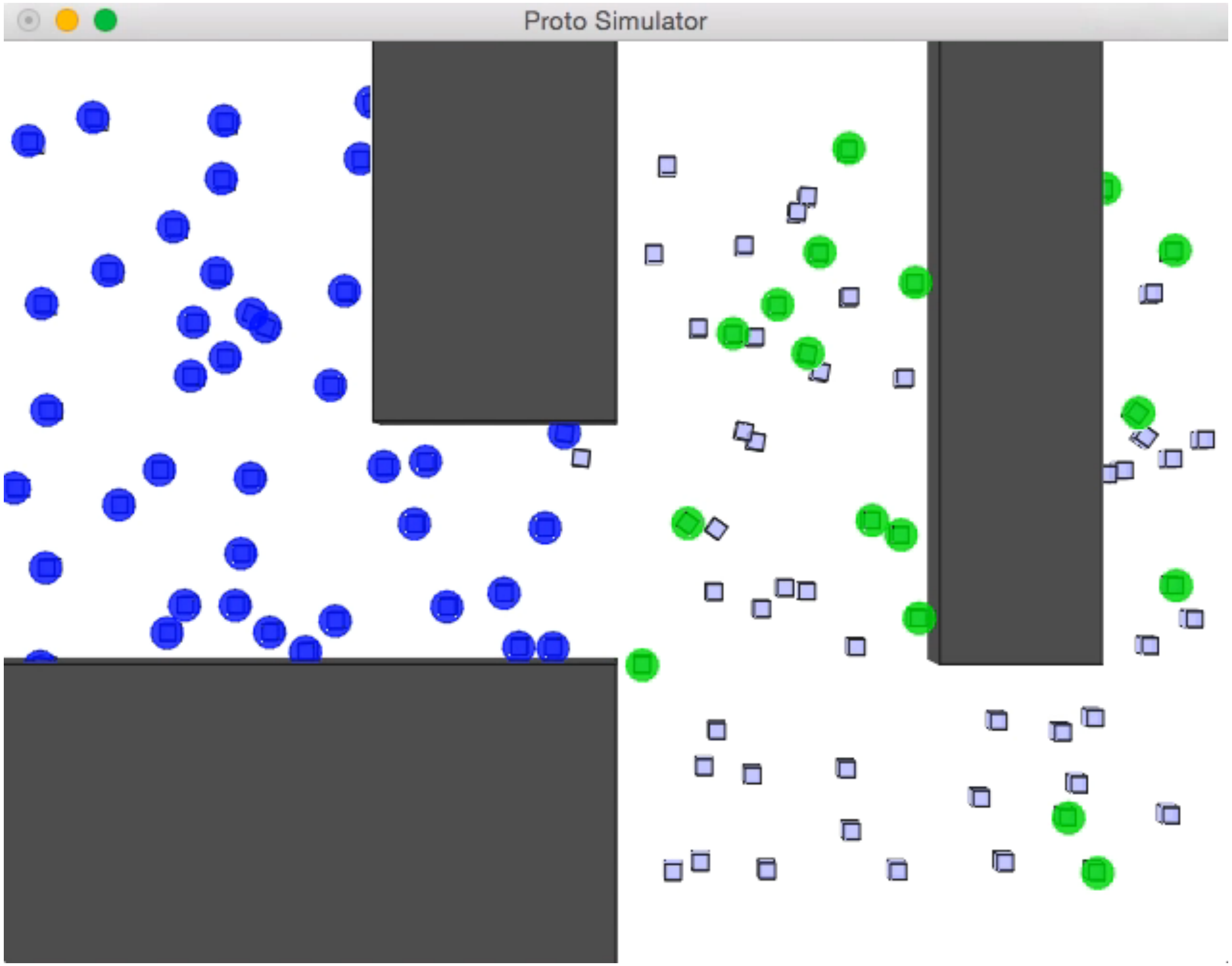


Naïve algorithm: when stationary, fine…

Optimization Example: Crowd Alert



… but dynamics can't keep up with fast mobility.

