Quantum Information Theory

Renato Renner
ETH Zurich

Do quantum computers exist?

Geordie Rose and his D-Wave quantum computer

Commercial devices

Quantum cryptography device by *id Quantique*, in operation at the FIFA World Cup competition in Durban

Commercial devices

Quantis Random Number Generator (4 Mbits/s)

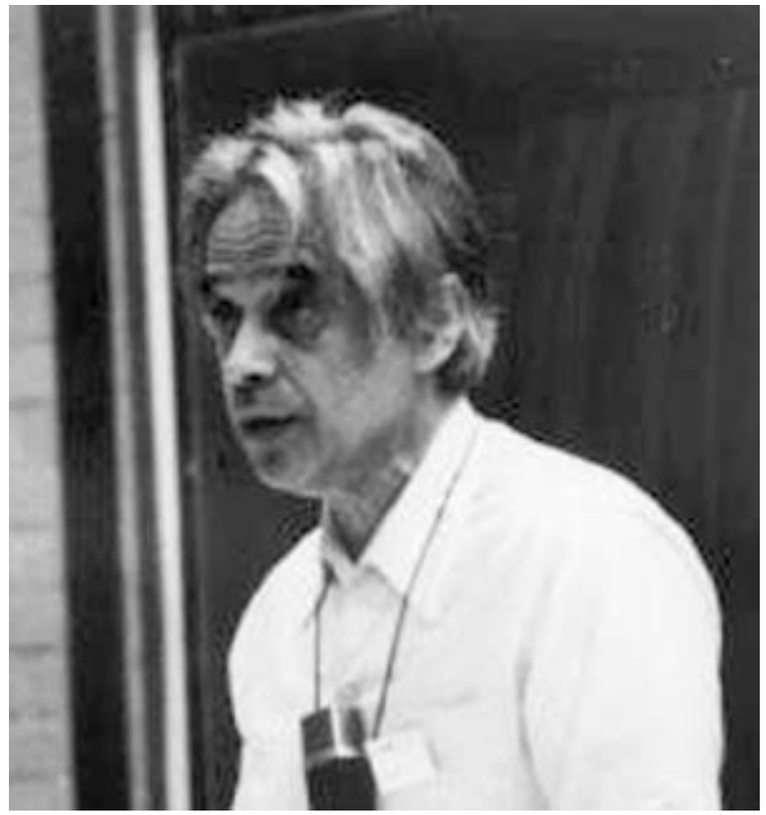
Is the world deterministic?

„Jedenfalls bin ich überzeugt, daß der nicht würfelt.“

Albert Einstein, in a letter to Max Born

It took several decades until the inherent non-deterministic nature of quantum theory was accepted as a “physical” fact.

„Mein Ziel war zu beweisen, dass niemand, nicht mal Gott, den Verlauf der Welt voraussagen kann.“

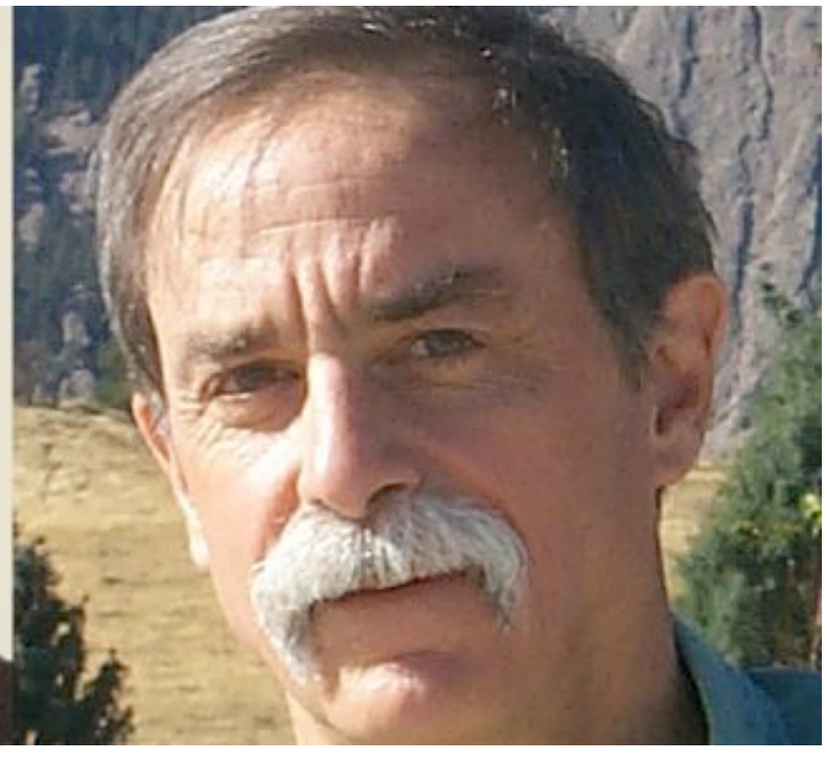


Ernst Specker (mathematician, ETH Zurich)

Research in quantum information

Nobel prize
2012

Serge Haroche



David Wineland

Research in quantum information

Research groups in quantum information science (from http://www.quantiki.org)

Research in quantum information

QSIT Quantum Science and Technology
National Centre of Competence in Research

Swiss network consisting of more than 300 scientists

YouTube™ → QSIT

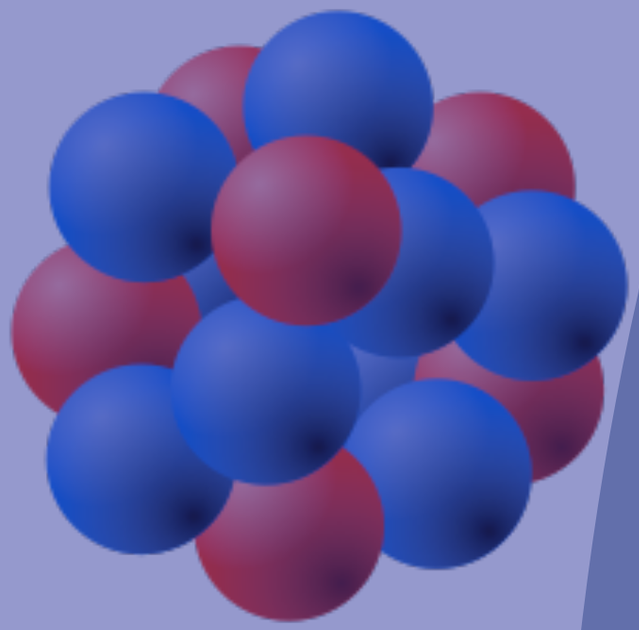
Quantum
Information Theory

What is “quantum”?

quantum physics

classical physics

general relativity

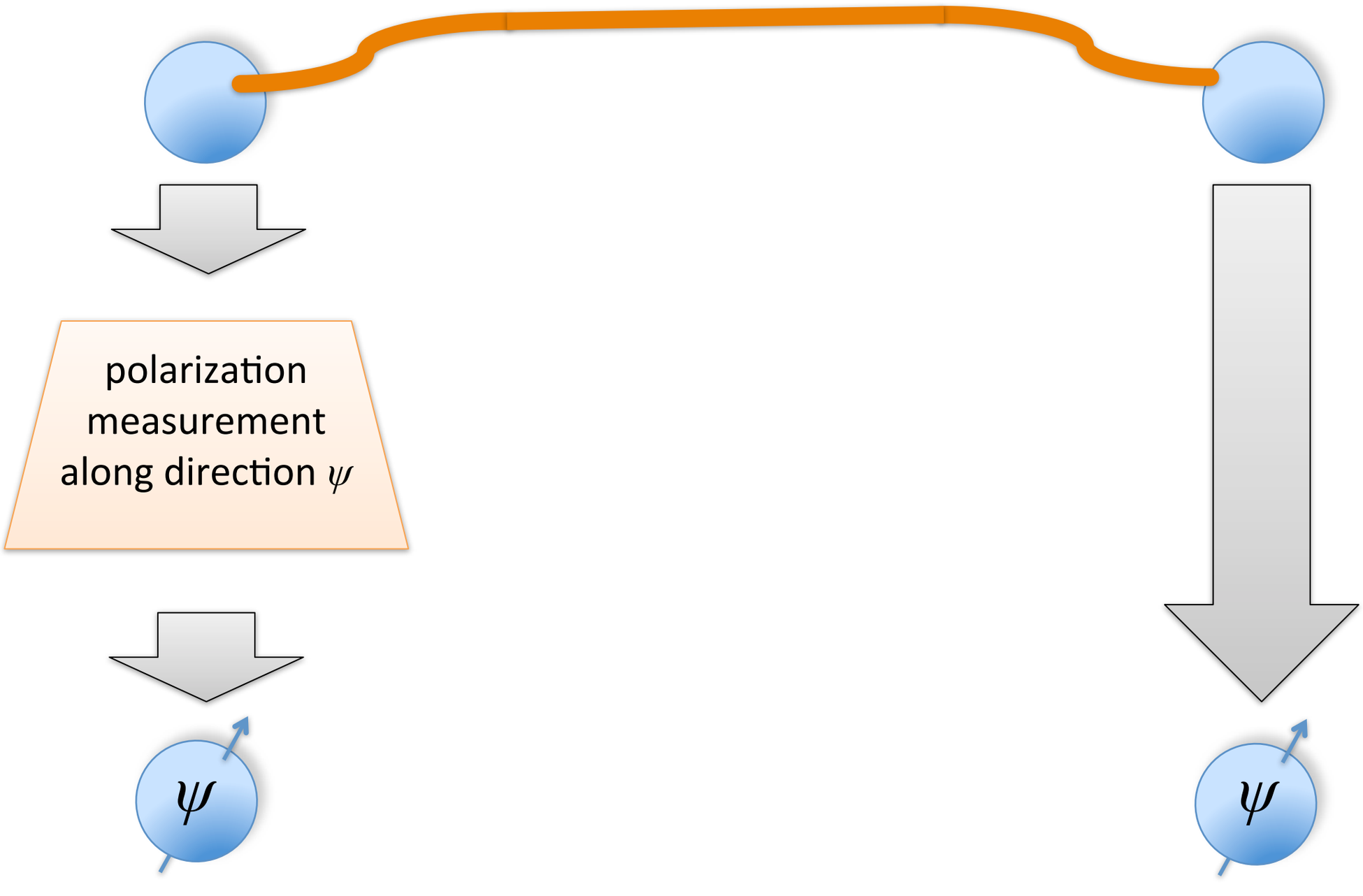


10^{-15} m 10^{-9} m 10^{-3} m 10^{3} m 10^{9} m 10^{15} m 10^{21} m

1 fm 1 pm 1 nm 1 µm 1 mm 1 m 1 km 1 Mm 1 Gm 1 Tm 1 Pm 1 Em 1 Zm

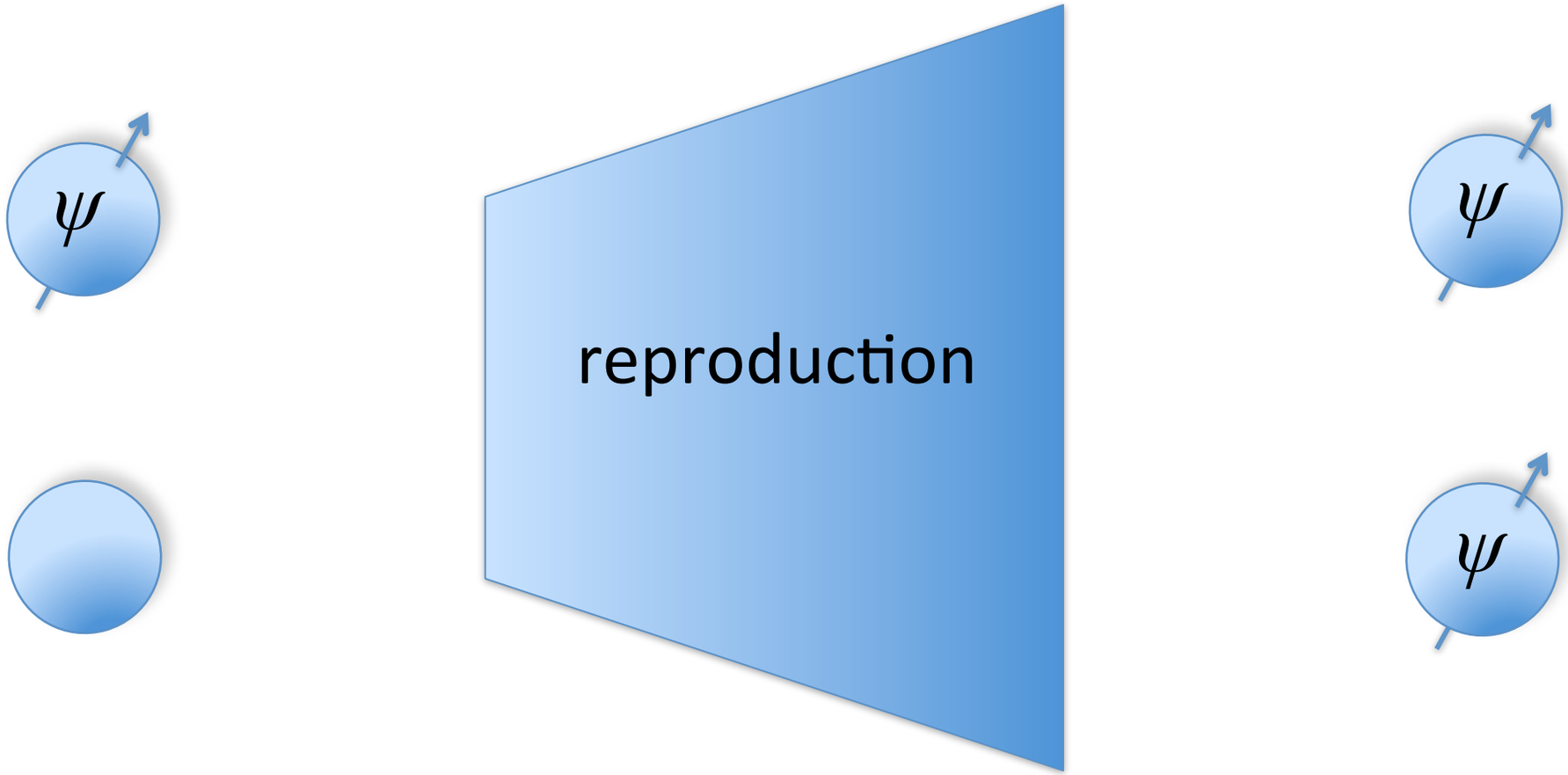
Quantum physics deviates from our day-to-day experience about the world around us

Example 1: Entanglement



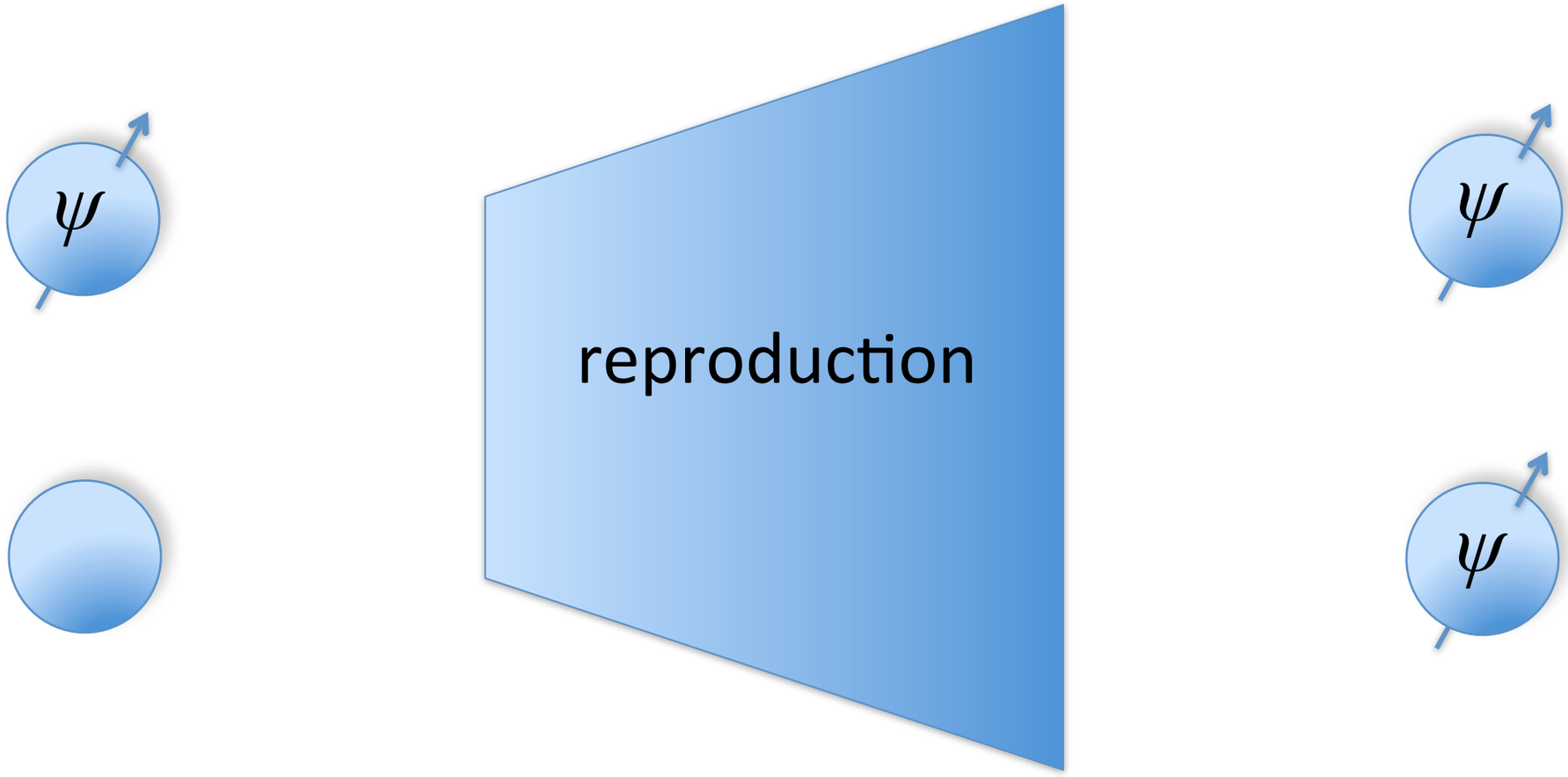
Quantum physics deviates from our day-to-day experience about the world around us

