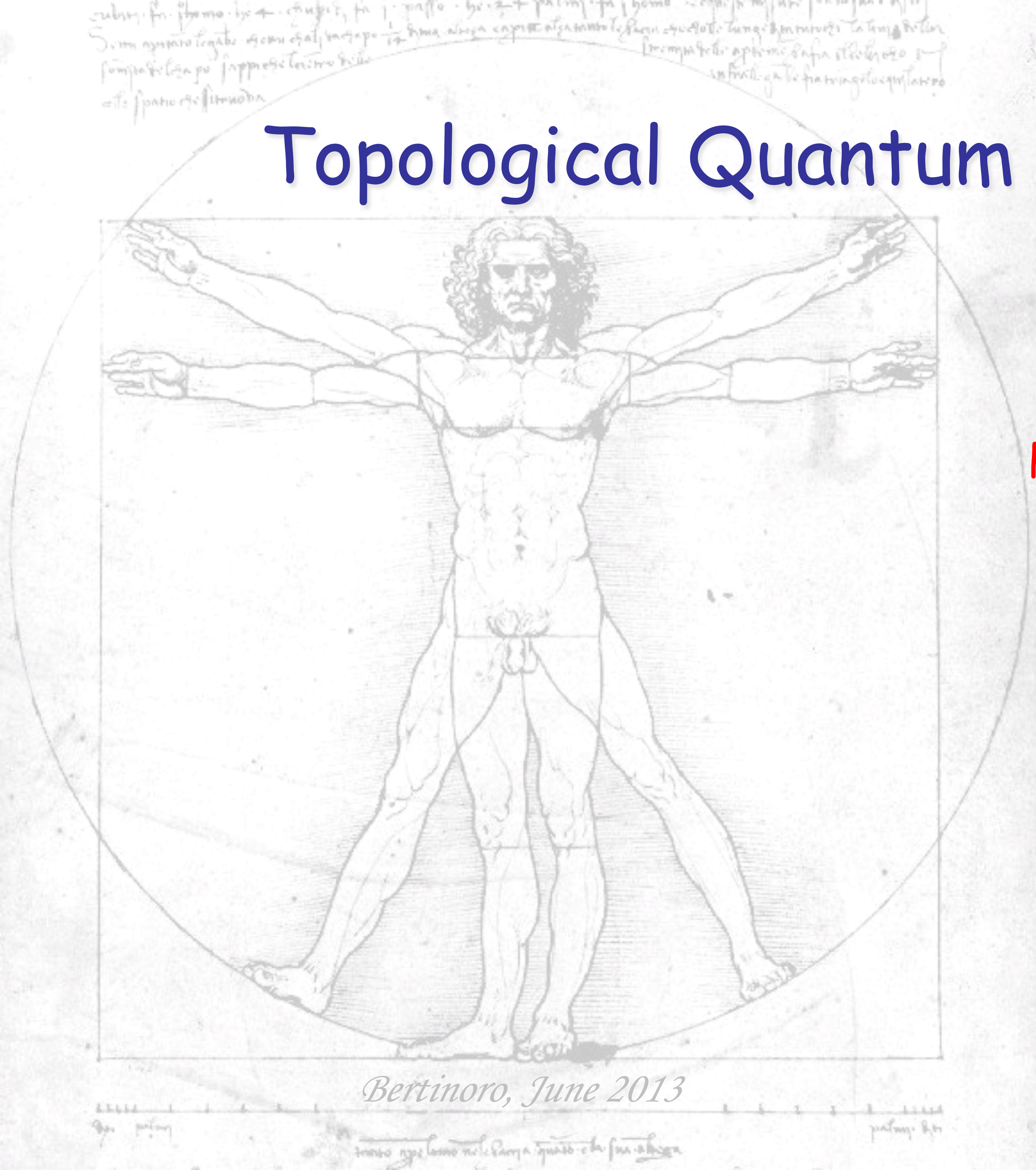
Topological Quantum Computation



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Properties of anyons

Bertinoro, June 2013

EPSRC
Engineering and Physical Sciences Research Council

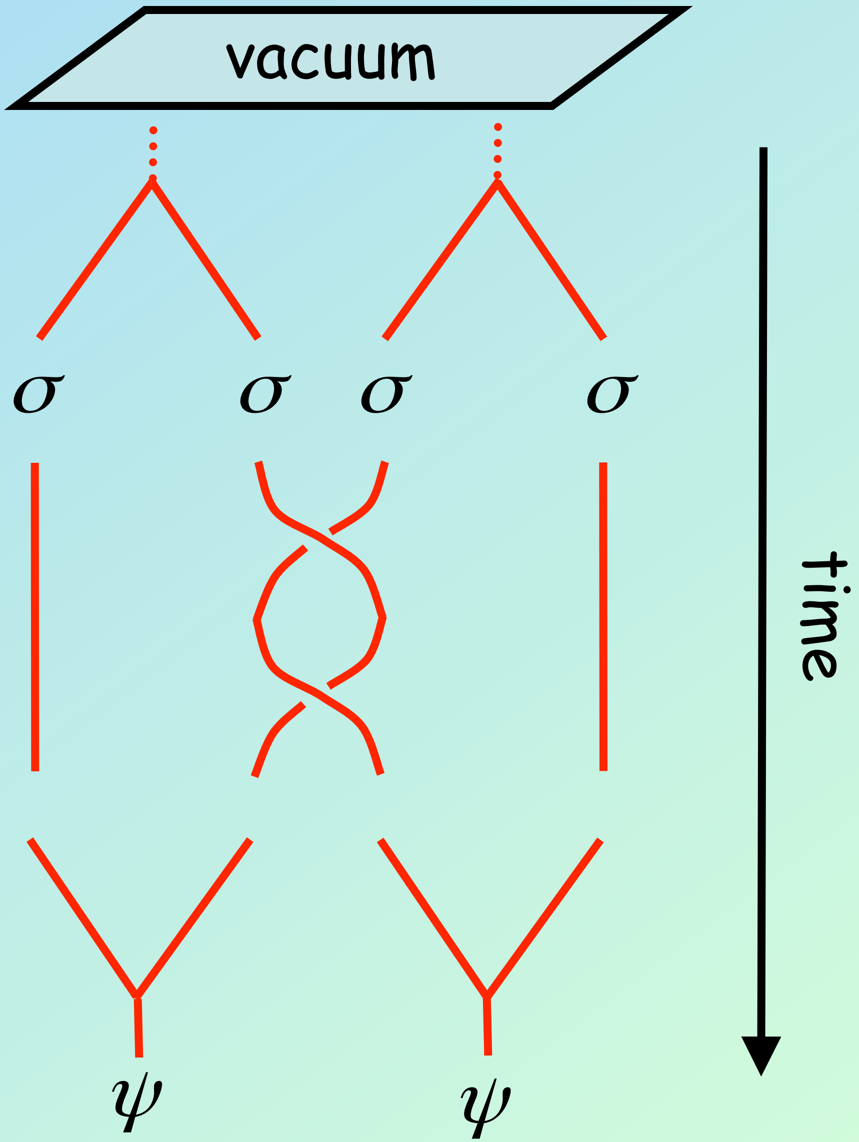
UNIVERSITY OF LEEDS

Use anyon for QC

- Assume we can:
  - **Create** identifiable anyons