Optimization Example: Crowd Alert

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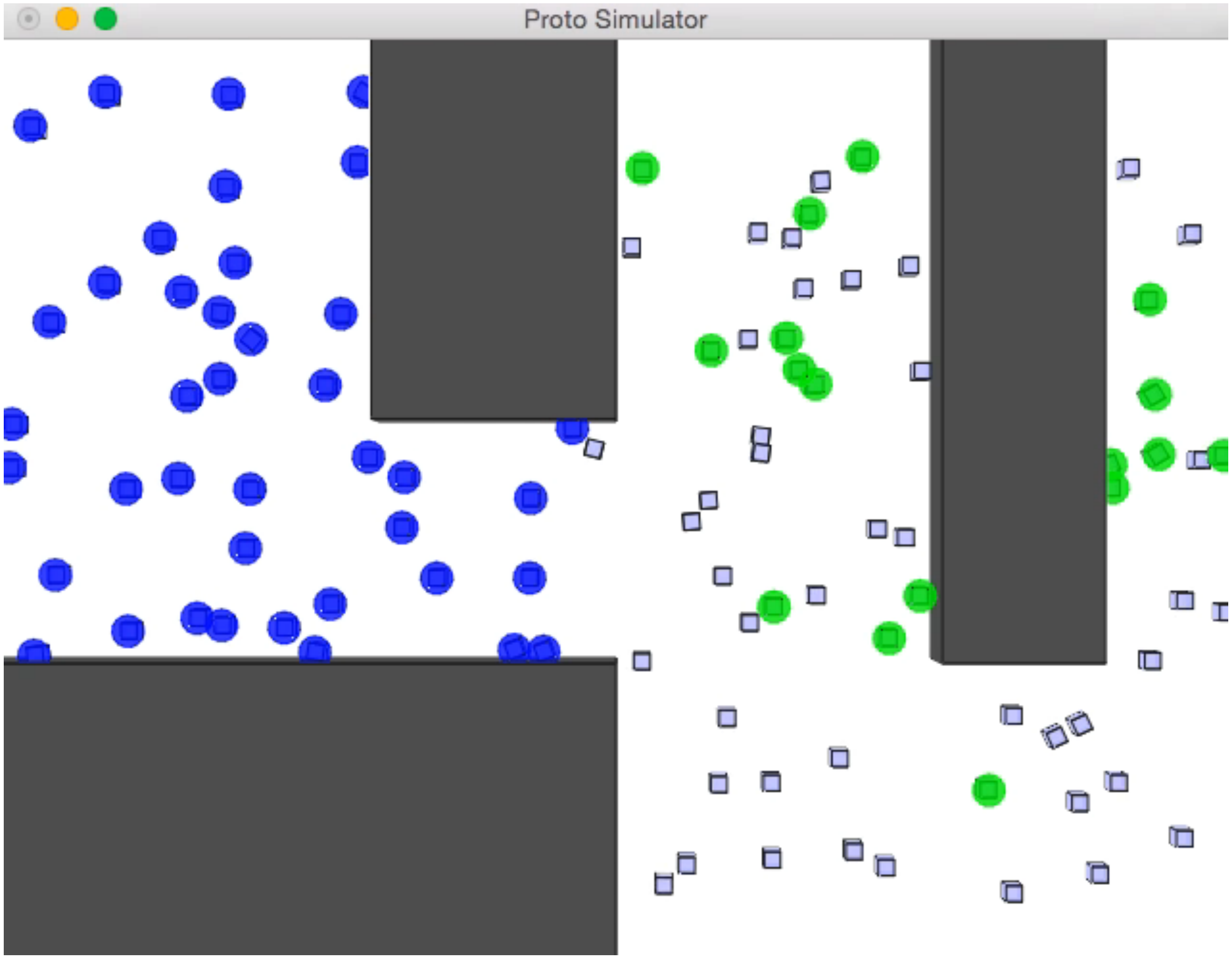

Optimized dynamics, however, work well.

Eventual Consistency

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- **Consistent Program:** Let *P* be a space-time program, *e* be an evaluation environment, and e_i a countable sequence of ε-approximations that approximate field *e*. Program *P* is consistent if $P(e_i)$ approximates $P(e)$ for every *e* and e_i.
- **Eventually Consistent Program:** Consider a causal program *P* evaluated on environment *e* with domain *M*. Program *P* is eventually consistent if, for any environment *e* in which there is a spatial section S_M such that the values of *e* do not change at any device in the time-like future $T^+(S_M)$, there is always some spatial section S_M' such that *P* is consistent on the time-like future $T^+(S_M')$

Intuition: resilience against scale, discretization, device location

[Beal et al., Submitted]

What are the threats to consistency?