Example 2: No-cloning principle



Quantum physics deviates from our day-to-day experience about the world around us

Example 2: No-cloning principle



No-cloning theorem [Wootters und Zurek, 1982]

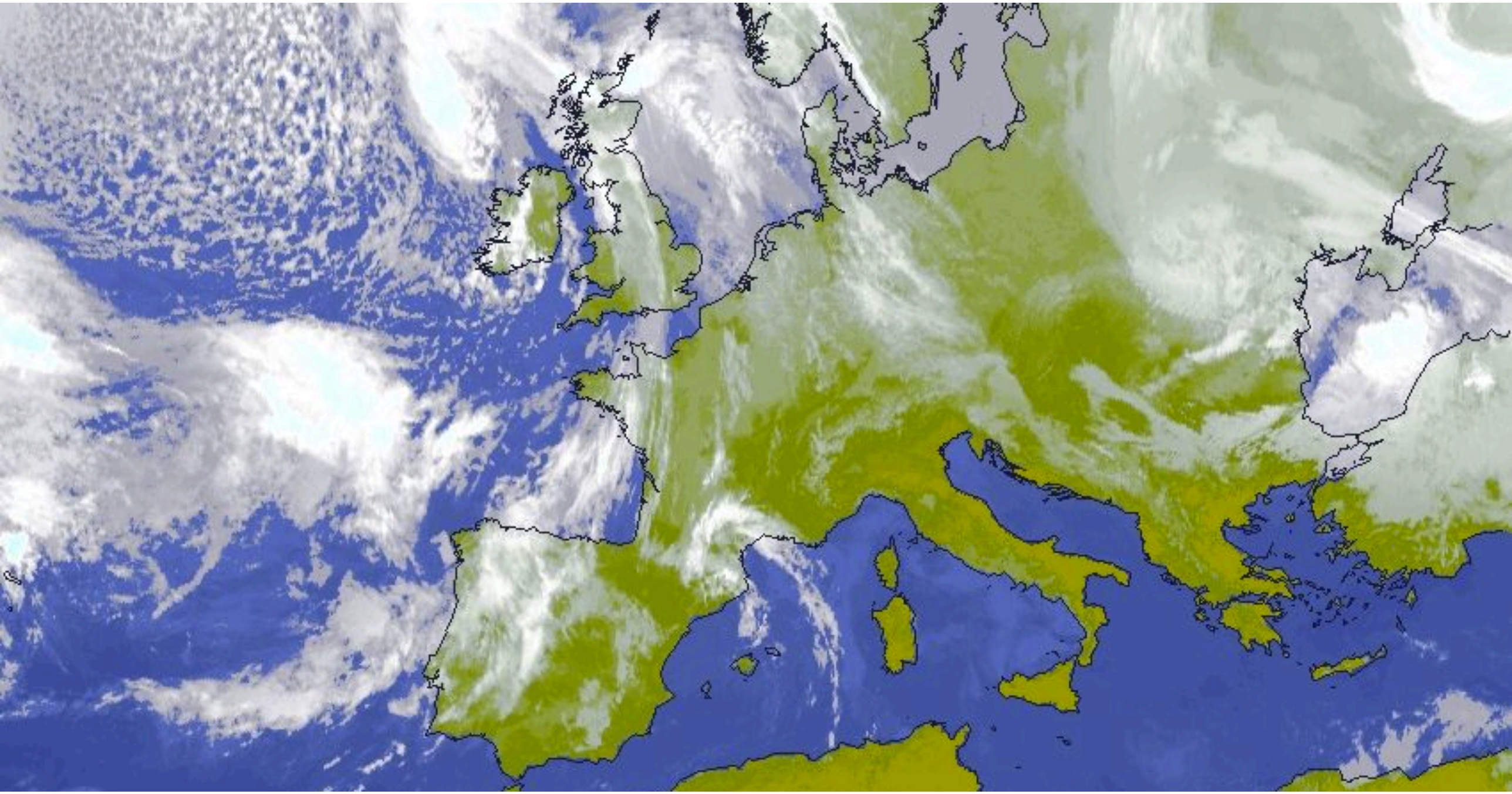
No physical device can copy the state of a quantum system (for arbitrary states ψ).

The no-cloning principle does not seem to be valid for macroscopic objects

reproduction

Quantum
Information Theory

What is “information”?



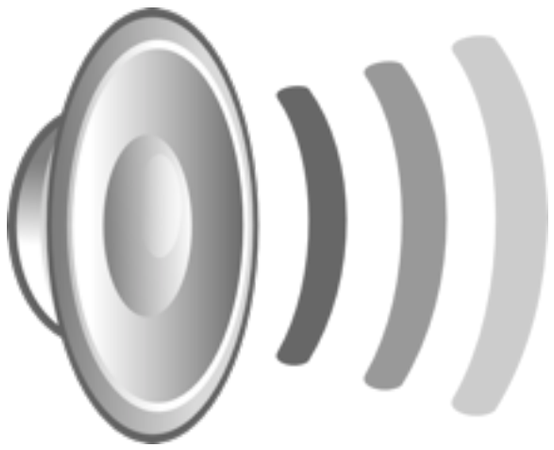
Two different approaches

Kolmogorov's notion:
based on the theory of computation

Shannon's notion:
based on probability theory

Common feature of both approaches

The mathematical theory should be independent of how information is represented.



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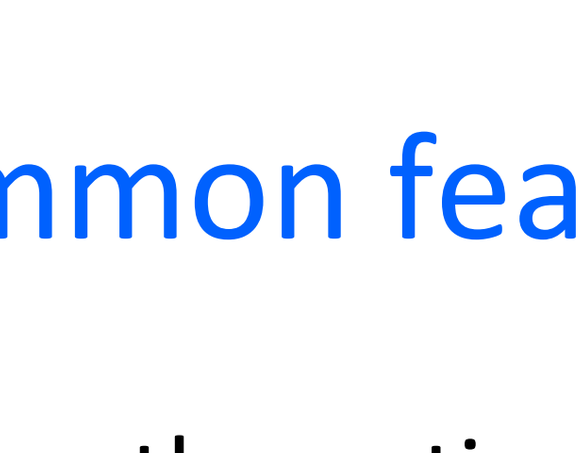
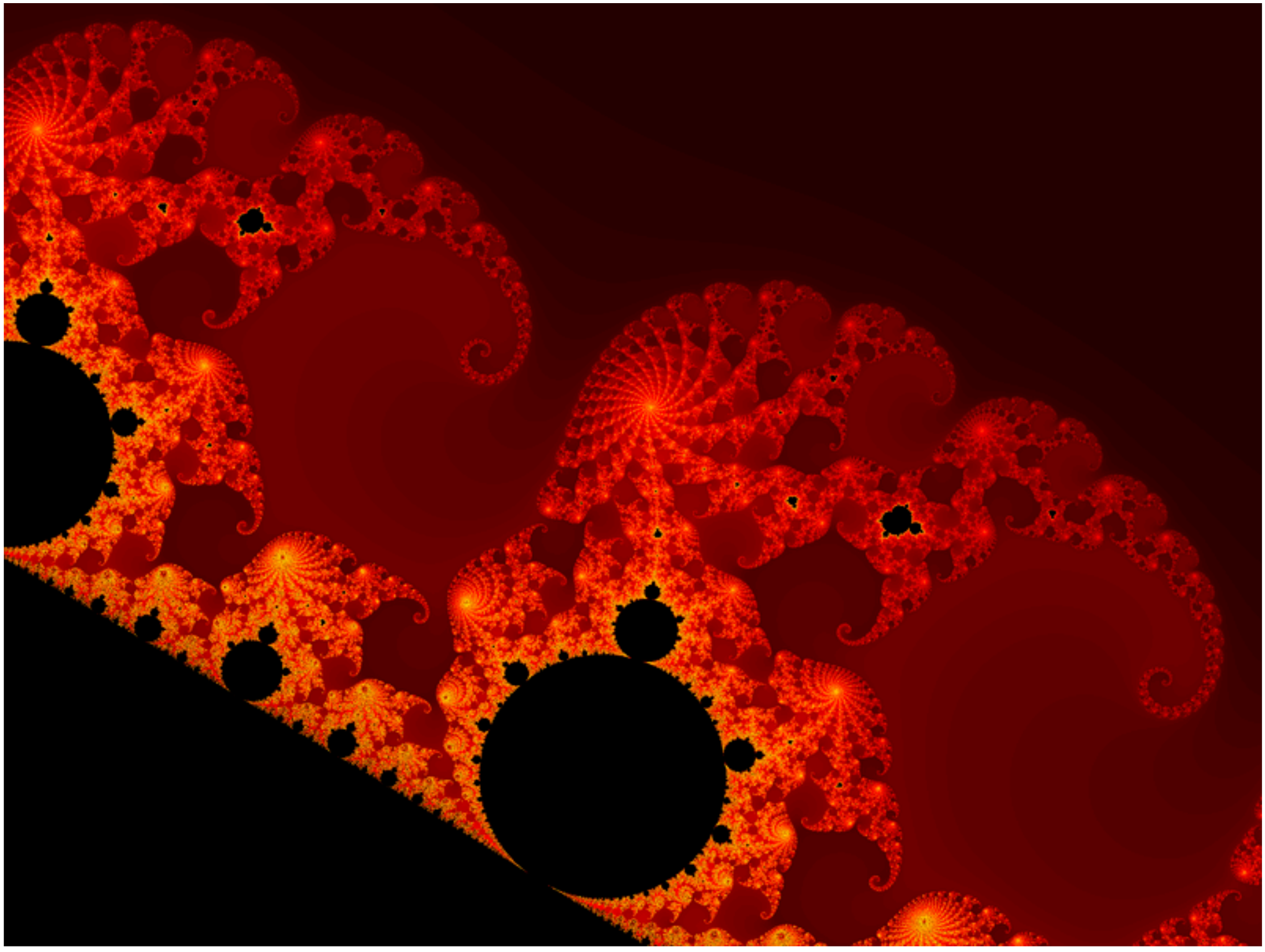


As we shall see, this idea is doomed to fail …

Kolmogorov’s notion of information



What is information?

```
For each pixel (x0,y0) on the screen do:
{  x = 0   y = 0
iteration = 0  max_iteration = 1000
while ( x*x + y*y <= (2*2)  AND  iteration < max_iteration )
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x = xtemp
iteration = iteration + 1      }
if ( iteration == max_iteration )
then color = black
else  color = iteration
plot(x0,y0,color)    }
```
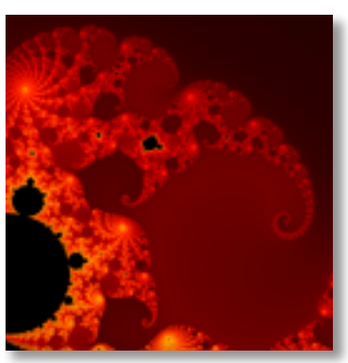
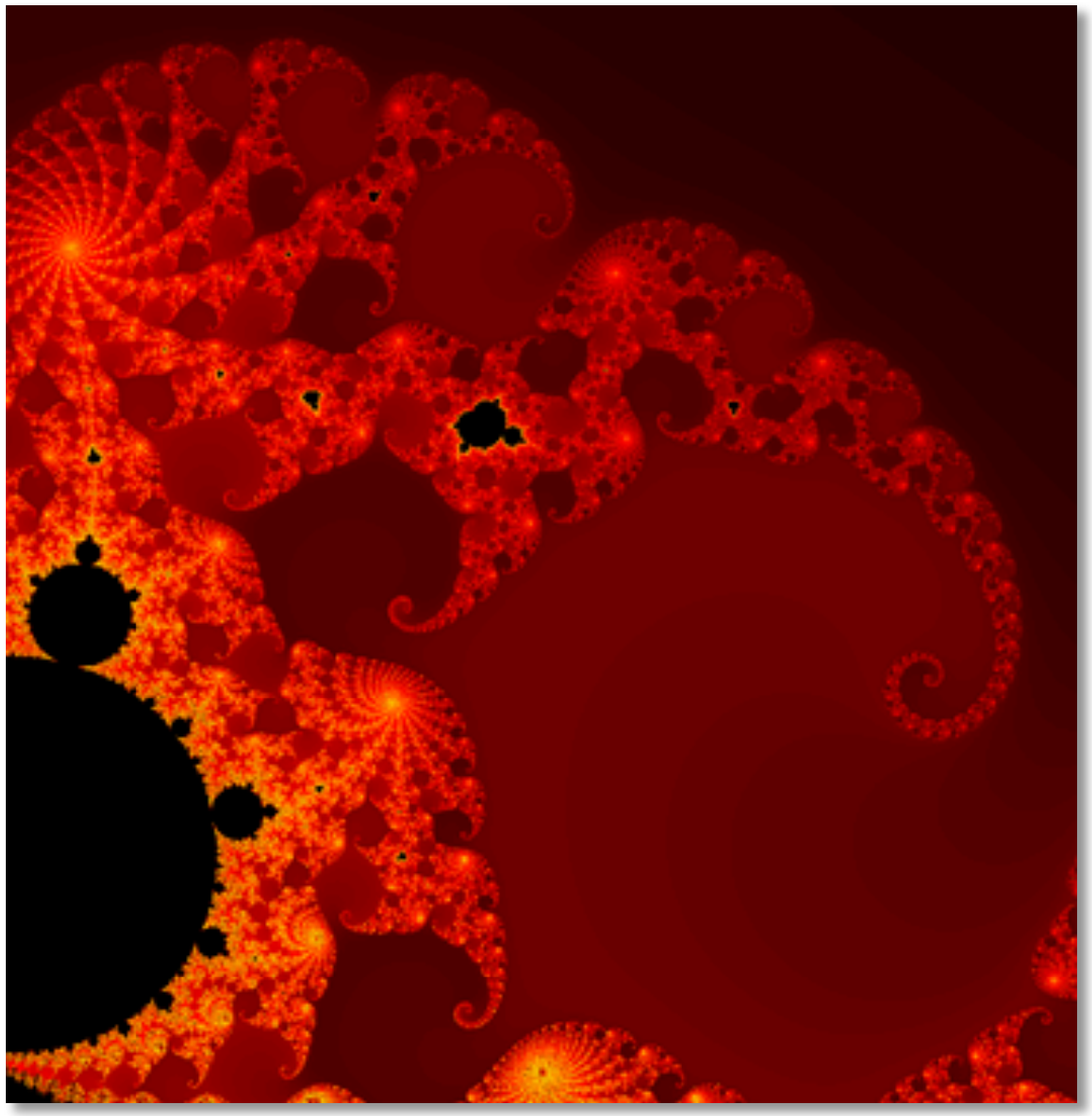



The algorithm reproduces this picture.

What is information?



≈

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Information can be represented in various equivalent ways.

Kolmogorov's notion of information

Definition [Kolmogorov]:

The "information content" of a "message" m is the length (in number of bits) of the shortest program that outputs m.

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- Model-independent: it is independent of the underlying "programming language" (up to an additive constant).
- Incomputable: There is no algorithm that takes as input a message m and outputs its information content.

Shannon’s notion of information

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Definition [Shannon]:

The "information content" $\mathrm{S}(m)$ of a "message" m is equal to the negative logarithm of its probability $\Pr[m]$, i.e., $\mathrm{S}(m) = -\log_2 \Pr[m]$.

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Examples

(1) m: the lottery numbers

(2) m: message whether you have won the lottery

(3) $m = \pi$

(4) m: random bitstring of length n

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Some remarks on Shannon's definition of "information content":

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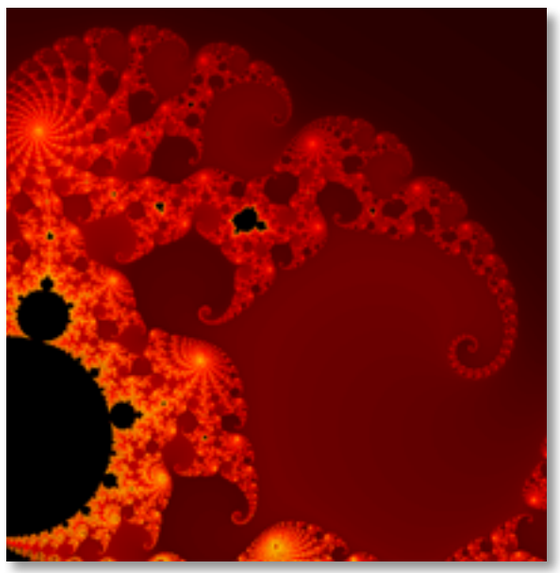
- Probabilistic definition: Requires an underlying probability distribution P_M on the set of messages M.
- Easily computable.
- Widely used in modern information theory (in theory and practice).