    vacuum pair creation
  - **Braid** anyons
    Statistical evolution:
    braid representation B
  - **Fuse** anyons
    e.g. $\sigma \times \sigma = 1 + \psi$

Fusion Hilbert space:

$$|\sigma, \sigma \rightarrow 1\rangle, |\sigma, \sigma \rightarrow \psi\rangle$$



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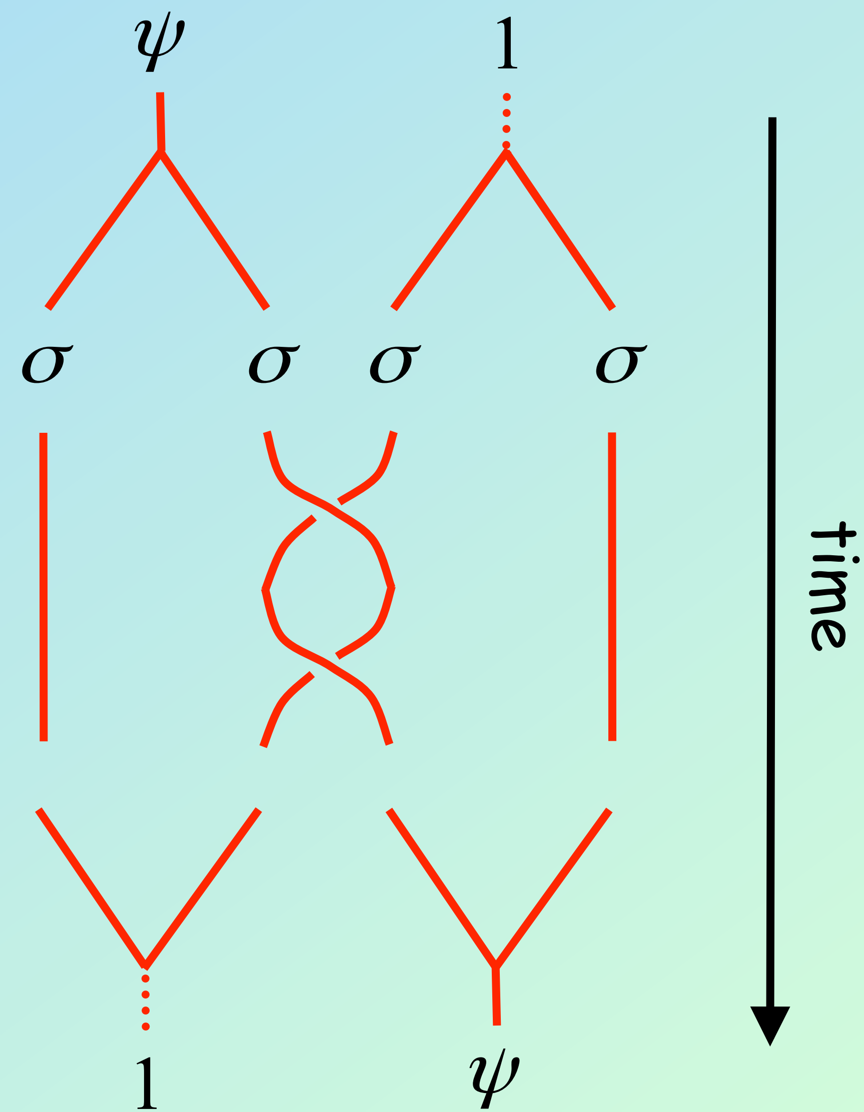