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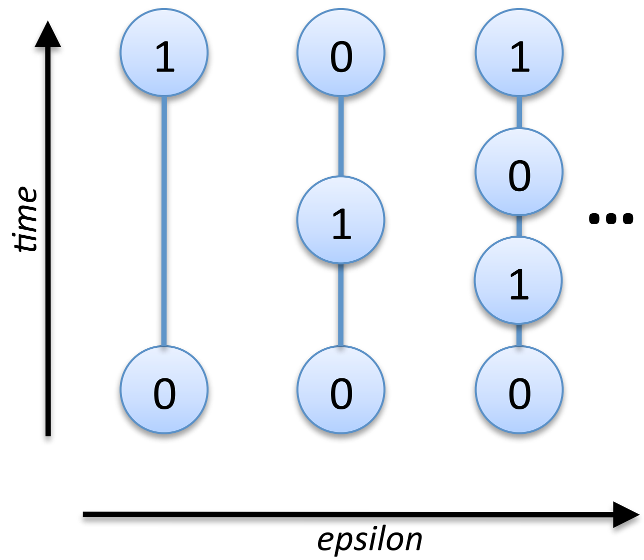
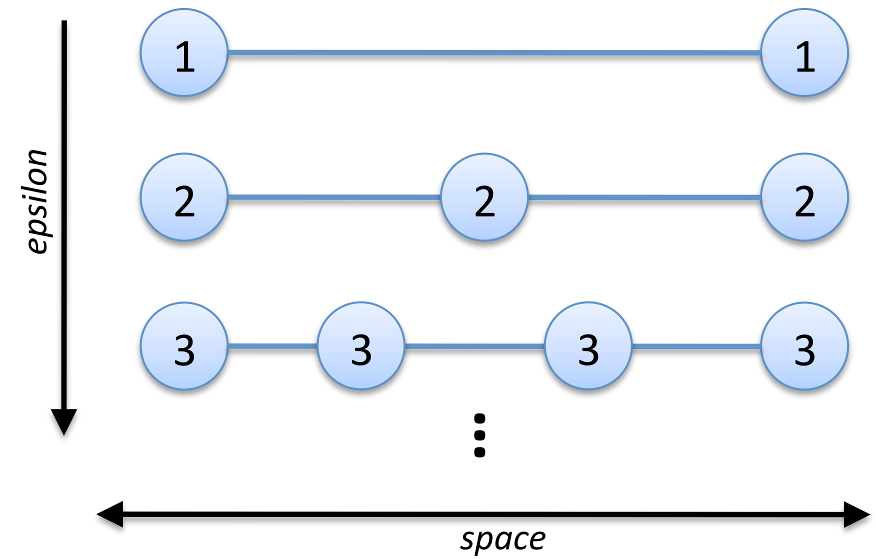
- Unbounded recursion
- Direct use of **rep**, **nbr** constructs

```
(rep x 0 (- 1 x))
```




```
(/ 1 (min-hood (nbr-range)))
```



What are the threats to consistency?

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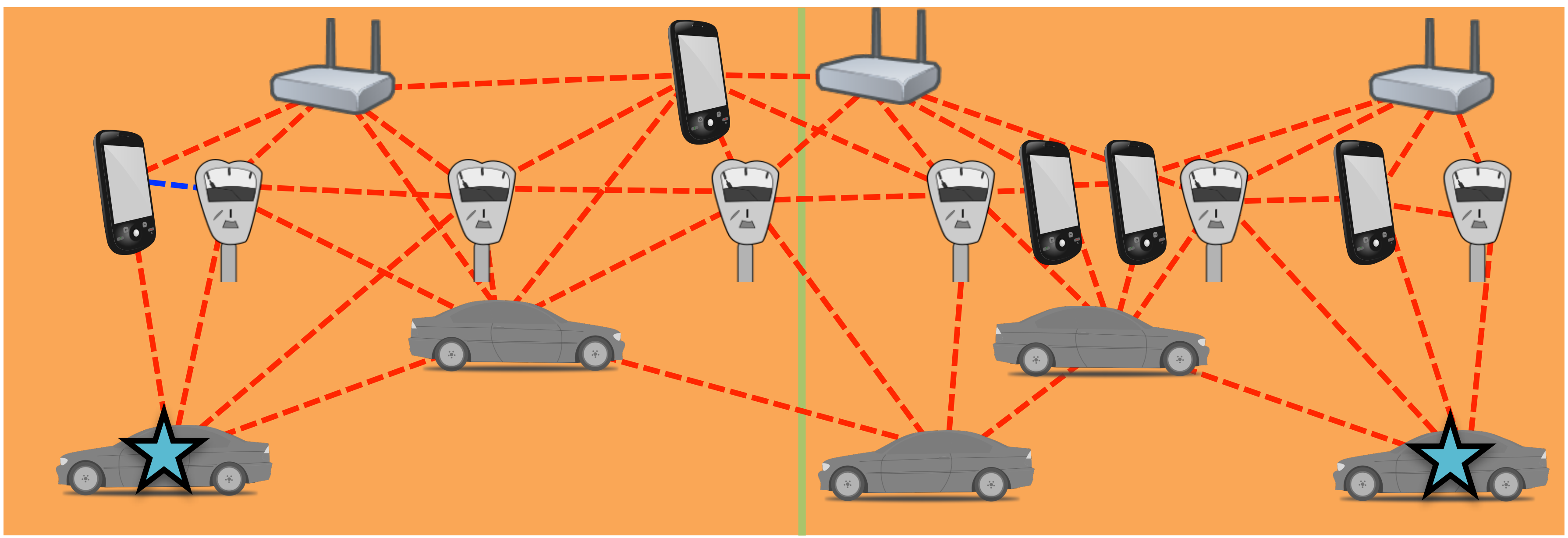
- Fragile values (measure zero sets)

```
(def bisector (a b)
  (= (distance-to a)
     (distance-to b)))
```