The idea of information compression

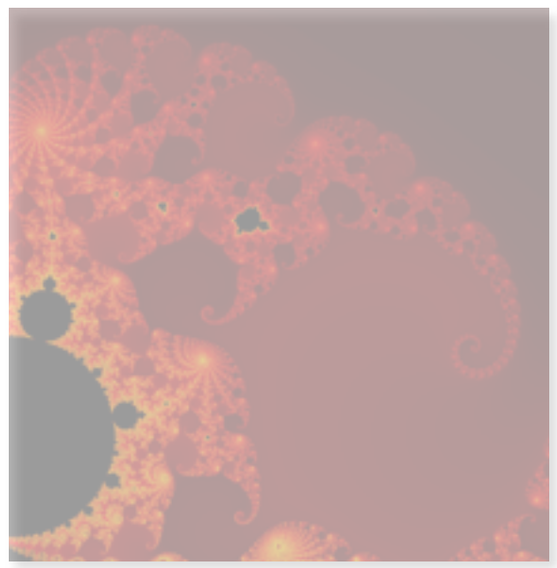
Suppose that we want to transmit the picture over a communication channel with limited capacity.

Sender



1000 bit channel

Receiver



The idea of information compression

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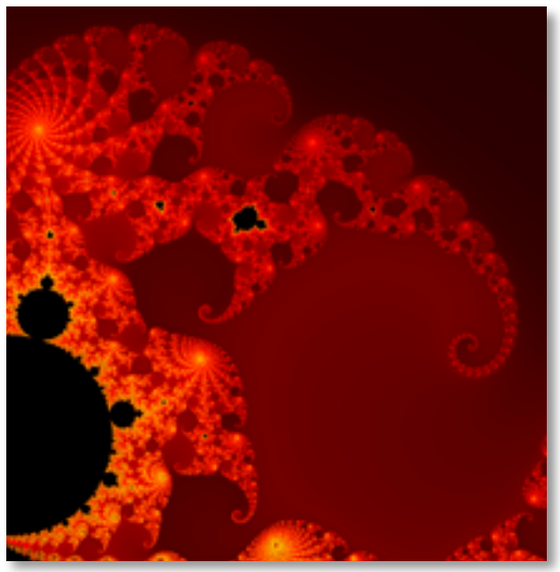

Compression

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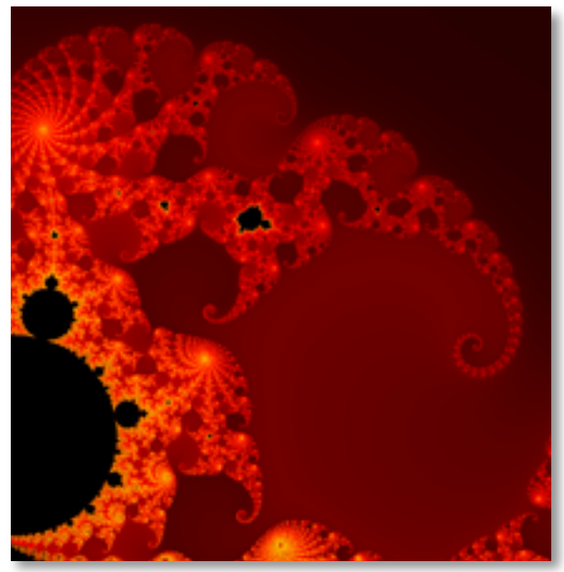

Decoding

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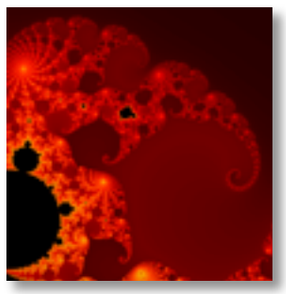
The idea of information compression

M



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1000 bits

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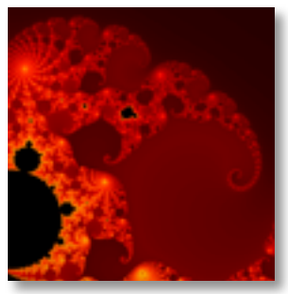
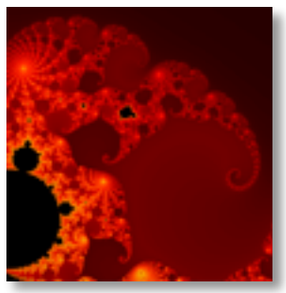


M



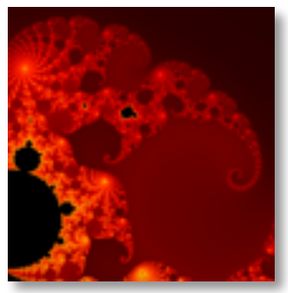
For abcdef
Hijklmnop
Qrstuvw
Xzy xyz

For abcdef
Hijklmnop
Qrstuvw
Xzy xyz

200 bits

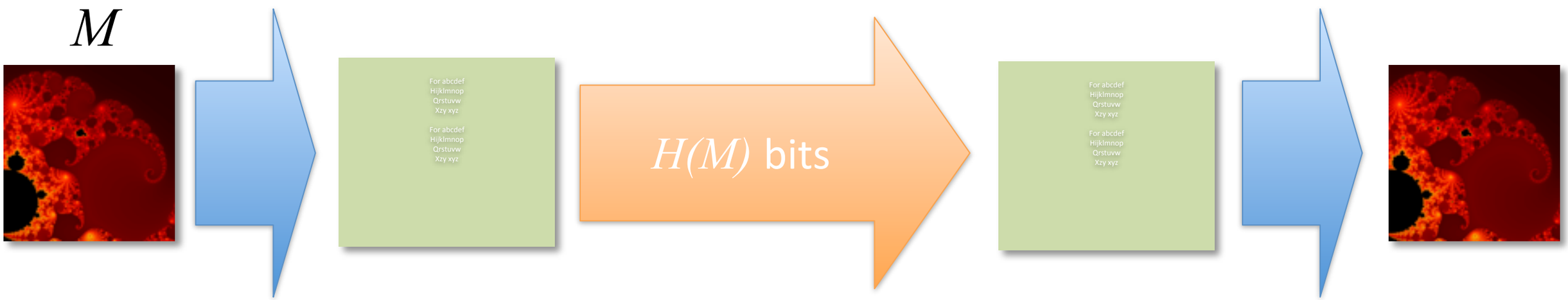
For abcdef
Hijklmnop
Qrstuvw
Xzy xyz

For abcdef
Hijklmnop
Qrstuvw
Xzy xyz



Quantify information content of a message M by the size (in # bits) of the minimal compression.

Shannon entropy and compression

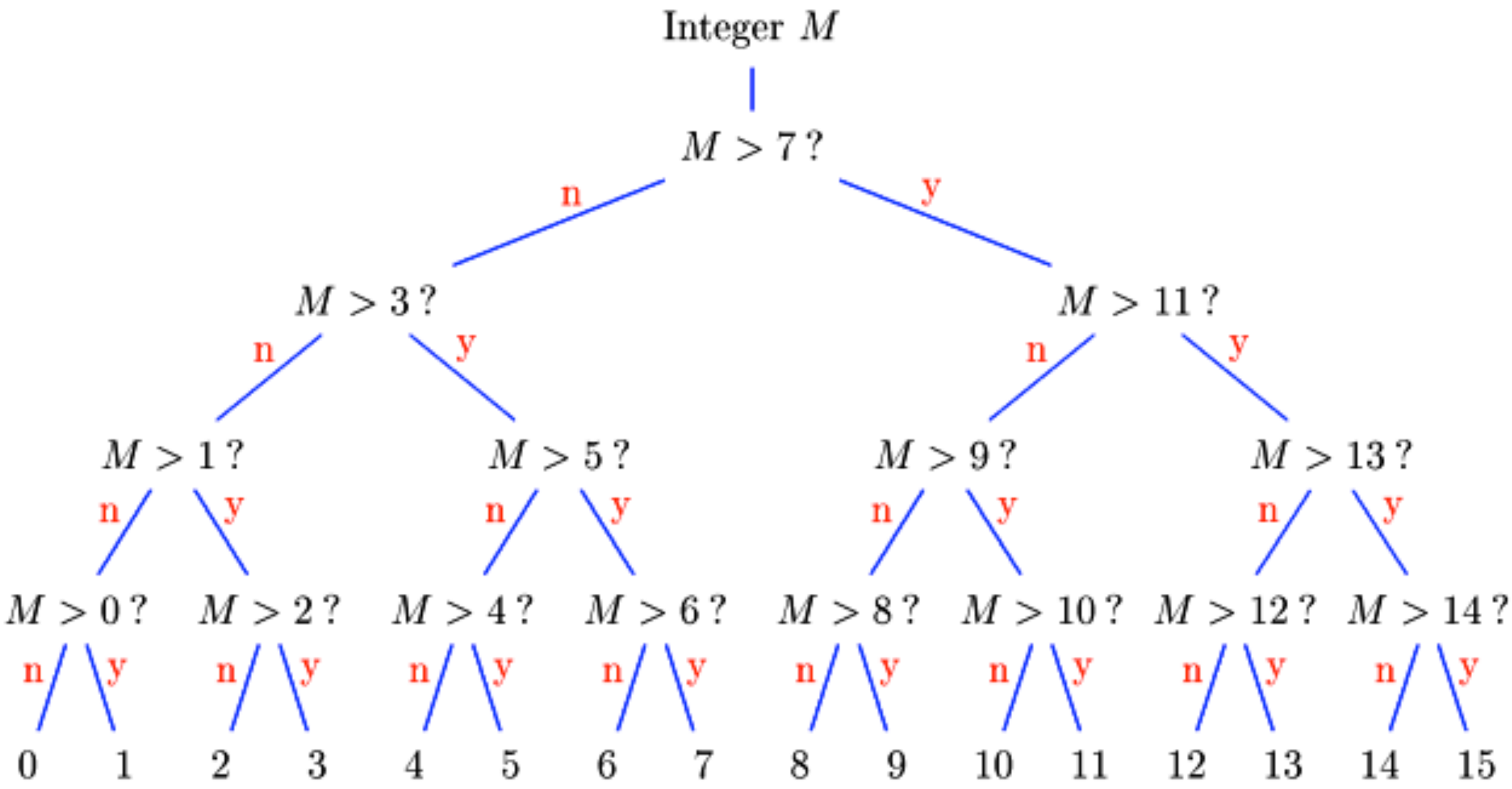


$$H(M) = -\sum_m p_m \log_2 p_m$$

Theorem [Shannon 1948]

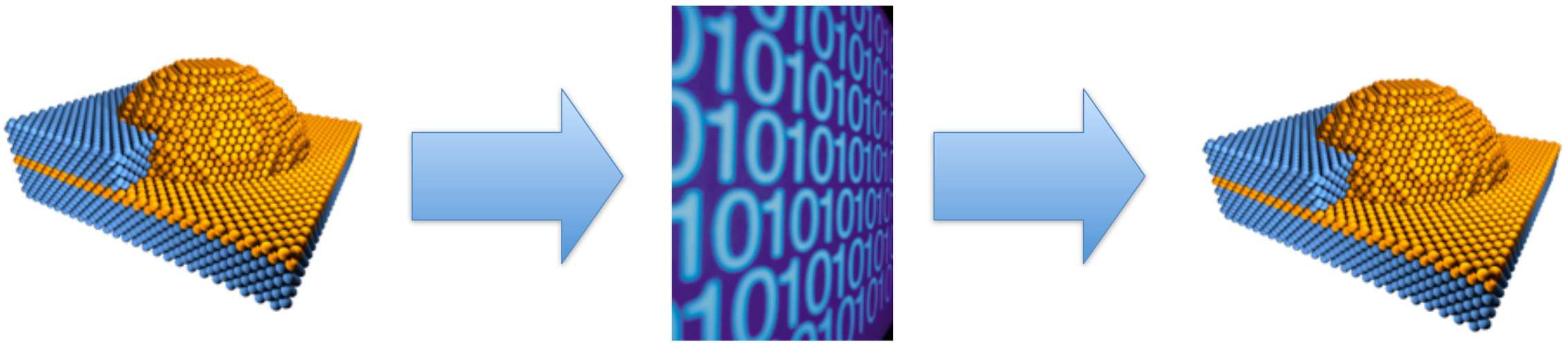
The Shannon entropy $H(M)$ corresponds to the minimum (average) compression length of M.

Compression according to Shannon

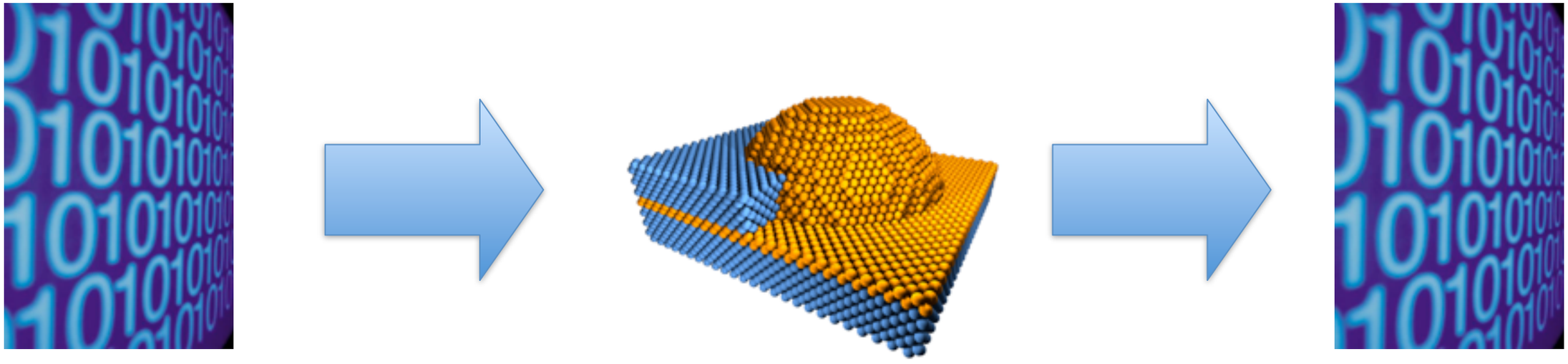


Operational relevance

- Given a physical object, how much information is required to describe it?



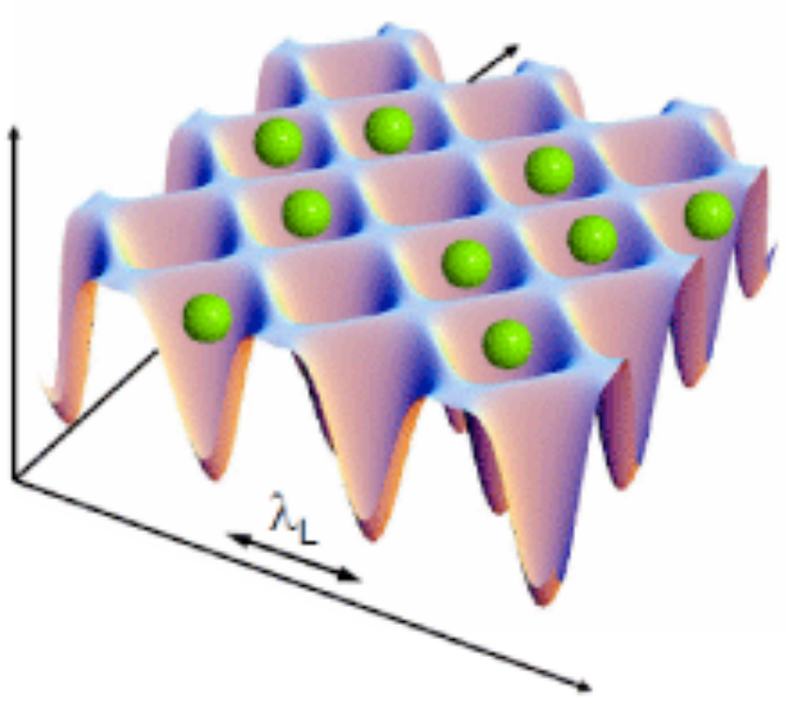
- Given a physical device, what is the maximum amount of information that can be stored reliably?



Why are such questions interesting?

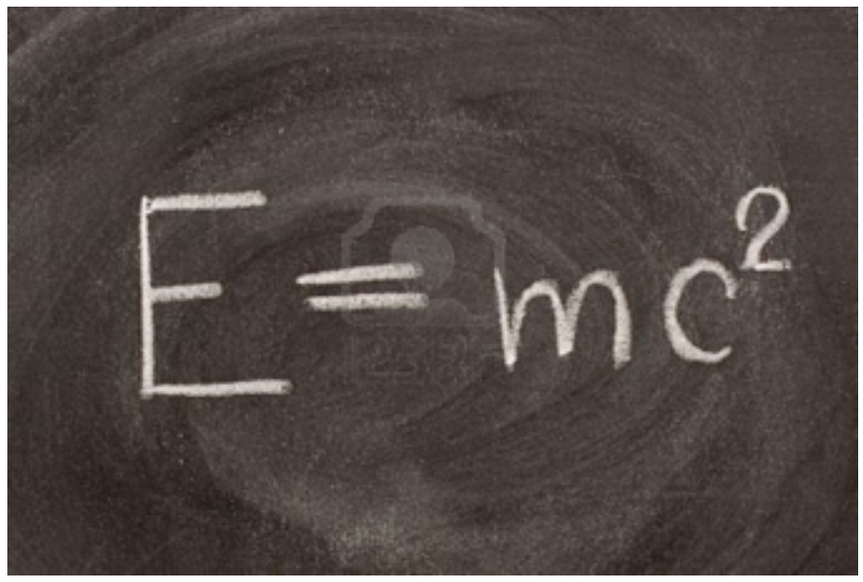
- Technological applications (information processing and transmission)

- Simulatability of physical systems



Why are such questions interesting? (cont'd)

- Development of physical theories



- Used in other areas of science (biology, finances, linguistics, …)

Linking Quantum Physics and Information Theory

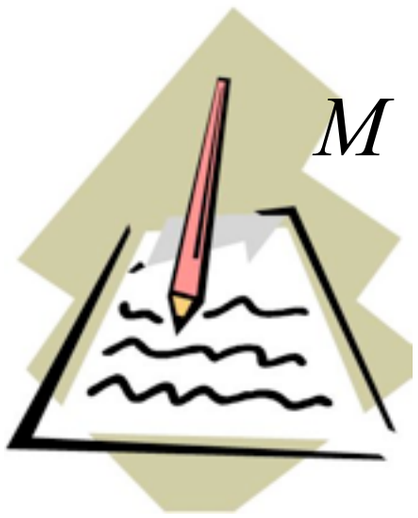
Information is physical

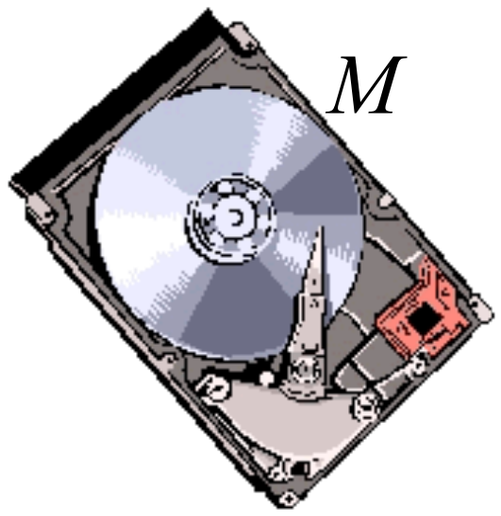
- Rolf Landauer:
  “information is always represented by the state of a physical system”.