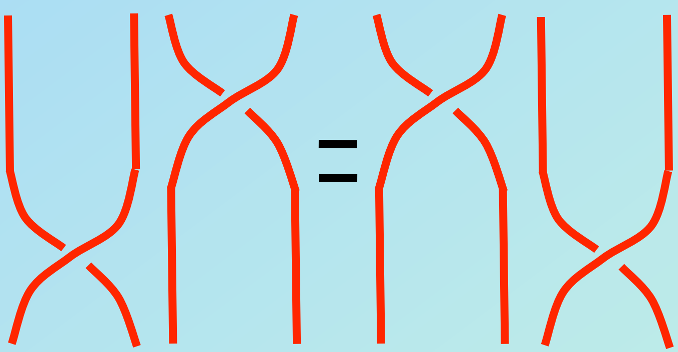
The braid group B_n

The braid group B_n has elements $b_1, b_2, \ldots, b_{n-1}$ that satisfy:

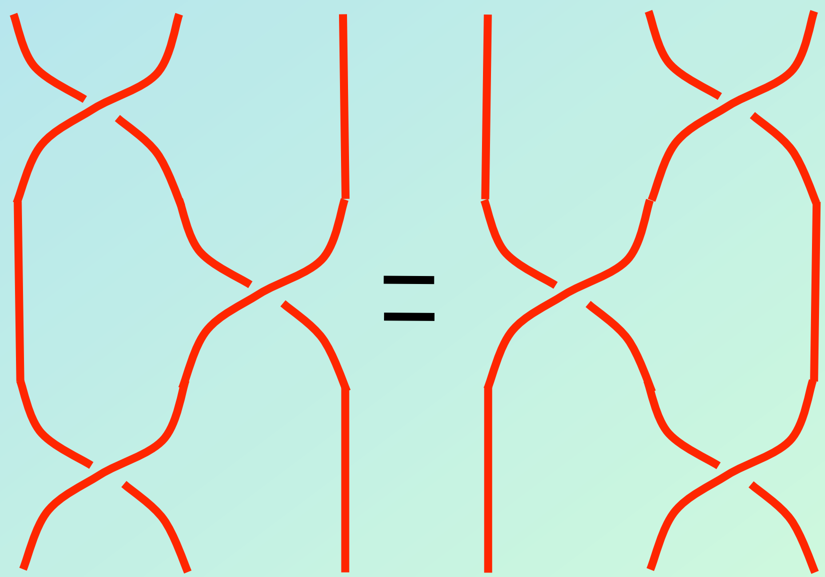
$$b_i b_j = b_j b_i, \ \ for\ |i-j| \geq 2$$

$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1} \ \ for\ 1 \leq i < n$$

Pictorially:



$$b_i b_j = b_j b_i$$

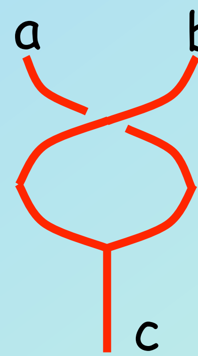


$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1}$$

Braiding and Fusion properties

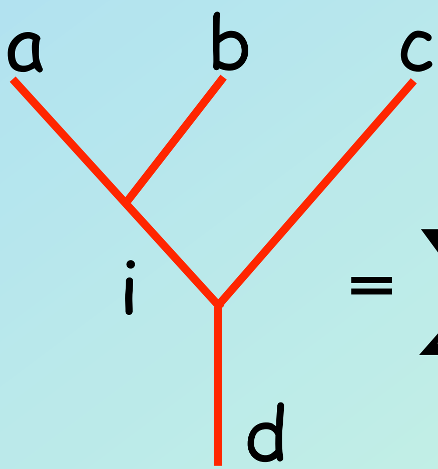
- The action of braiding of two anyons depends on their fusion outcome:

R^c_{ab} is a phase factor

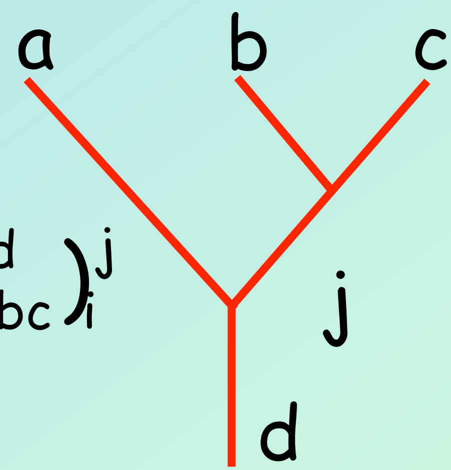


$=R^c_{ab}$

- Changing the order of fusion is non-trivial:



$$= \sum_j \left(F^d_{abc}\right)^j_i$$



Inception of Anyonic Models

1. Take a certain number of **different anyons**

   1, a, b, ...