GPI-calculus

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Special "Boundary" value

Only integers and reals

Restricted built-in ops

GPI replaces nbr, rep

Semantics prohibits recursion

$$
\begin{array}{llr}
\mathtt{l} ::= \mathbb{B} \mid \mathbb{Z} \mid \mathbb{R} & & \text{;; Literals} \\
\mathtt{b} ::= \mathtt{m} \mid \mathtt{mux} \mid \mathtt{<} & & \text{;; local operators} \\
\mathtt{e} ::= \mathtt{x} \mid \mathtt{l} \mid (\mathtt{b}\ \overline{\mathtt{e}}) \mid (\mathtt{f}\ \overline{\mathtt{e}}) \mid (\mathtt{sense}\ \mathbb{Z}^{+}) & & \text{;; expression} \\
\quad \mid (\mathtt{if}\ \mathtt{e}\ \mathtt{e}\ \mathtt{e}) \mid (\mathtt{GPI}\ \mathtt{e}\ \mathtt{e}\ \mathtt{e}\ \mathtt{e}) & & \text{;; special constructs} \\
\mathtt{F} ::= (\mathtt{def}\ \mathtt{f}(\overline{\mathtt{x}})\ \mathtt{e}) & & \text{;; function} \\
\mathtt{P} ::= \overline{\mathtt{F}}\ \mathtt{e} & & \text{;; program}
\end{array}
$$

- Further restriction of self-stabilizing calculus
  - Real # comparison produces "Boundary" for equality
  - GPI = Gradient-Path-Integral