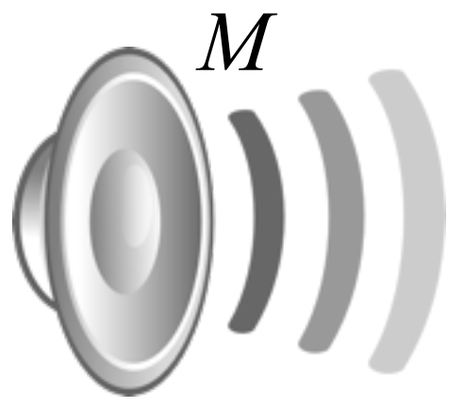
- If information is represented by a quantum system then it is by definition “quantum information”.

Independence of information carriers

According to Shannon's theory, information is independent of the "physical information carriers".







Representation of a message M

Each value $M=m$ is represented by a different physical state of the system.

Independence of information carriers

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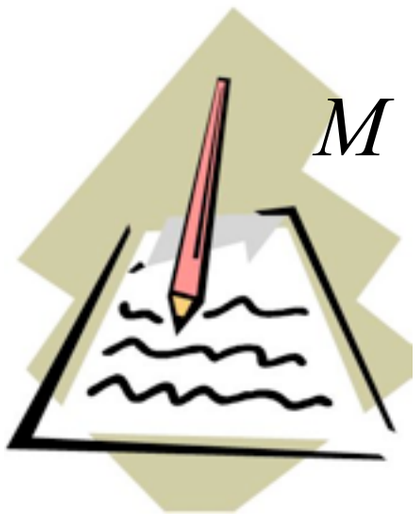

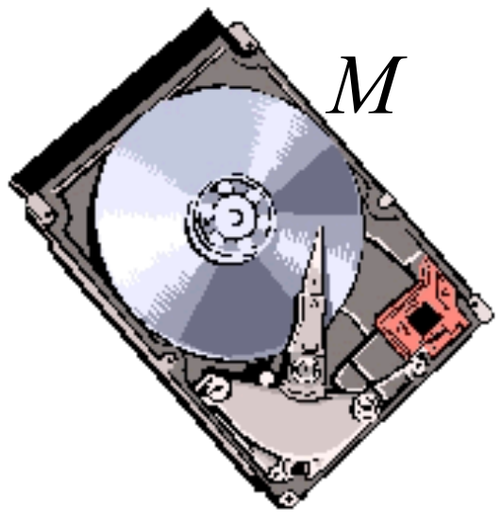

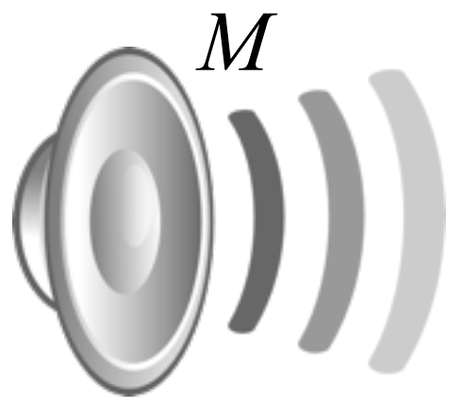

Representation of a bit

Each value of a bit ("0" or "1") is represented by two different (perfectly distinguishable) states of the information carrier.

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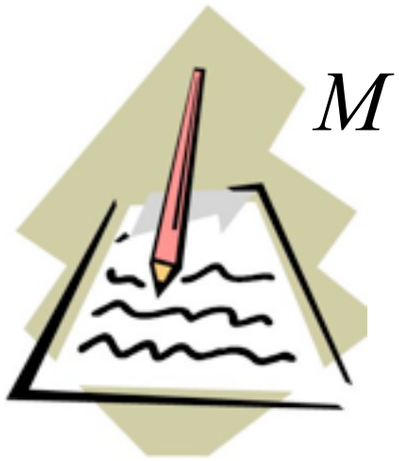

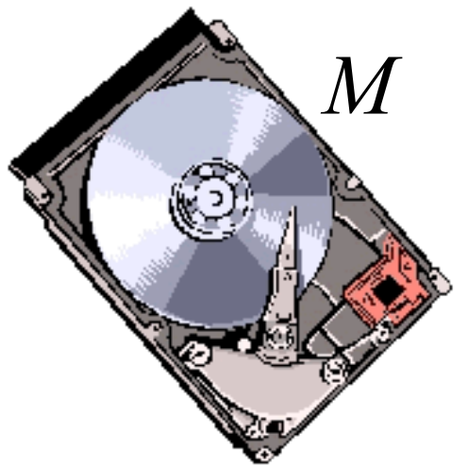

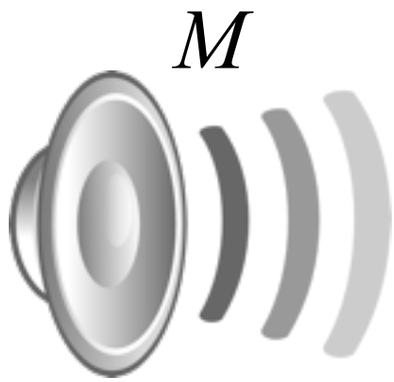

Independence of information carriers?

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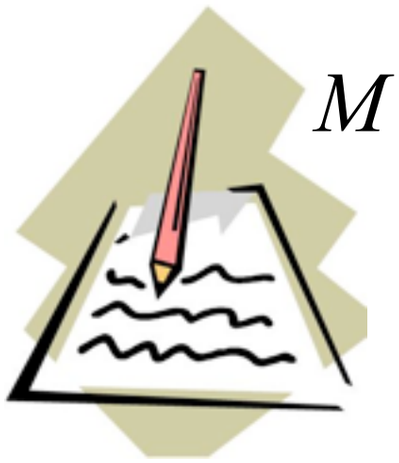

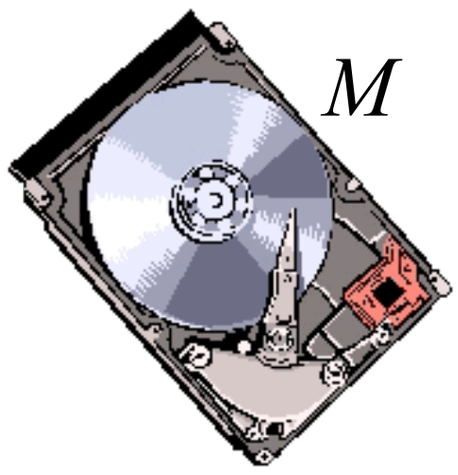

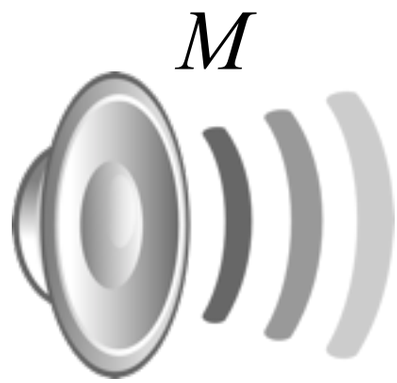

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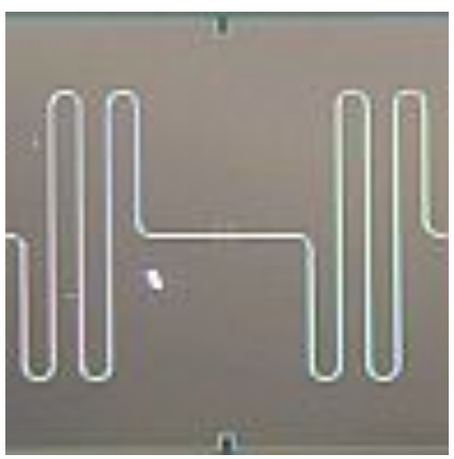
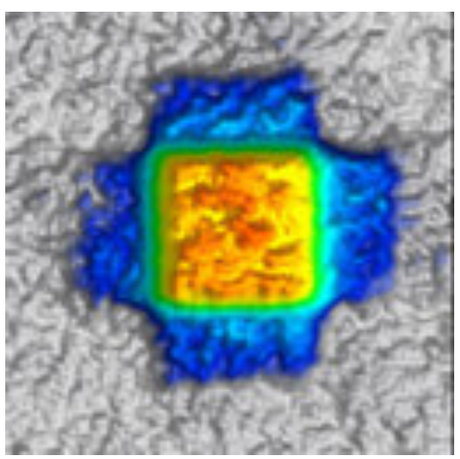
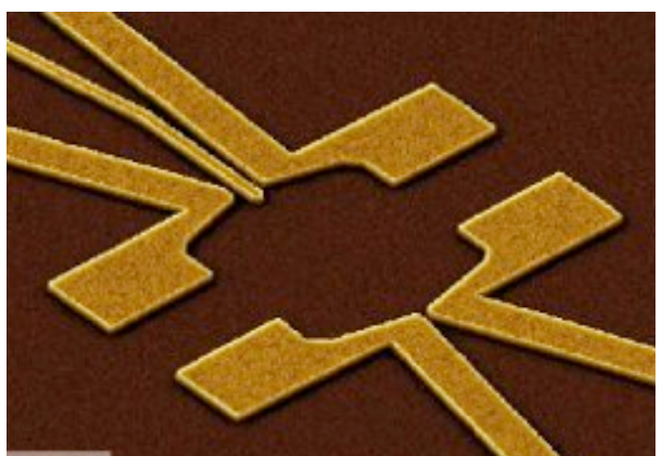






[Photos: QSIT]

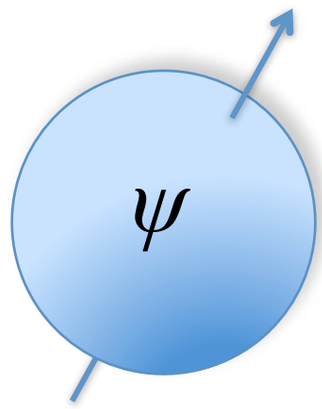
Classical information

Classically, information may always be represented as a sequence of binary numbers (the bits).

0 - 1

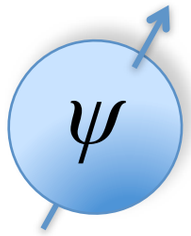
Quantum information

Quantum information is represented as the state of a quantum system, such as the polarization degree of freedom of a photon.



Qubit

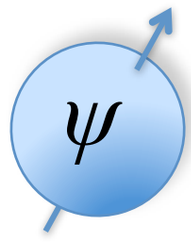
Although the smallest possible unit of quantum information is a that represented on a two-level system (a qubit), there is a continuum of states.



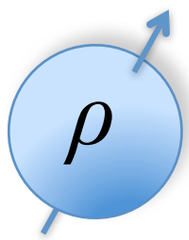
The state of a qubit is generally represented as a vector in C^2.

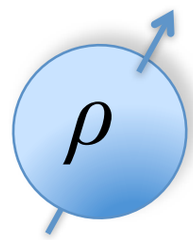
Qubits

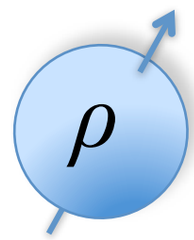
The state of a *single* system is specified by a 2-dimensional vector $\psi \in C^2$



The state of n qubits is specified by a 2^n-dimensional vector $\psi \in C^{2^n}$

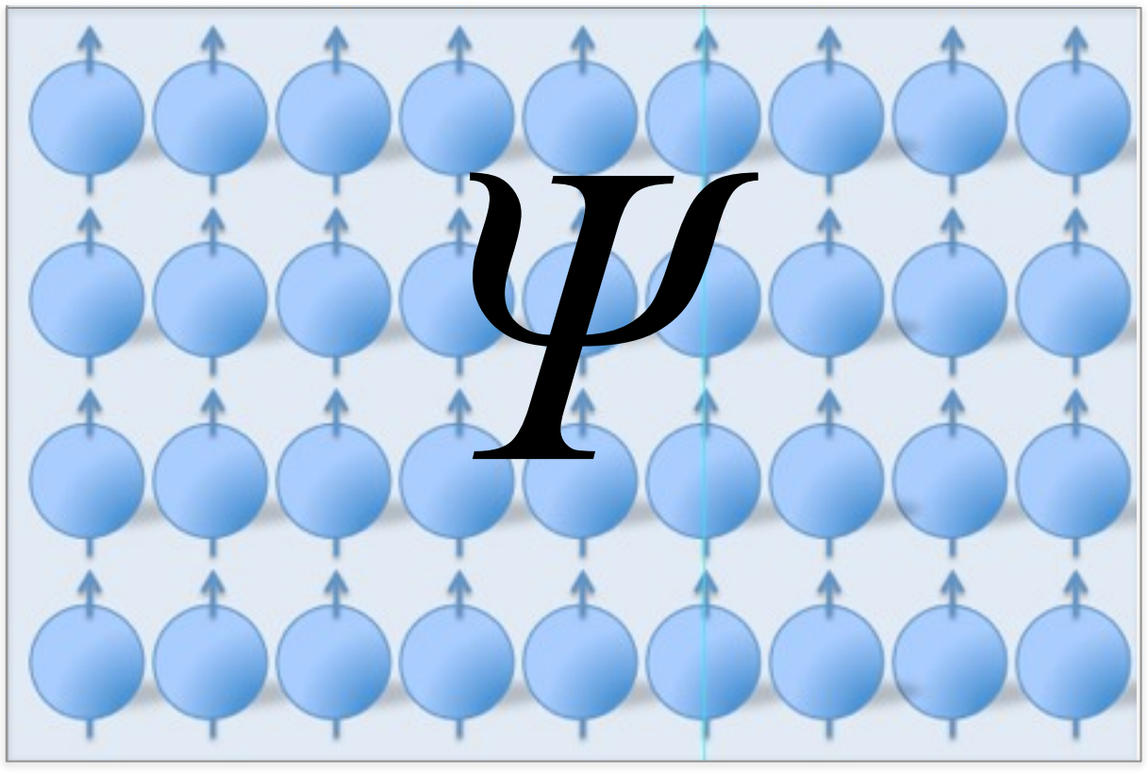






Comparison: bits vs qubits

36 qubits



2^{36} coordinates

> 100 GByte

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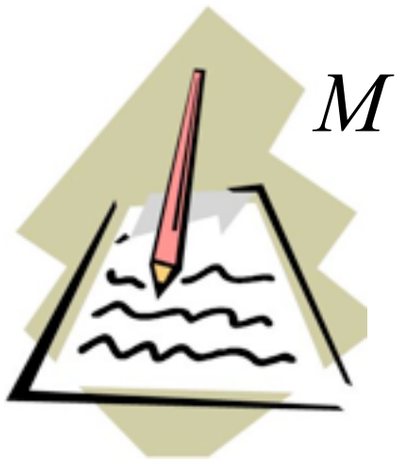