   the vacuum (1) and one or more non-trivial particles

2. Define **fusion rules** between them

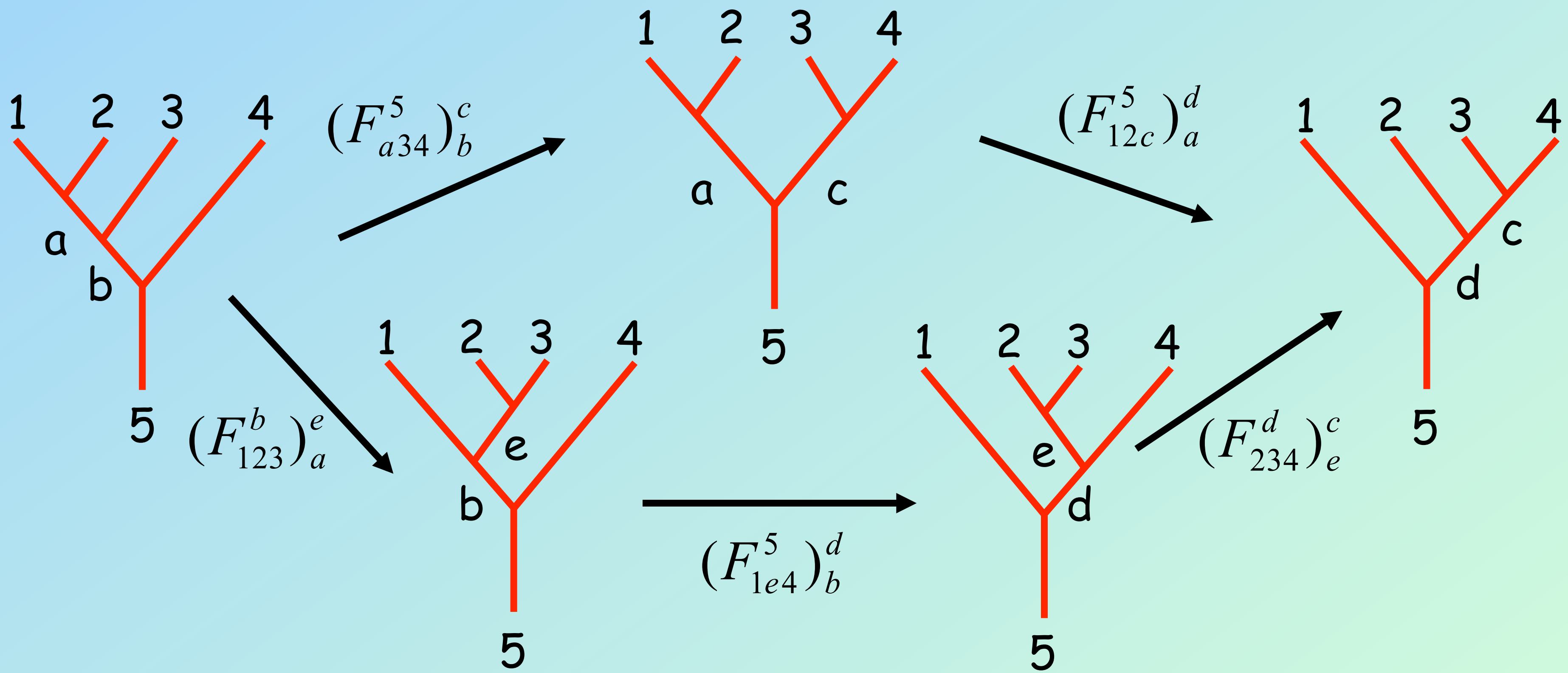
   1×a=a, a×b=c+d+..., a×a=1+...

   The **vacuum** acts trivially. Each particle has an **anti-particle** (might be itself or not).

- Abelian anyons axb=c
- Non-Abelian anyons axb=c+d+...