    G, except accumulation always integral, Boundary discarded

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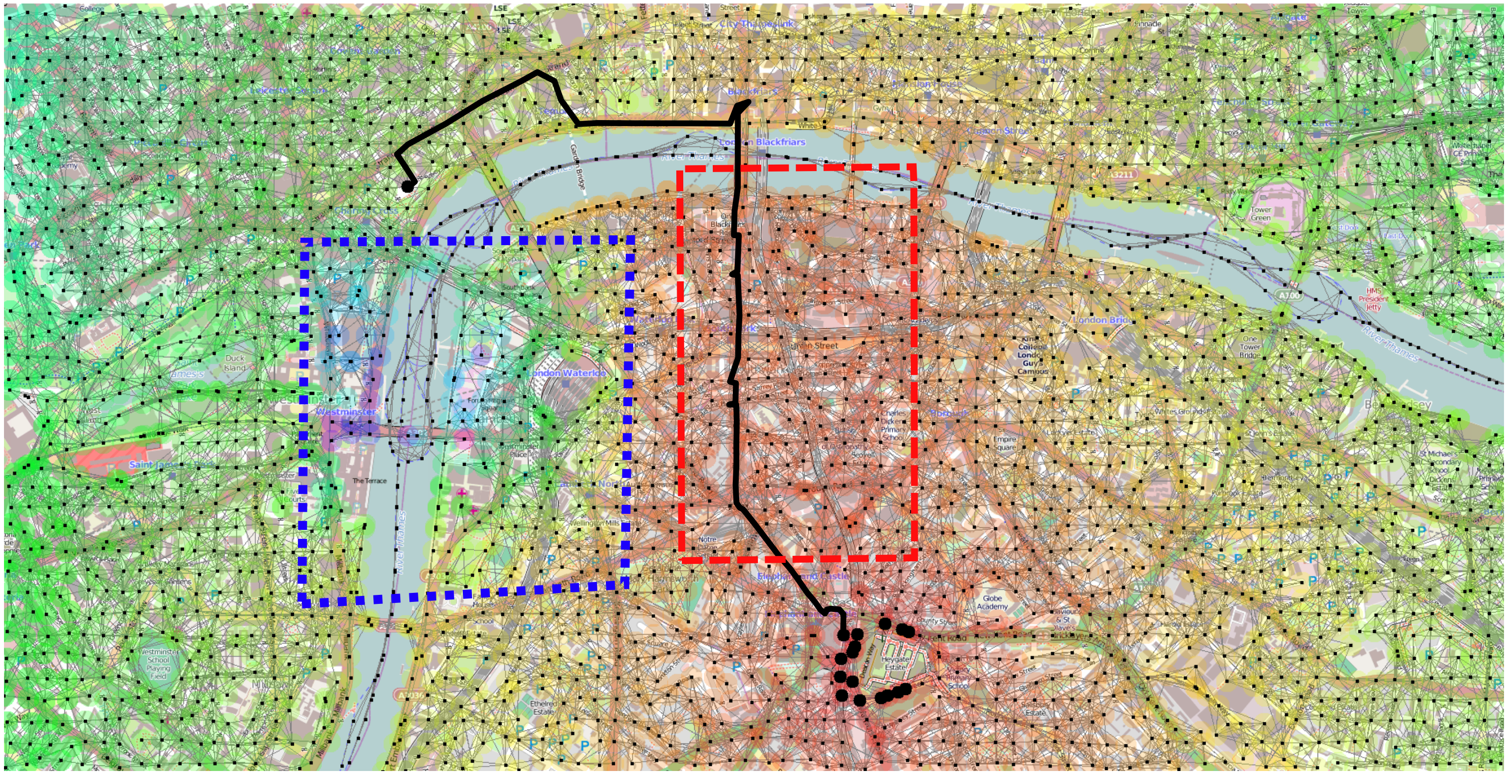
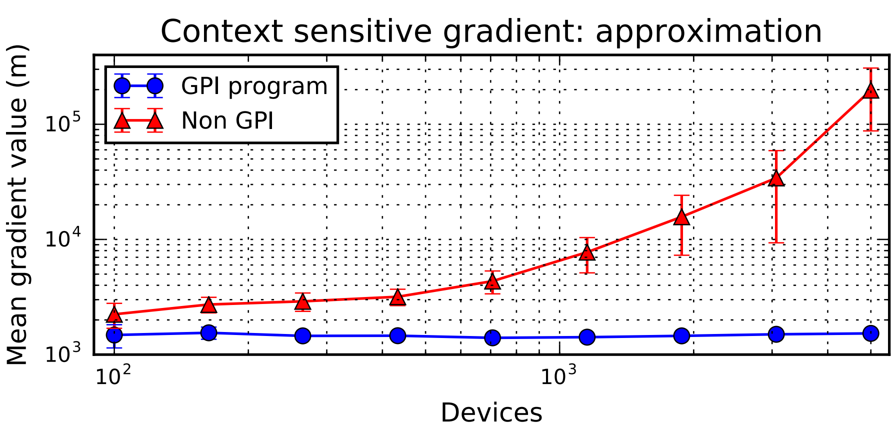
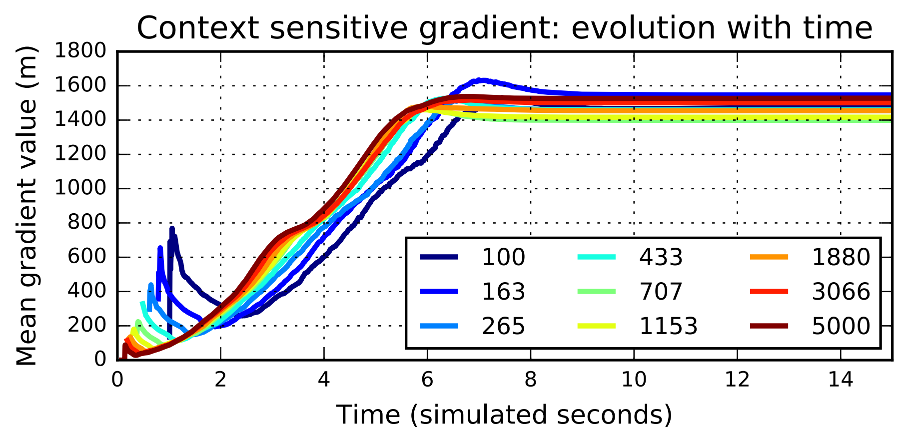
Example: Context-sensitive distance







Summary

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- Resilient convergence: self-stabilization
- Dynamics of resilience: substitution
- Resilience to location: eventual consistency