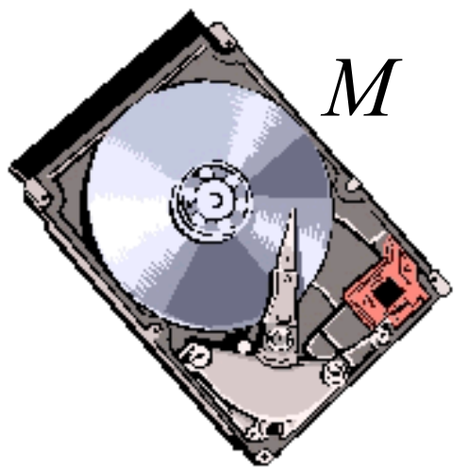

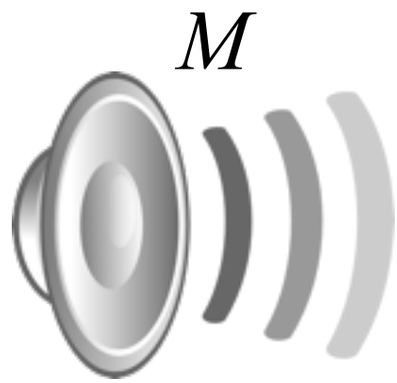

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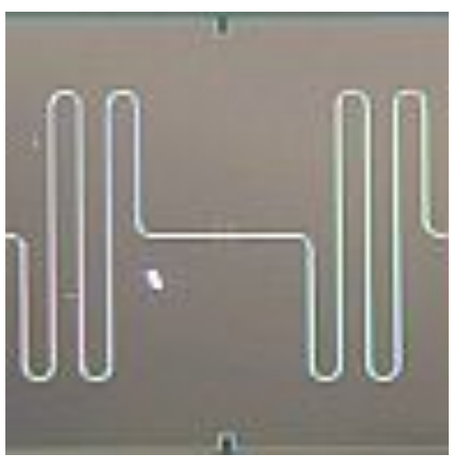
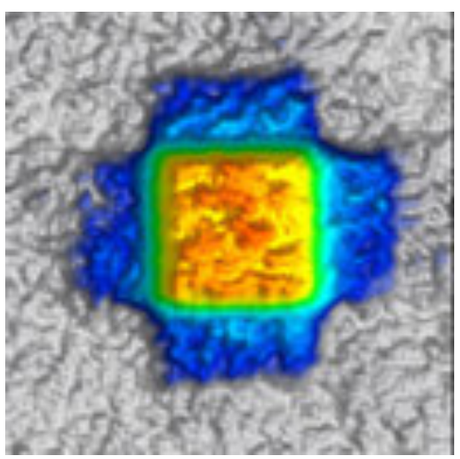
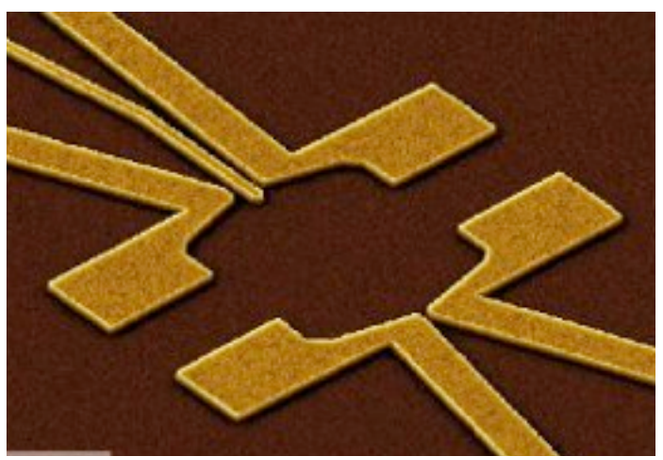
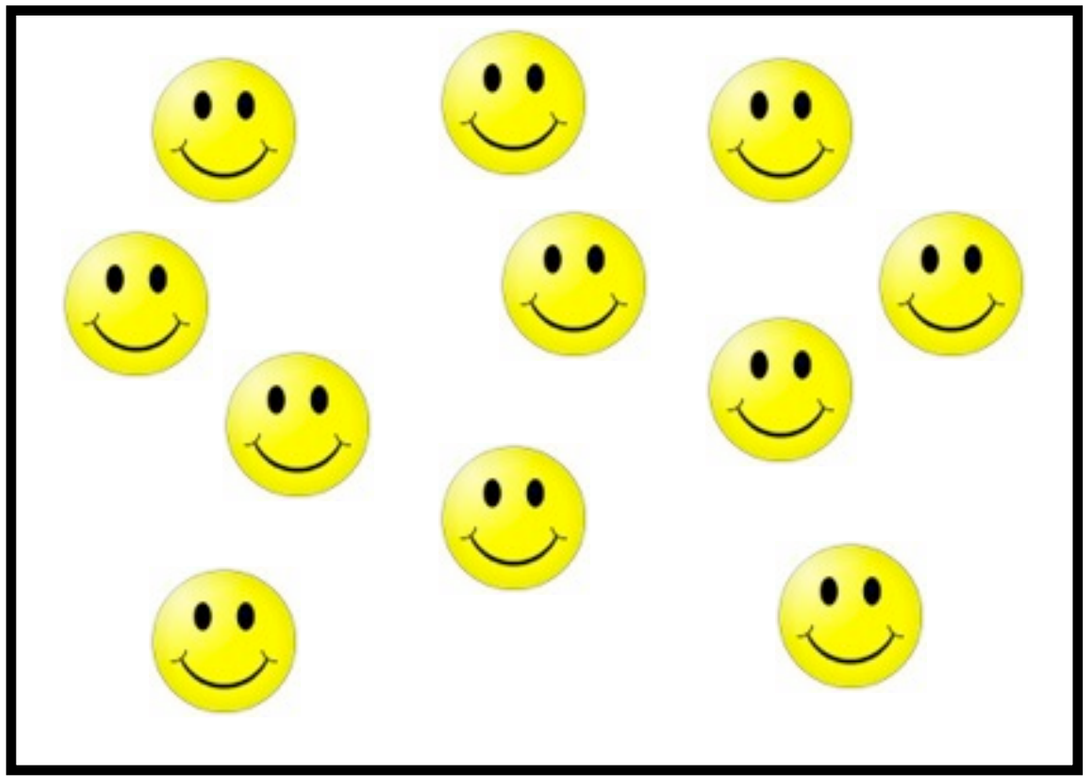


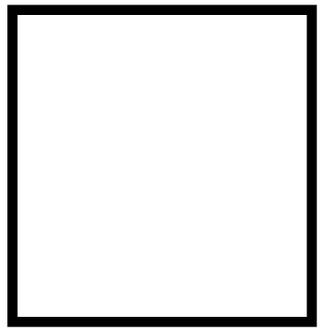


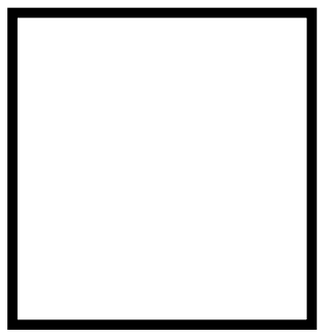


[Photos: QSIT]

Toy example







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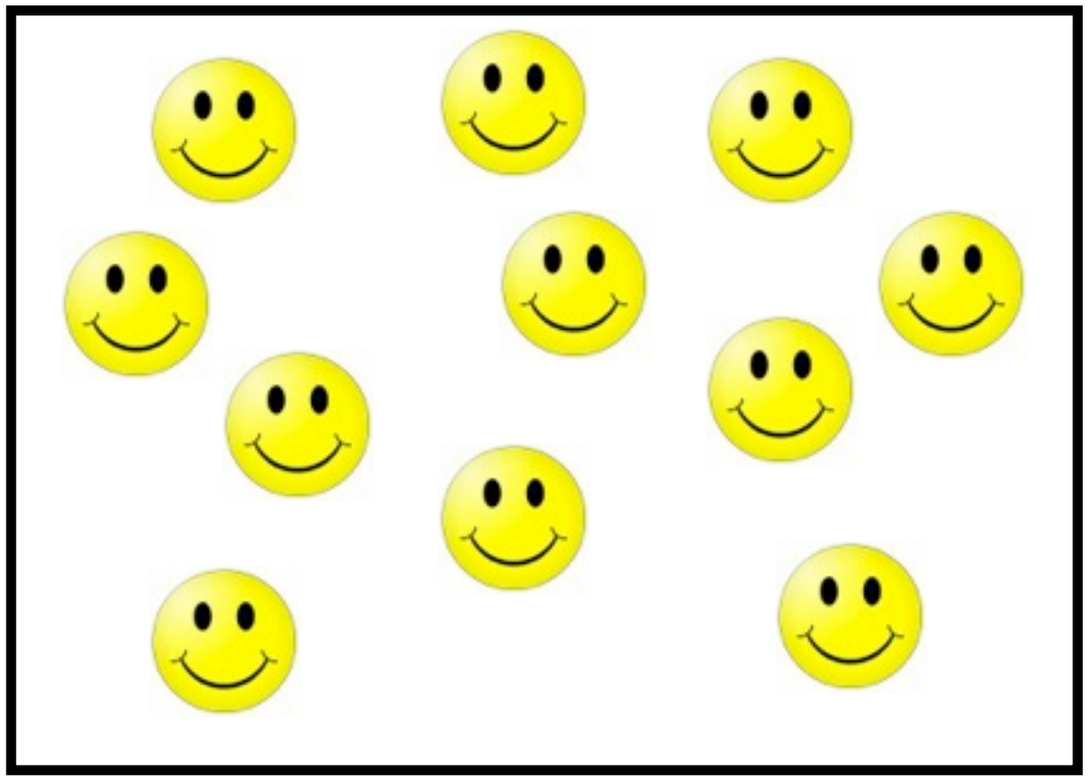

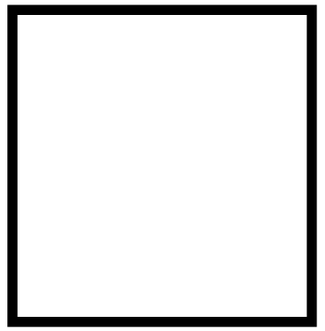

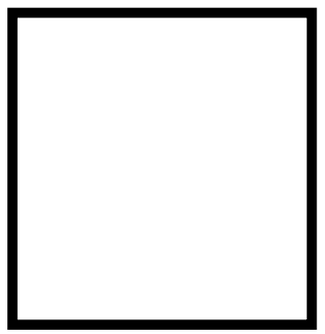

1. N collaborating players sitting in a room

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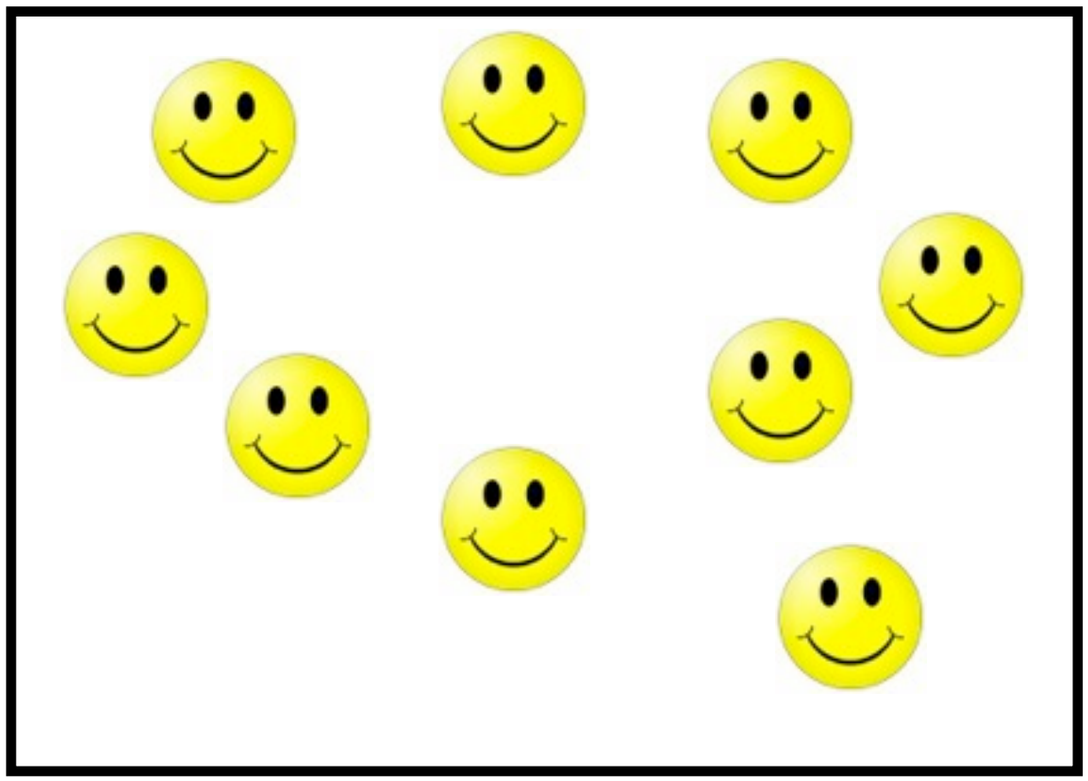

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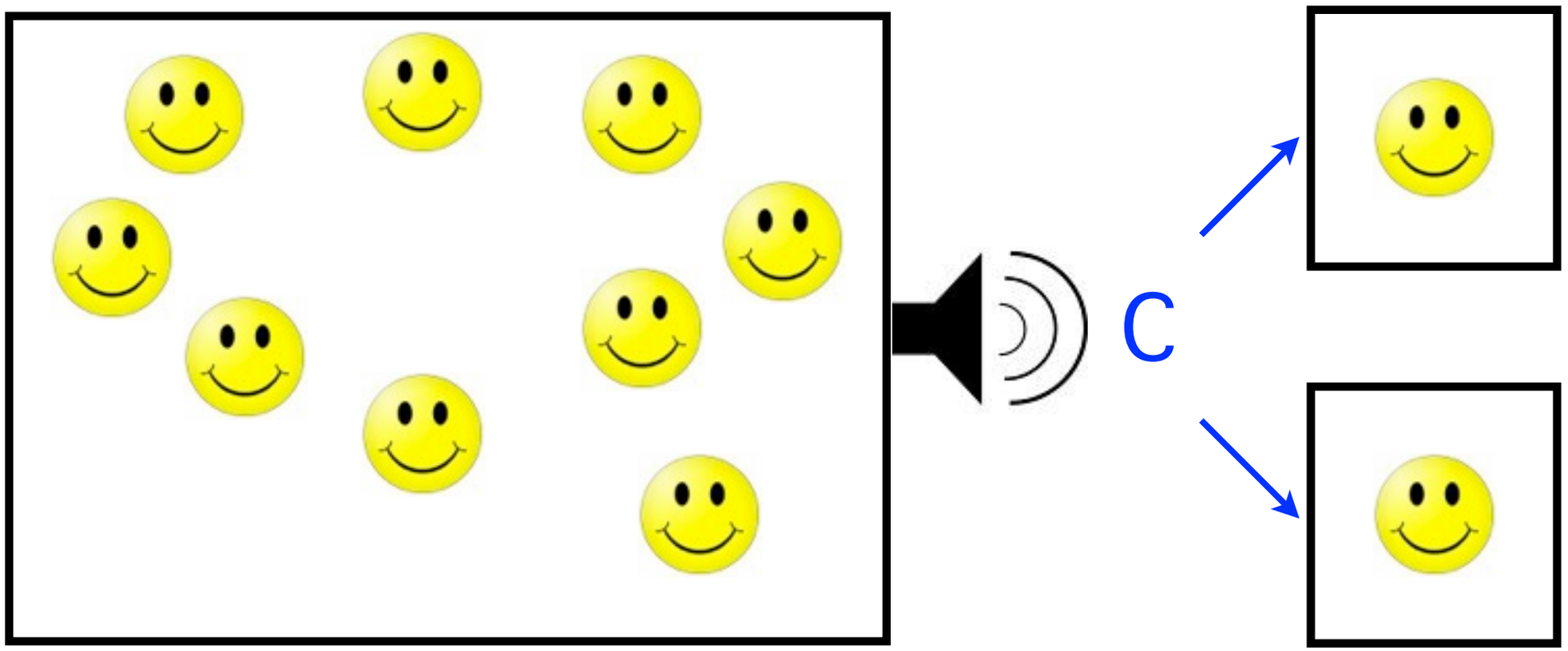

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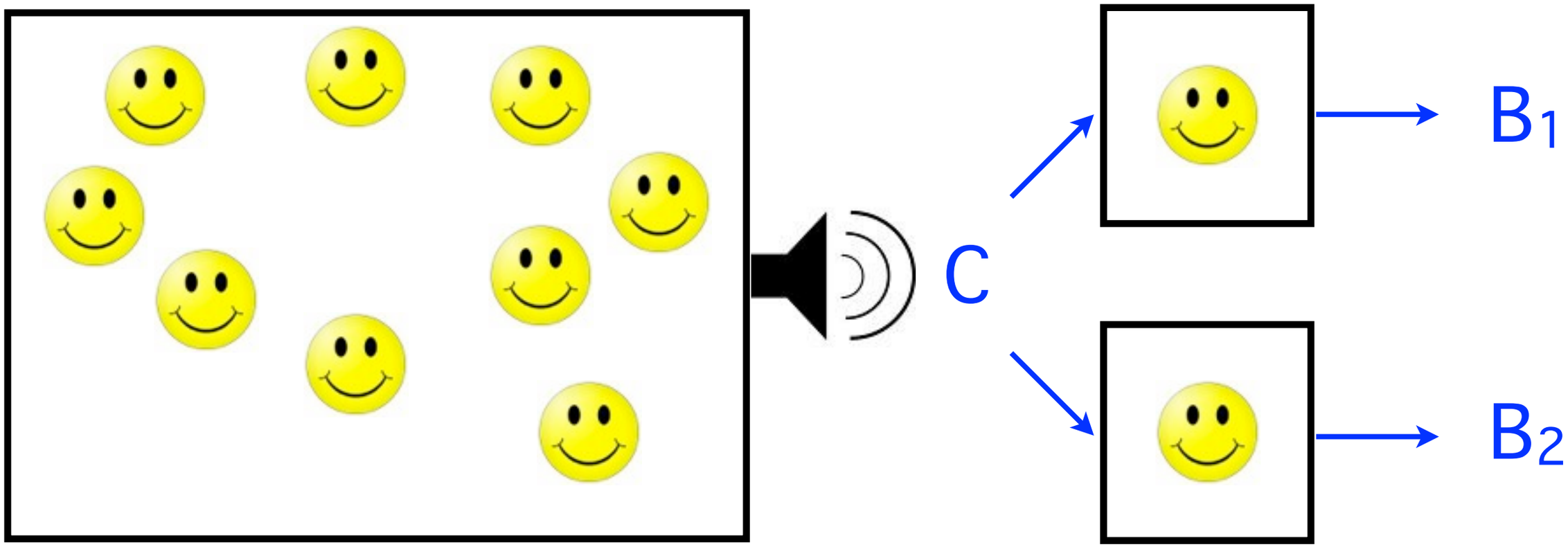