Inception of Anyonic Models

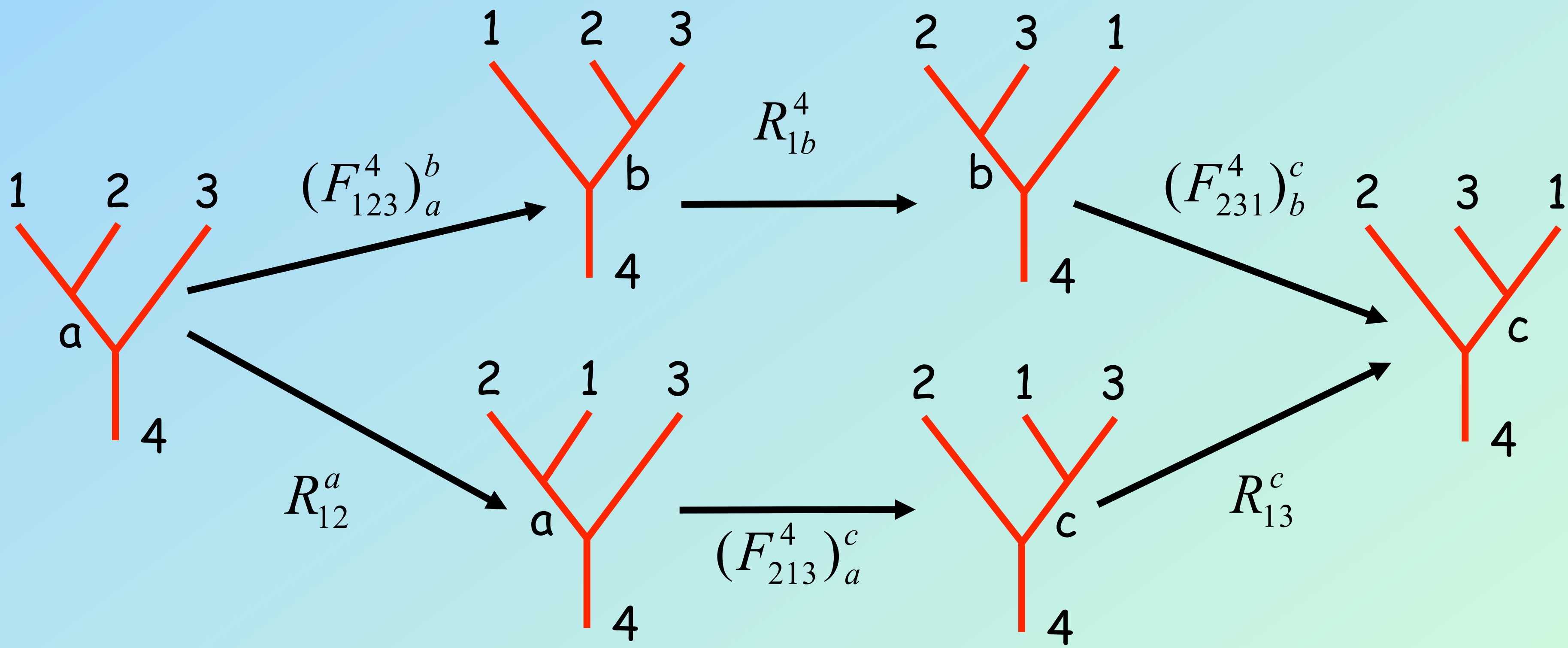
3. The F and B matrices are determined from the **Pentagon** and Hexagon identities



$$(F^5_{12c})^d_a (F^5_{a34})^c_b = \sum_e (F^d_{234})^c_e (F^5_{1e4})^d_b (F^b_{123})^e_a$$