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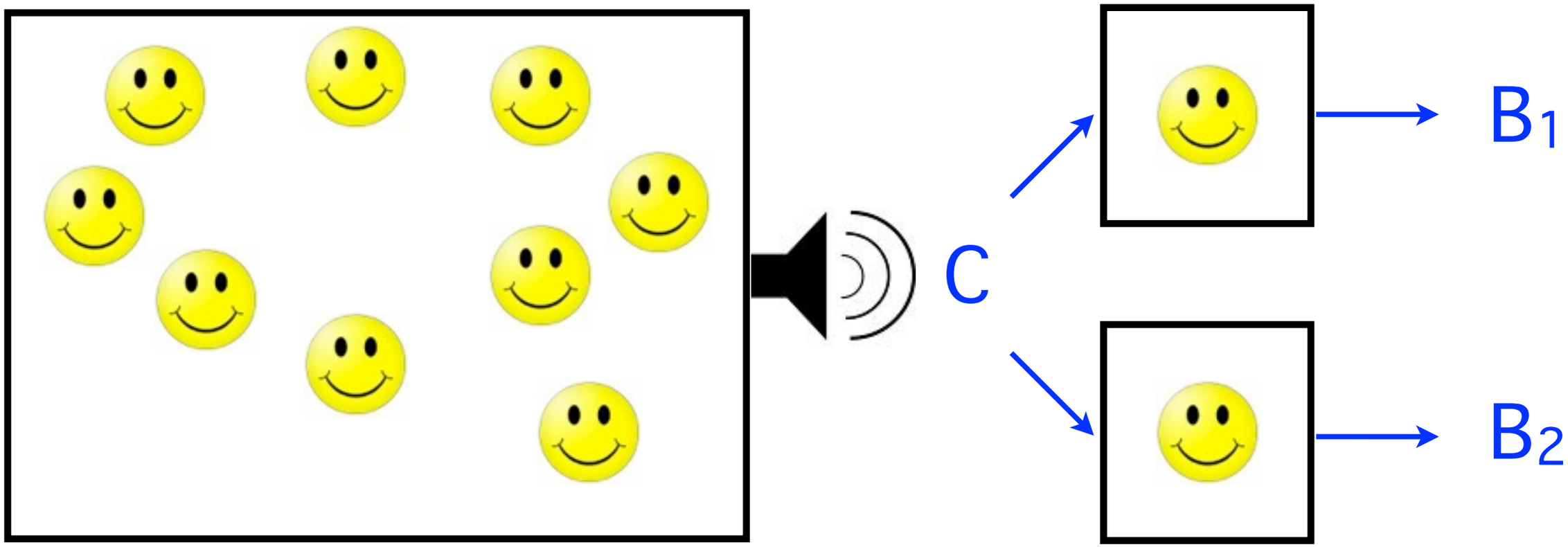

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Game is won if $B_1 \neq B_2$.

Maximum winning probability

Strategies	B=0	B=1	B=C	B=1-C

Maximum winning probability

- Each player may choose one of the following four strategies (in case he is selected).

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- Hence, the game is lost with probability (at least) 1/4.

What did we prove?

Claim

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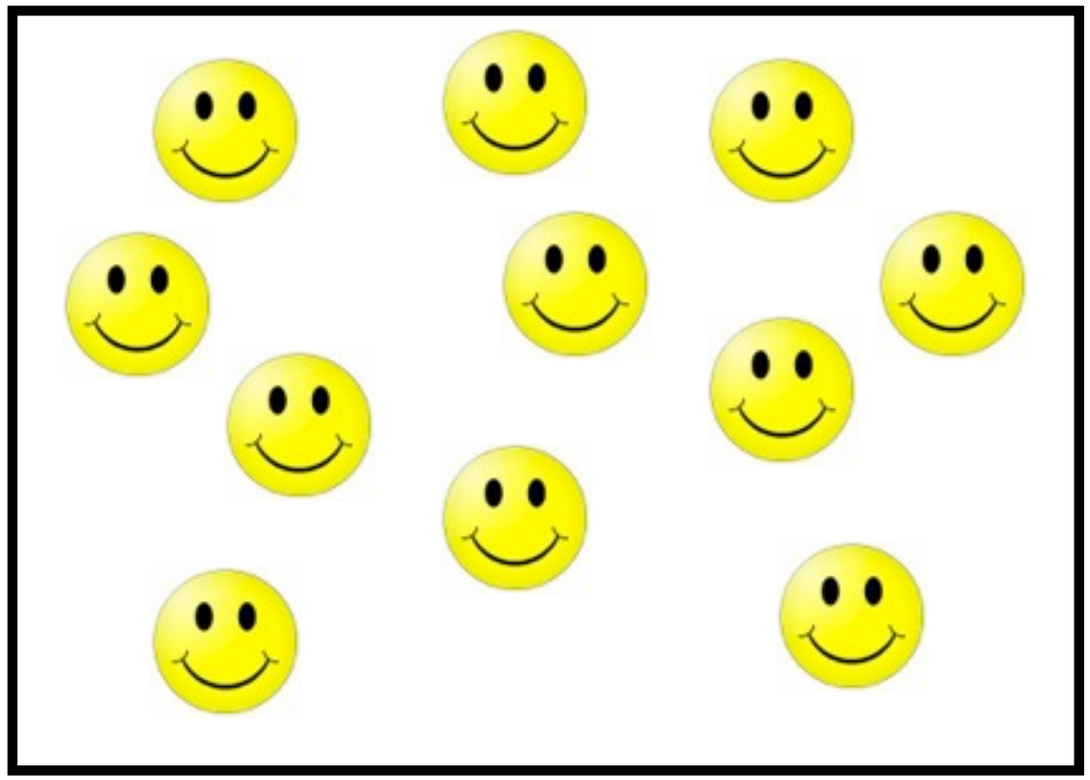
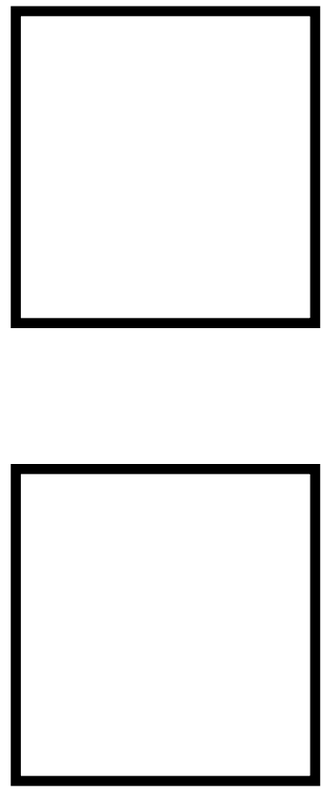
Additional implicit assumption

All information is encoded and processed classically.

Quantum strategies are stronger





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Note: all communication during the game is still purely classical.

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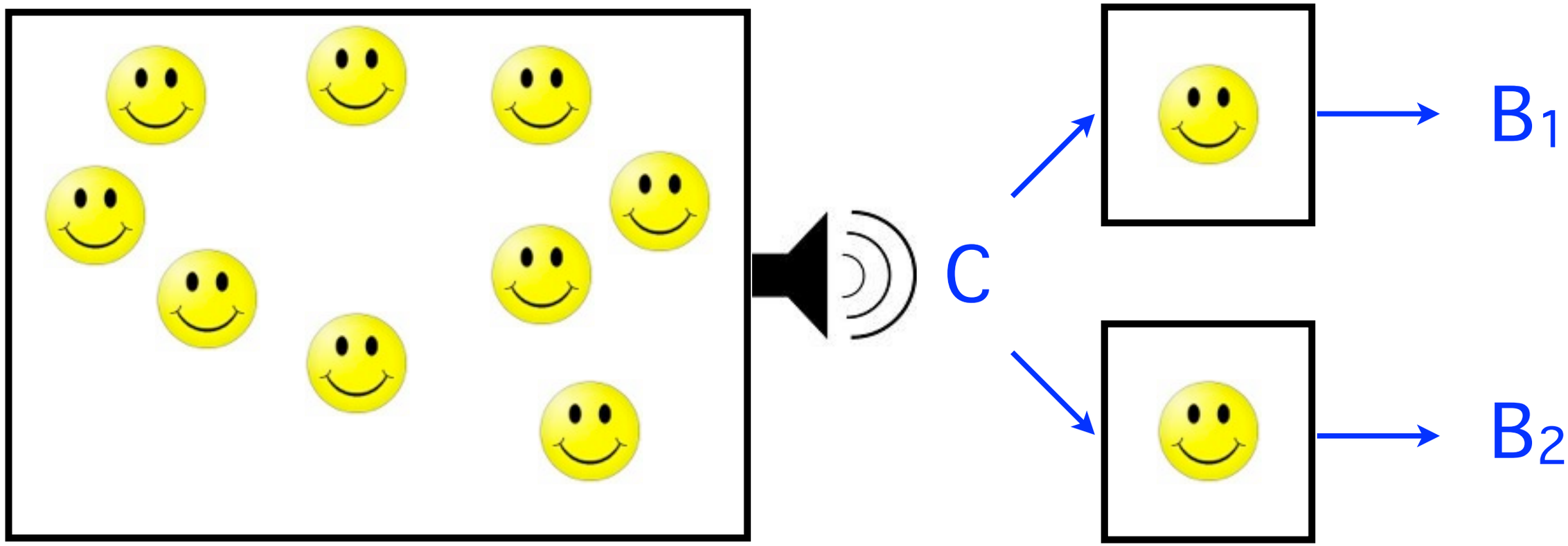

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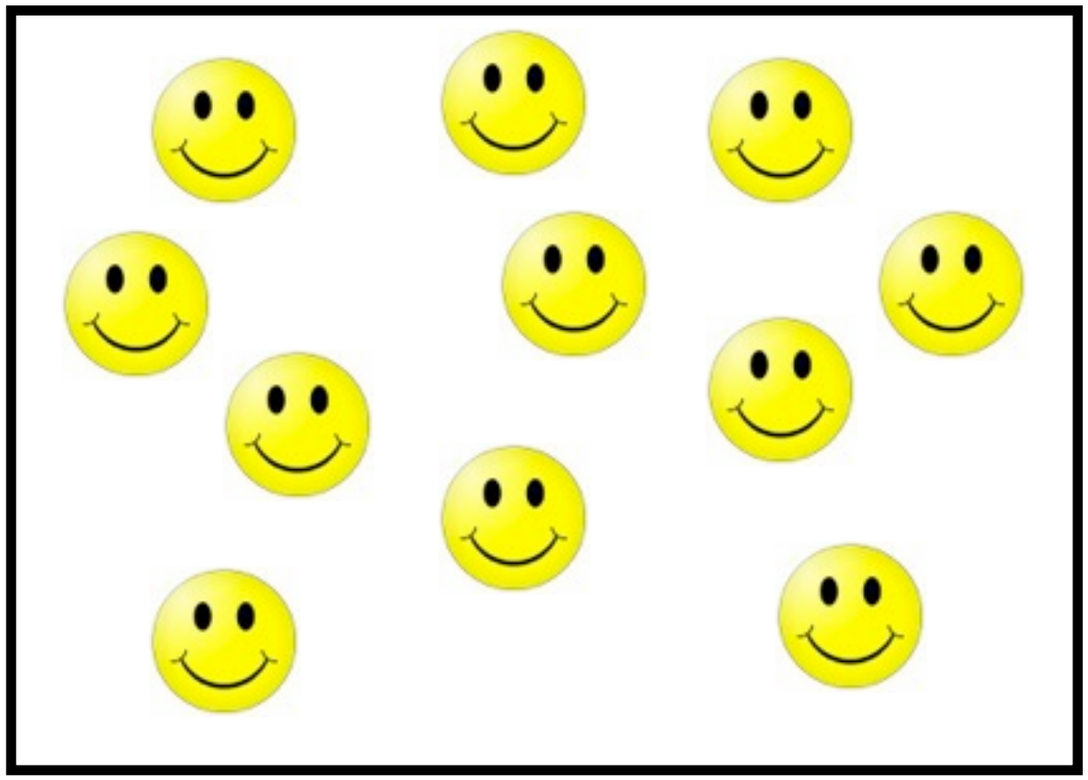

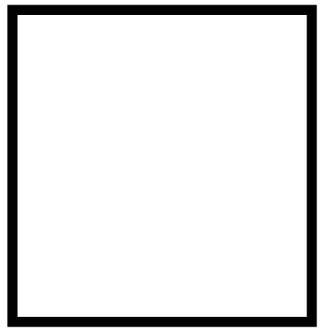

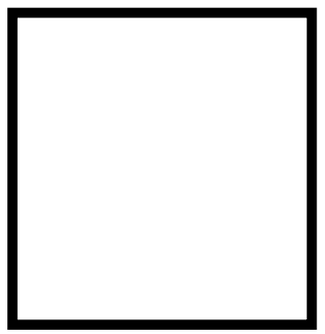

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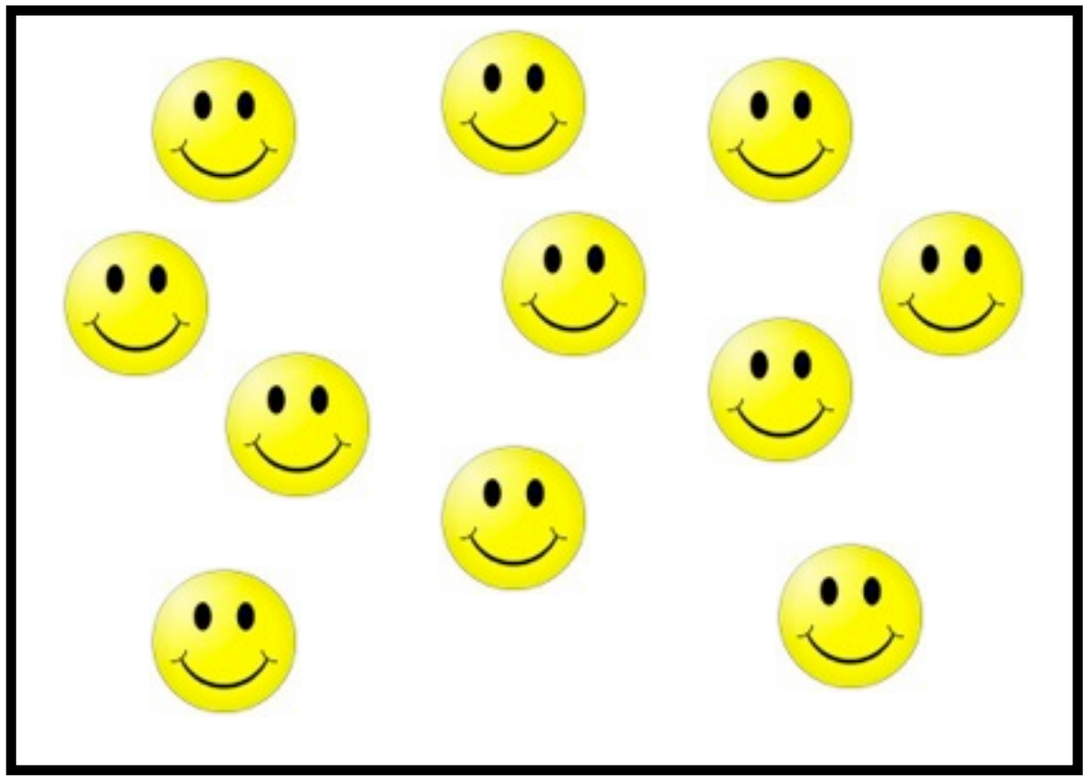

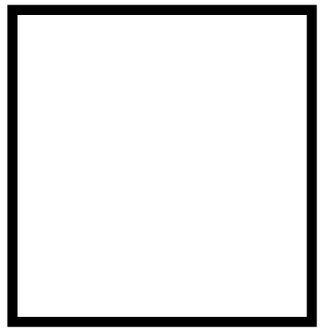

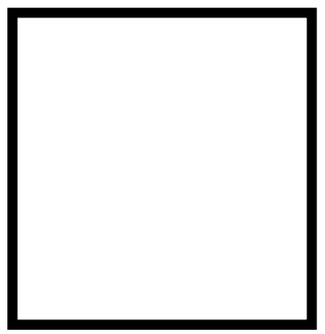

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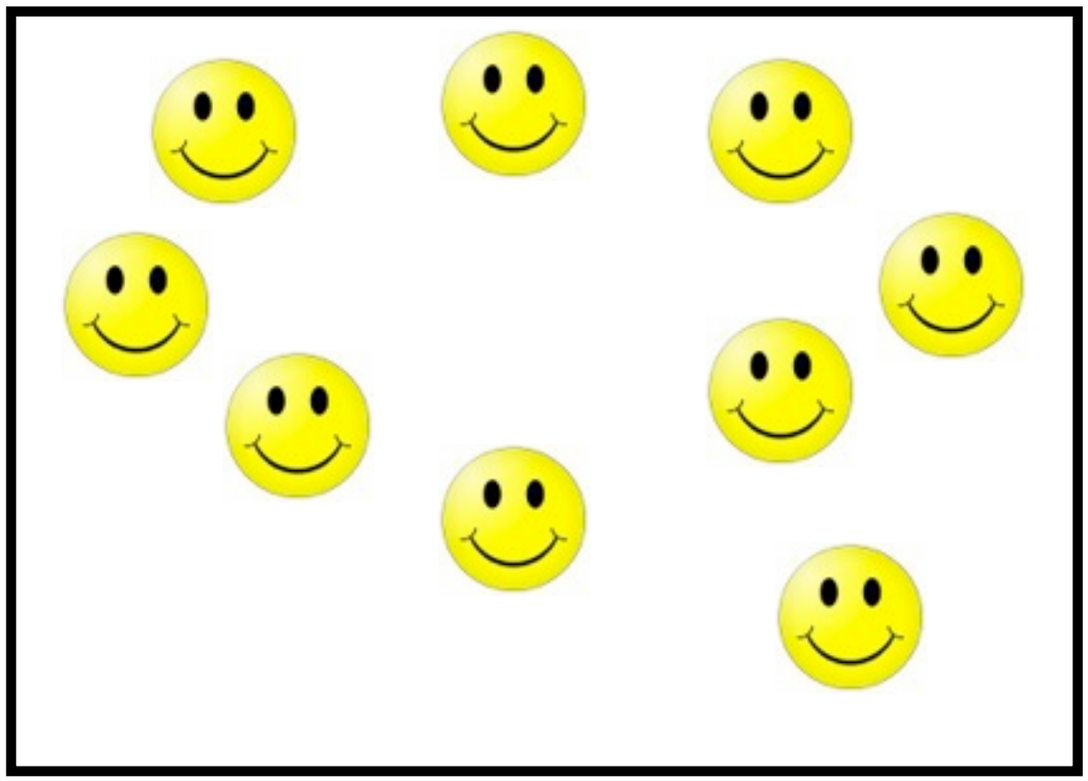
Quantum strategy