Inception of Anyonic Models

3. The F and B matrices are determined from the Pentagon and **Hexagon** identities

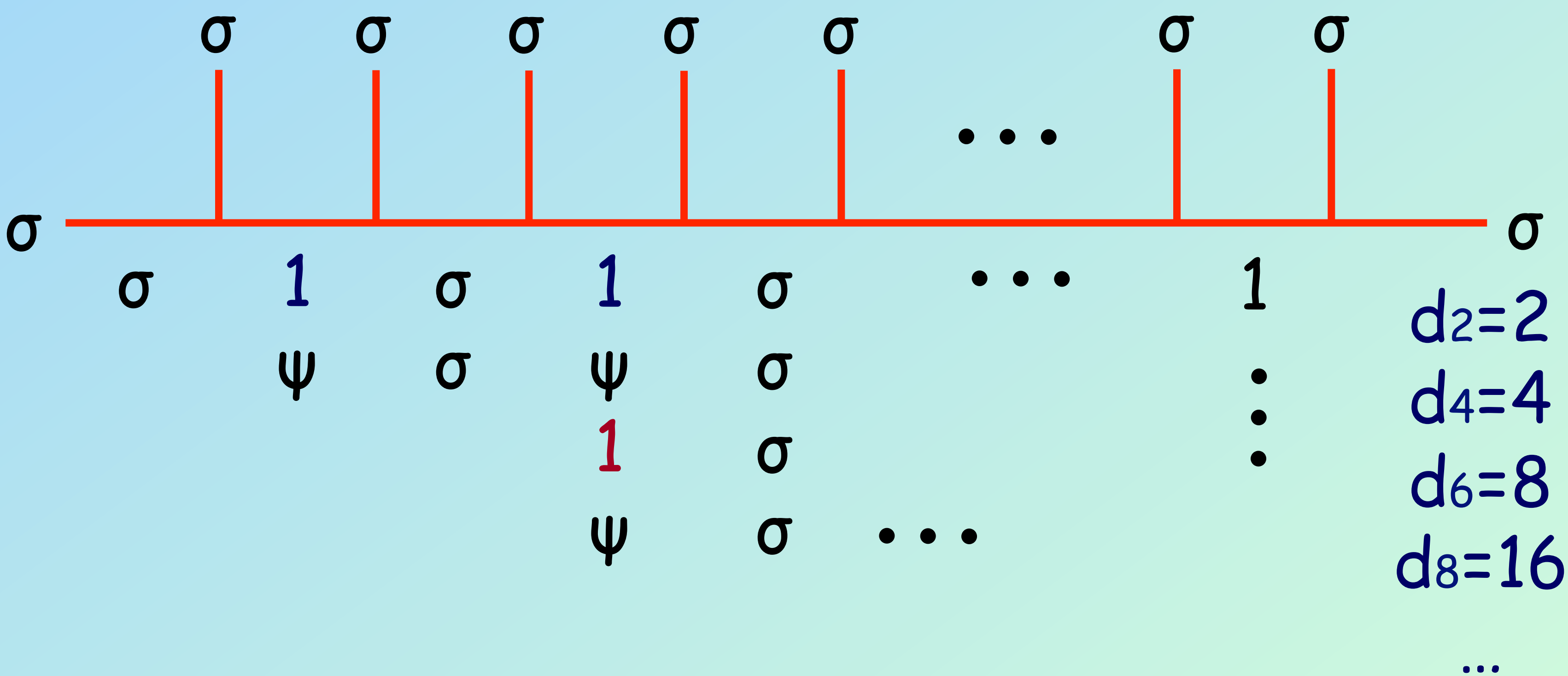


$$\sum_b (F^4_{231})^c_b R^4_{1b} (F^4_{123})^b_a = R^c_{13} (F^4_{213})^c_a R^a_{12}$$

Ising Anyons

Consider the particles: 1, σ and ψ

Fusion rules: σ×σ=1+ψ, ψ×ψ=1, σ×ψ=σ

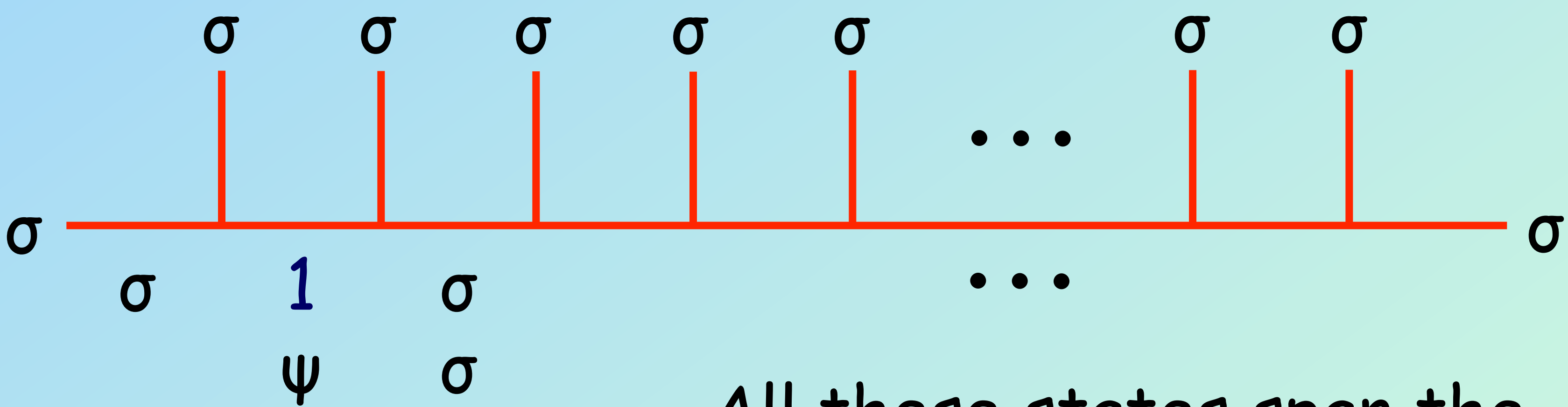


$d_n=2^{n/2}$ increase in dim of Hilbert space

Ising Anyons

Consider the particles: 1, σ and ψ

Fusion rules: $\sigma\times\sigma=1+\psi$, $\psi\times\psi=1$, $\sigma\times\psi=\sigma$



$$|\Psi\rangle = |1,1,\ldots\rangle$$

$$|\Psi\rangle = |1,\psi,\ldots\rangle$$

All these states span the **fusion Hilbert space**.

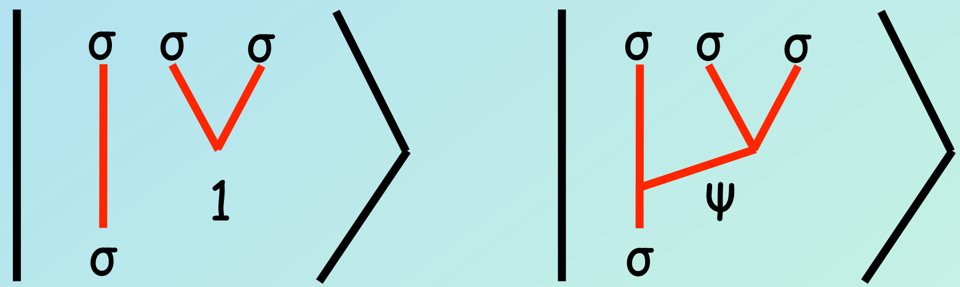
Braiding neighboring anyons transforms states