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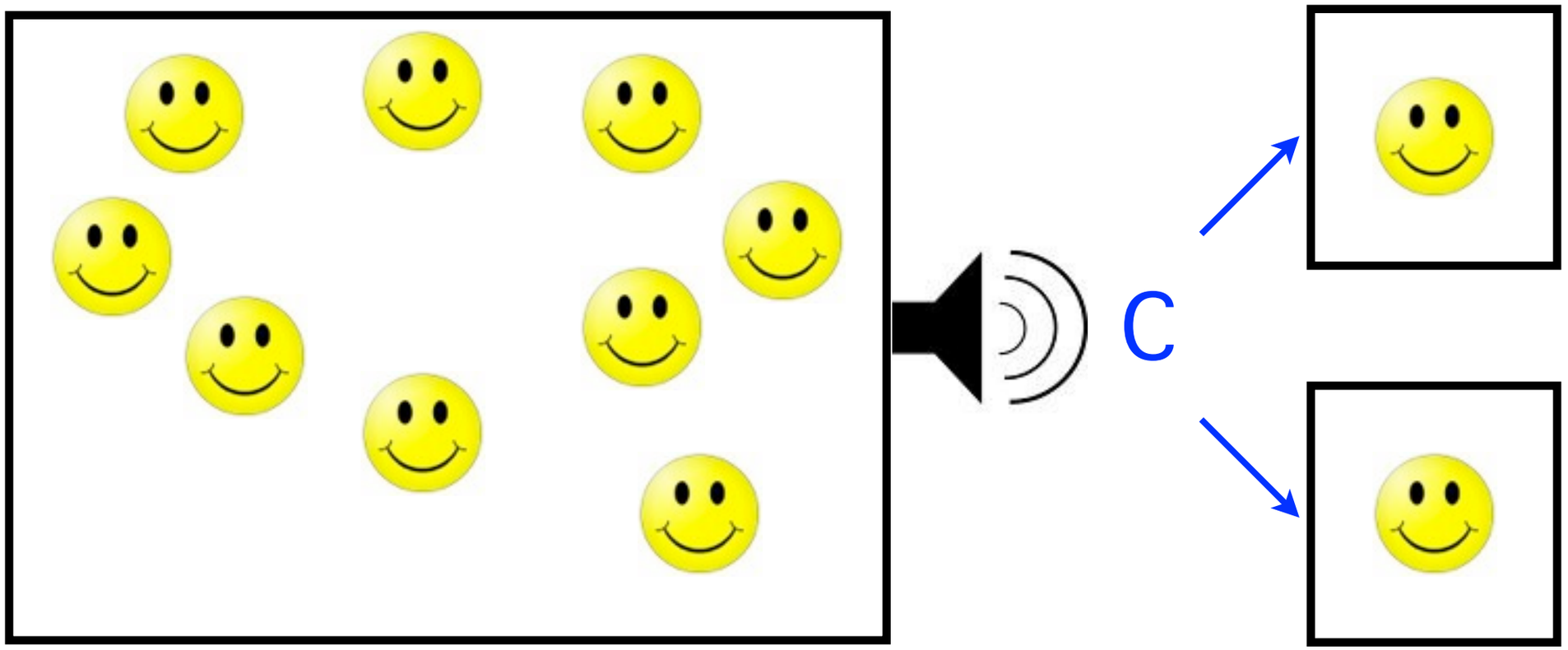

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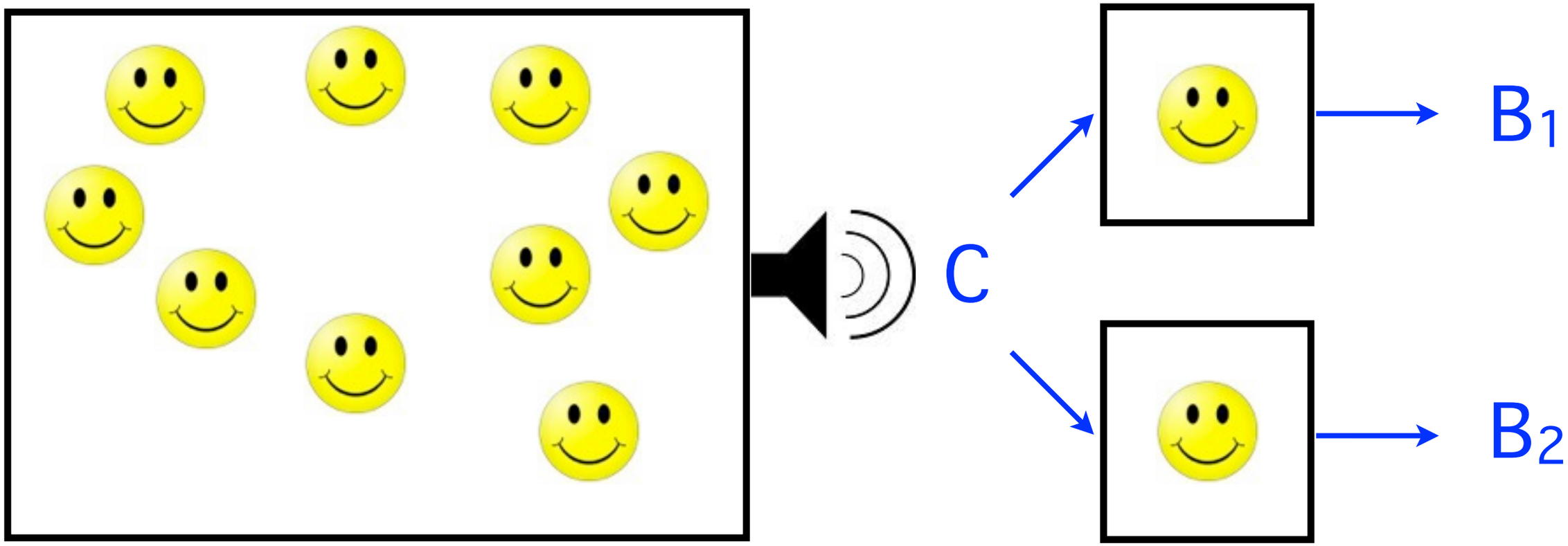

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4. separated players determine B_1 and B_2 by measuring in either the diagonal or the circular basis, depending on C.

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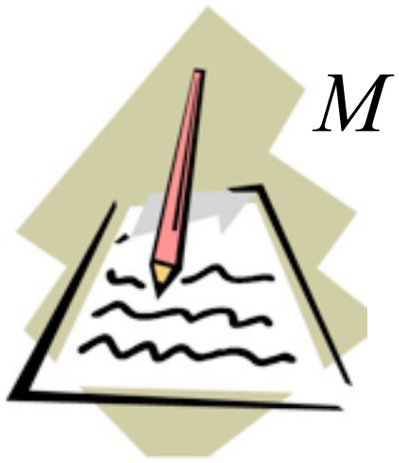
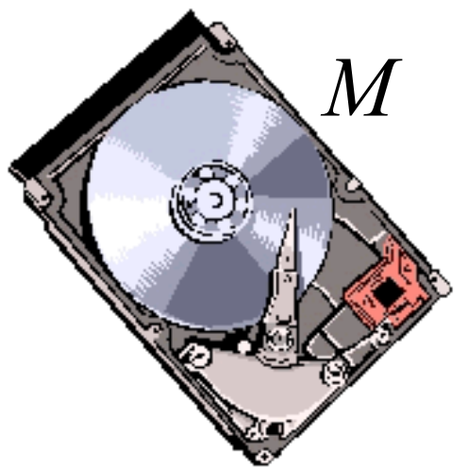
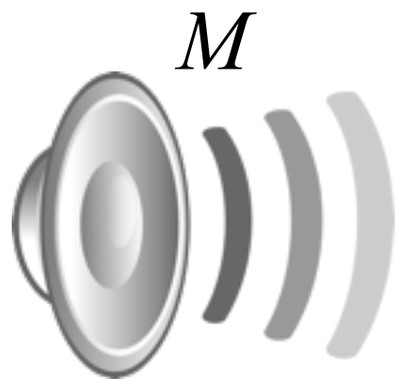
It is, in general, unavoidable to take into account quantum effects.

Independence of information carriers?

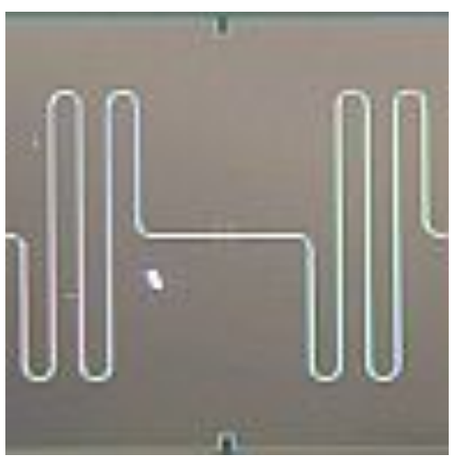
According to Shannon's theory, information is independent of the "physical information carriers".

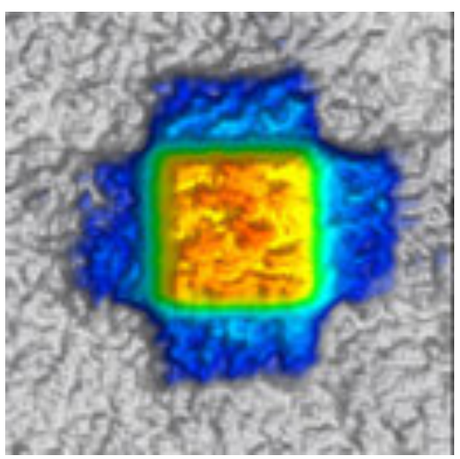


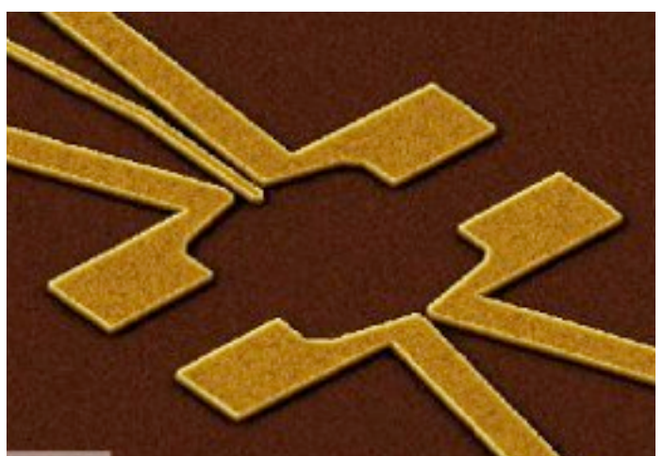




But does this paradigm also apply to information stored in quantum devices? **No!**







[Photos: QSIT]

Shannon's "impossibility result"

C = enc(M,S)

C

M = dec(C,S)

Theorem

For information-theoretically secure encryption, the key S needs to be at least as long as the message M.

In particular, *One-Time-Pad encryption* is optimal.

Proof of Shannon's theorem

Let M be a uniformly distributed n-bit message,
S a secret key, and C the ciphertext.

Requirements

- H(M | SC) = 0, since M is determined by S, C.
- H(M | C) = H(M) = n, since M is indep. of C.

Hence

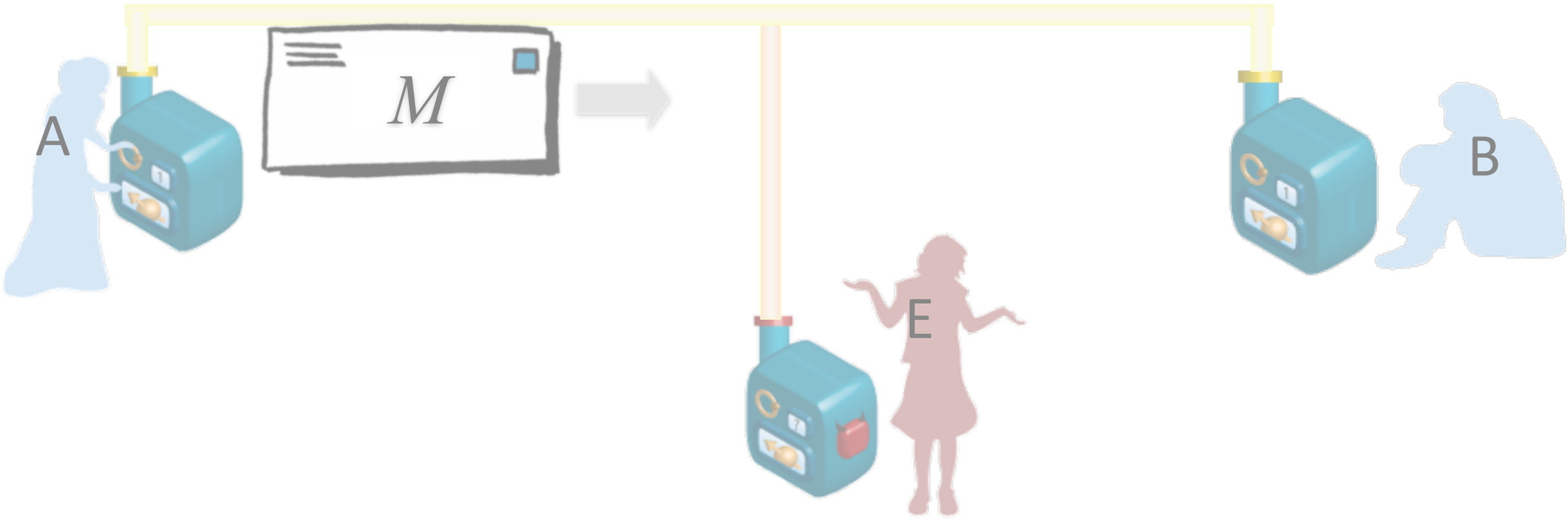
H(S) ≥ I(M : S | C) = H(M | C) - H(M | SC) = n.

Shannon's impossibility result

Theorem [Shannon, 1949]

Two parties connected via an insecure channel cannot exchange any messages secretly

(even if they have methods for authentication).



Bennett and Brassard's possibility result

C.H. Bennett

[Photo: ETH Zurich]

G. Brassard

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If information cannot be cloned, then it can also not be stolen (without leaving traces).

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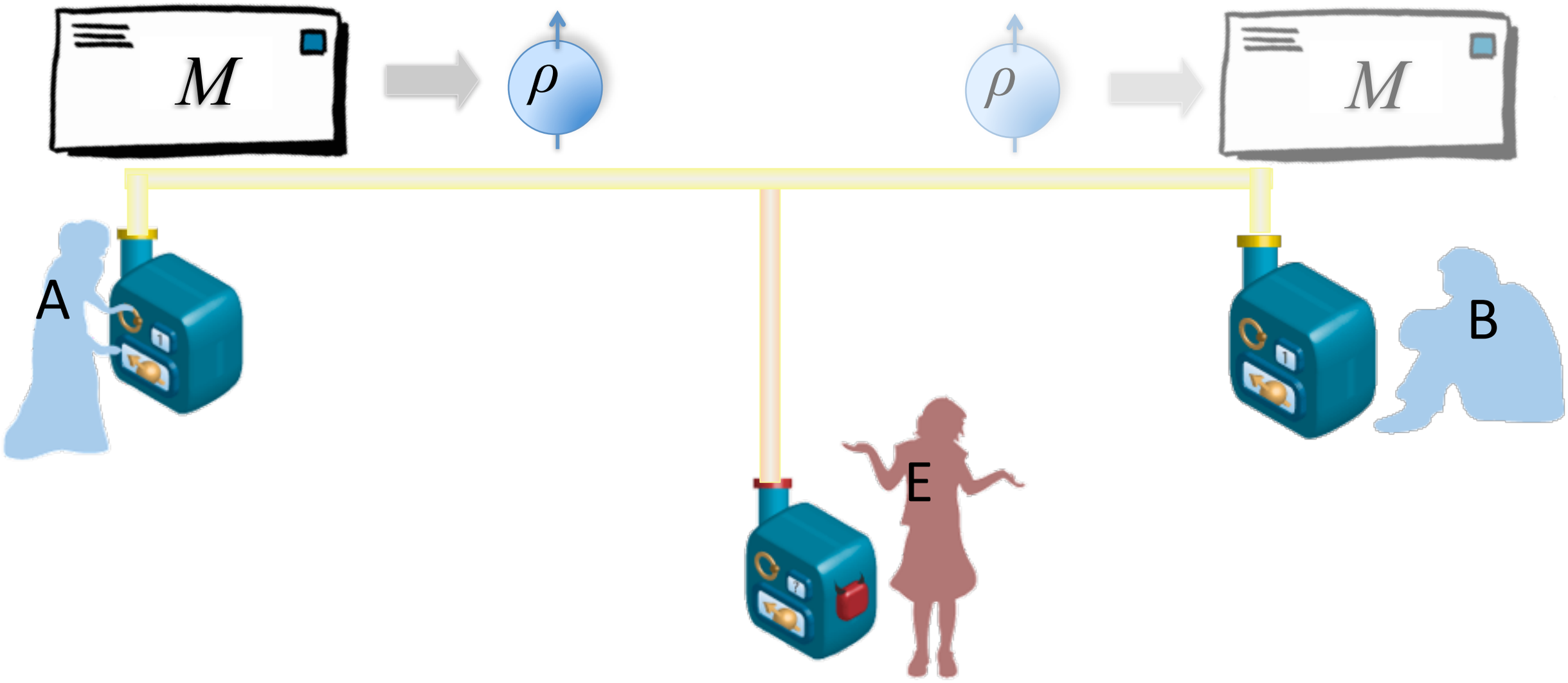
This was the invention of quantum cryptography.