Ising Anyons

Consider the particles: 1, σ and ψ

Fusion rules: σ×σ=1+ψ, ψ×ψ=1, σ×ψ=σ

Qubit:



State |0> State |1>

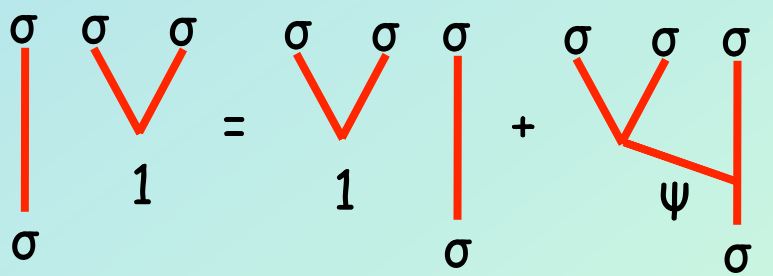
Ising Anyons

Consider the particles: 1, σ and ψ

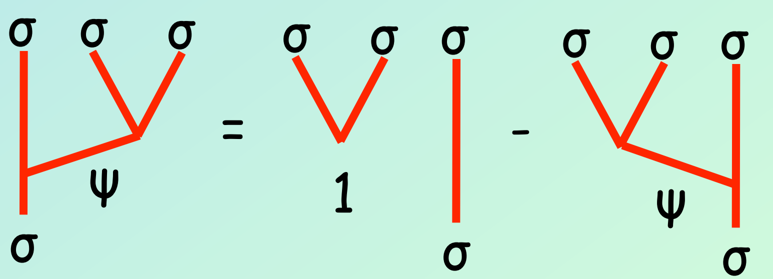
Fusion rules: σ×σ=1+ψ, ψ×ψ=1, σ×ψ=σ

From 5-gon and 6-gon identities we have:

$$F^{\sigma}_{\sigma\sigma\sigma} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = H$$