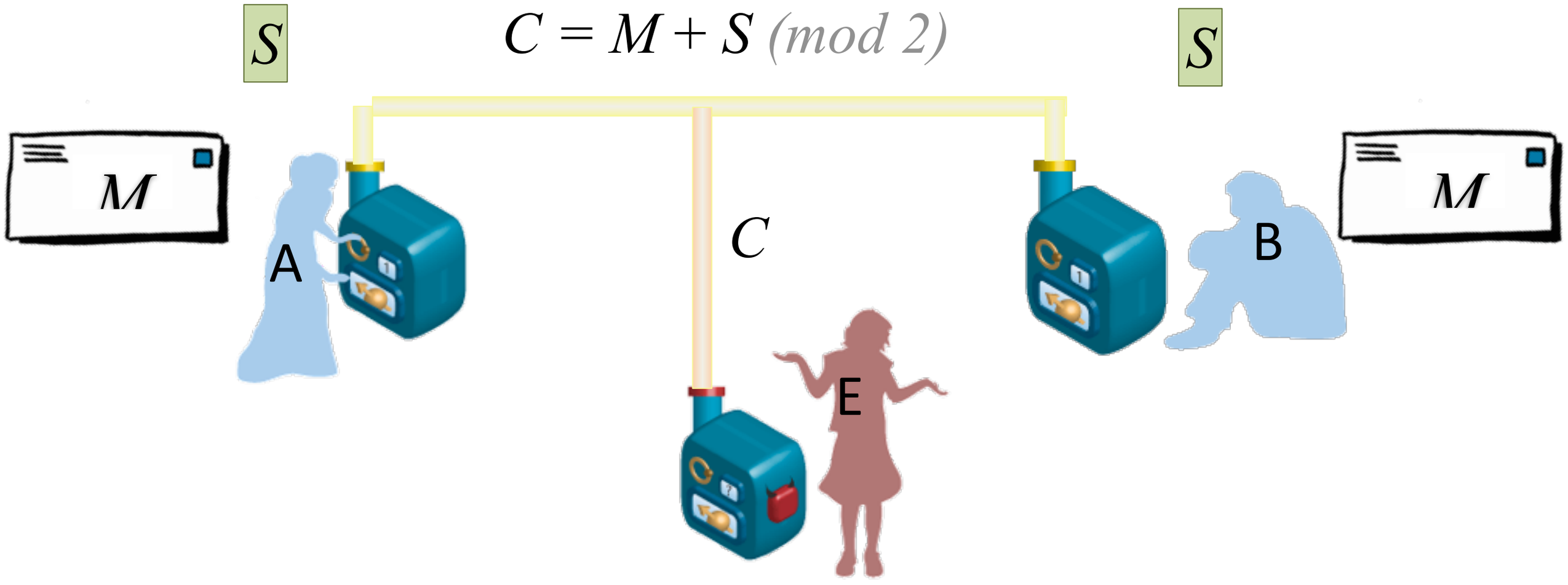
Quantum cryptography

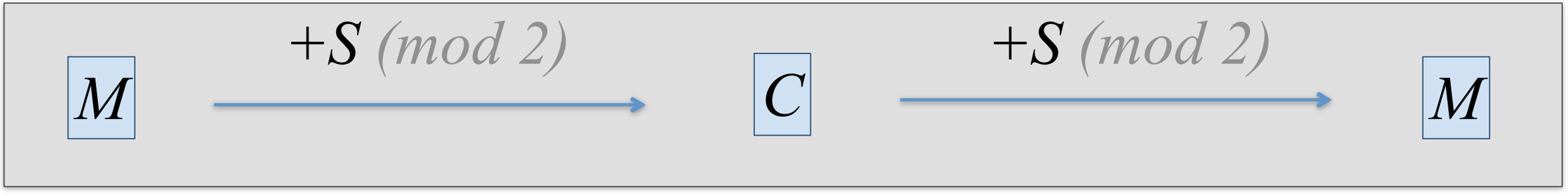


Idea: Use no-cloning principle to verify secrecy.

One-time-pad encryption

Let $M \in \{0,1\}$ be a message bit and $S \in \{0,1\}$ a "key bit".





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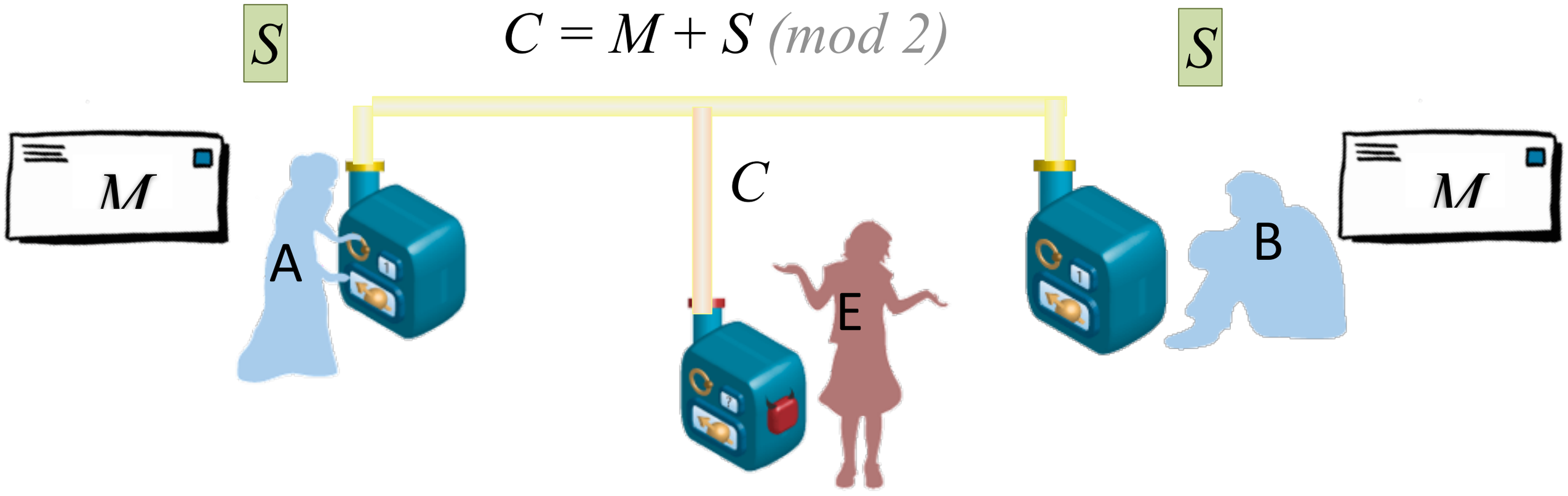
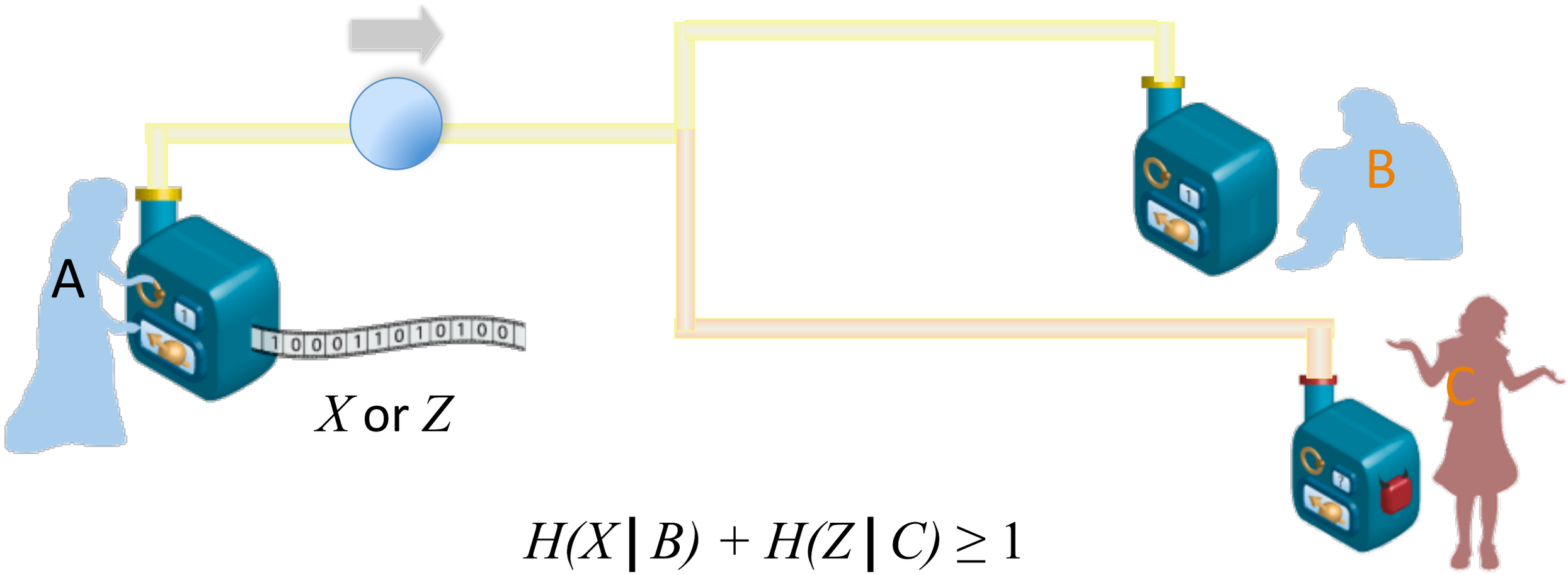


Theorem

If S is uniformly distributed then C is uncorrelated to M.

No-cloning principle provides security



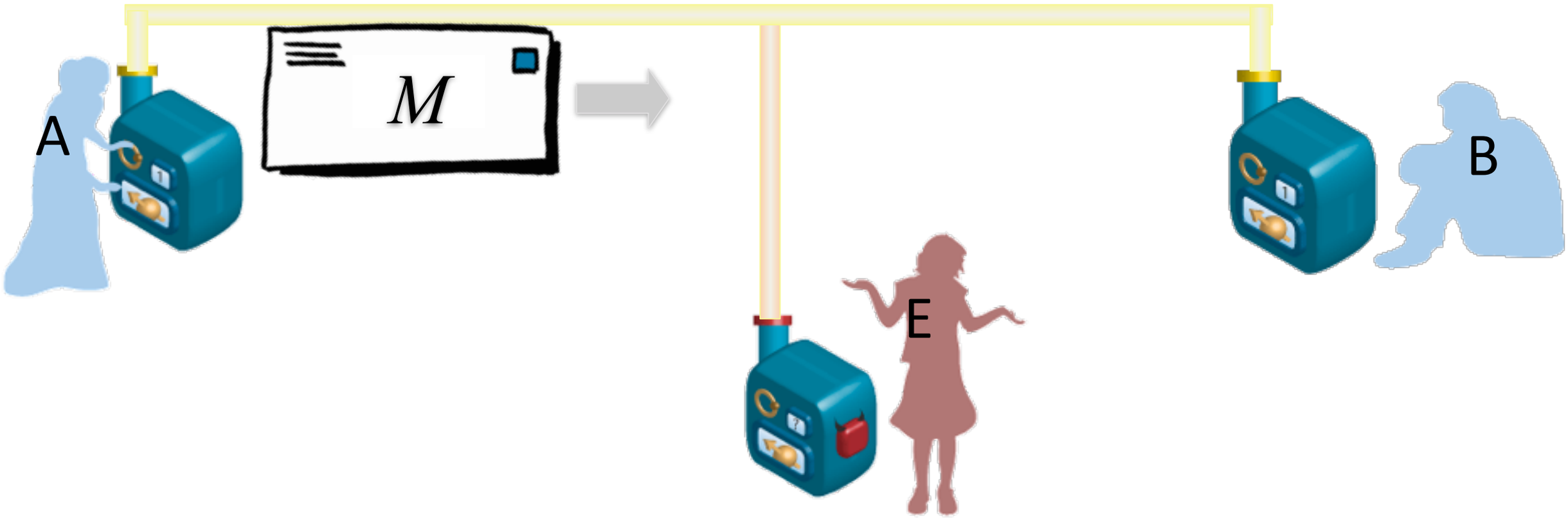
$$H(X|B) + H(Z|C) \geq 1$$

Idea

Check statistically that $H(X|B)$ is small. The generalized uncertainty principle then implies that $H(Z|C)$ is large.

[Tomamichel *et al.*, Nature Communications, 2012]

Quantum cryptography



Protocol

1. Use quantum communication to generate a key
   (the no-cloning principle guarantees that it is secure)
2. Use one-time-pad encryption to send message M.

An apparent contradiction

Theorem [Bennett and Brassard, 1984]

Two parties connected via an insecure channel can exchange messages secretly

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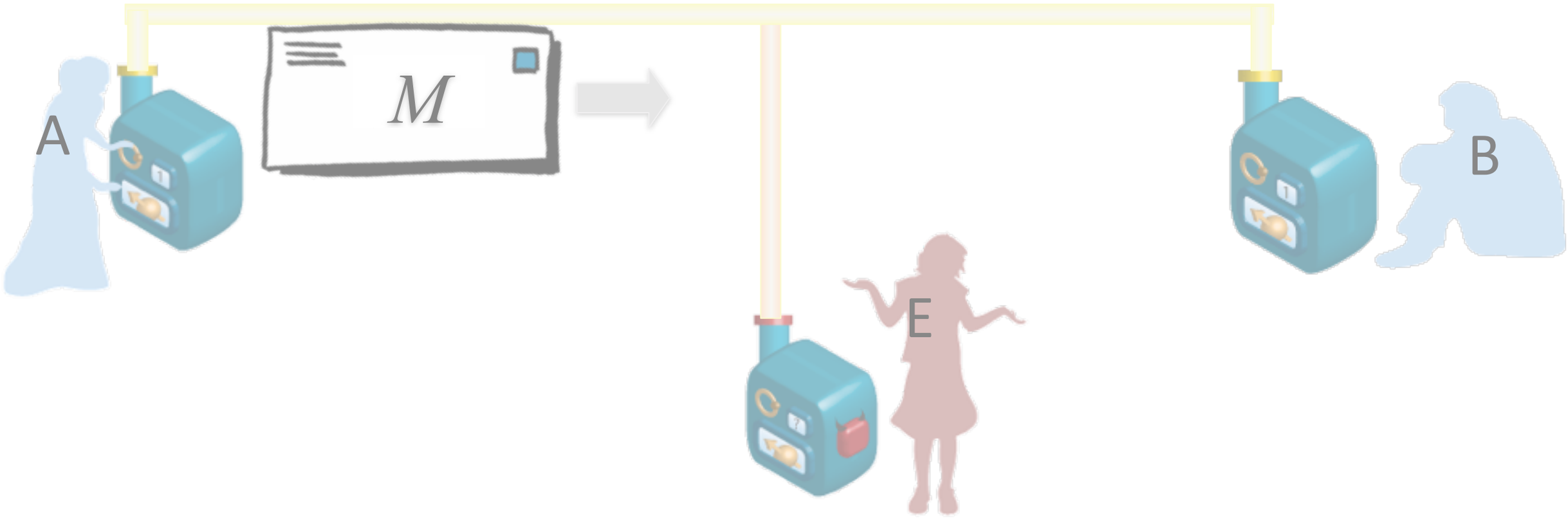

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(even if they have methods for authentication).

Proof of Shannon's theorem

Let M be a uniformly distributed n-bit message, S a secret key, and C the ciphertext..

Requirements

- H(M | SC) = 0, since M determined by S, C.
- H(M | C) = H(M) = n, since M indep. of C.

Hence

H(S) ≥ I(M : S | C) = H(M | C) - H(M | SC) = n.

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Proof of Shannon's theorem

No cloning:
$C_{Bob} \neq C_{Eve}$ in general

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Properties of entangled qubits

$\Pr[X = Y] = \cos^2(\alpha-\beta)$

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Note: If the left particle is measured with angle α and gives output 0 (or 1) then the right particle behaves as if it was prepared along α (or $\alpha+\pi/2$).

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- Information is physical: Since information is physical, the physical properties of the underlying information carriers have to be taken into account when describing the laws of information.

- Implications: The resulting laws of information are fundamentally different from the corresponding classical laws. Examples include the no-cloning principle, which has applications, e.g., in cryptography.

Many thanks for your attention