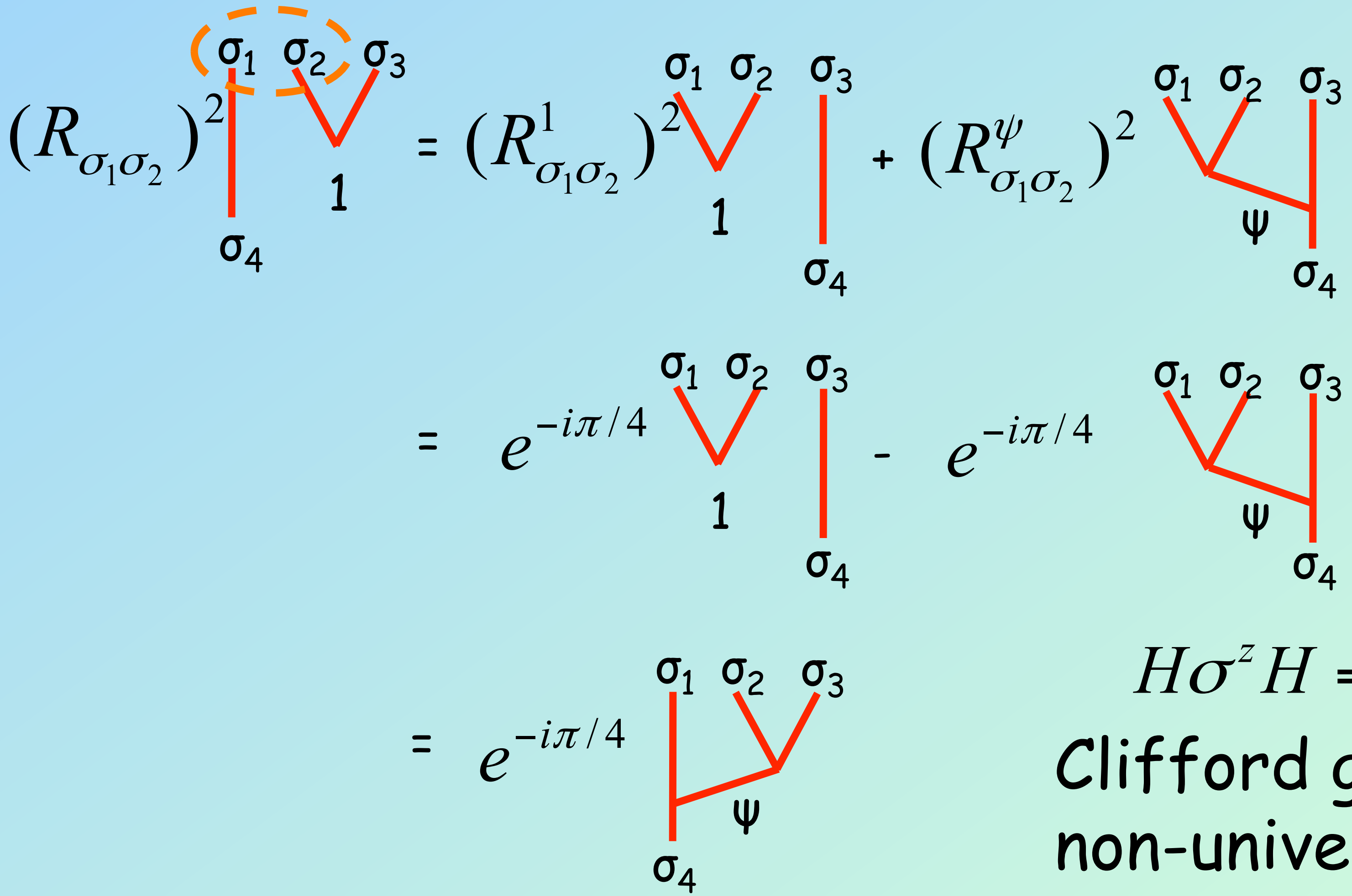


Rotation of basis states



Ising Anyons

Braiding $R^{1}_{\sigma\sigma} = e^{-i\pi/8}$ and $R^{\psi}_{\sigma\sigma} = ie^{-i\pi/8} \Rightarrow R_{\sigma\sigma} = e^{-i\pi/8}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

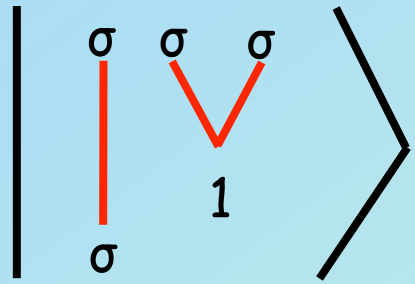


$$H\sigma^z H = \sigma^x$$

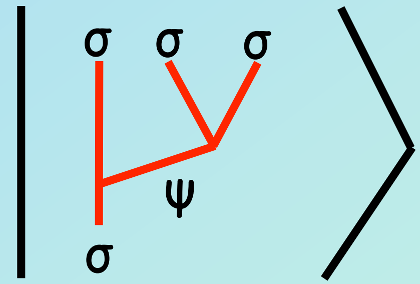
Clifford group: non-universal!

Ising Anyons

Qubit initialization:



State |0>



State |1>

Measurement: Outcome of pairwise fusion, 1 or ψ

$$H\sigma^z H = \sigma^x$$

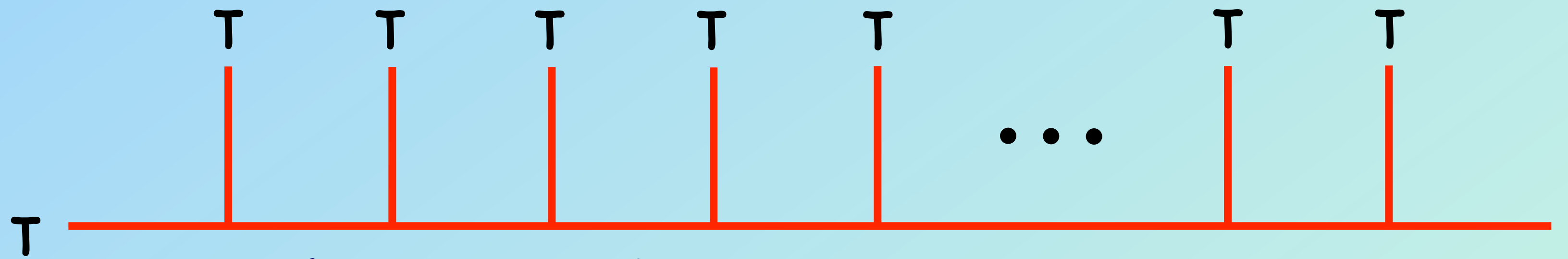
Gates: Clifford group. Non-universal!

One needs a **phase gate**: employ interactions between anyons.

Can be employed as a quantum memory.

Fibonacci Anyons

Consider anyons with labels 1 or τ with the fusion properties: **1×1=1, 1×τ = τ, τ×τ =1+τ**



τ	1	τ	1	τ	
	τ	1	τ	1	
		τ	τ	τ	
			1	1	…
			τ	τ	
				τ	
				1	
				τ	

$d_1=1$
$d_2=2$
$d_3=3$
$d_4=5$
$d_5=8$
…

Fibonacci sequence!

Dimension of Hilbert space

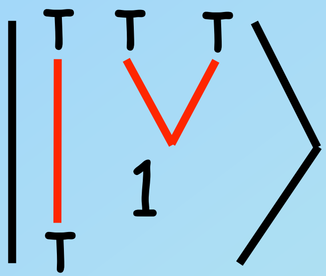
$$d_n \propto \phi^n$$

$$\phi = (1+\sqrt{5})/2$$

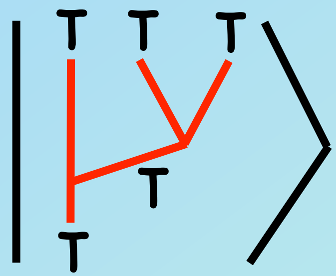
Golden mean

Fibonacci Anyons and QC

Qubit encoding:



State |0>

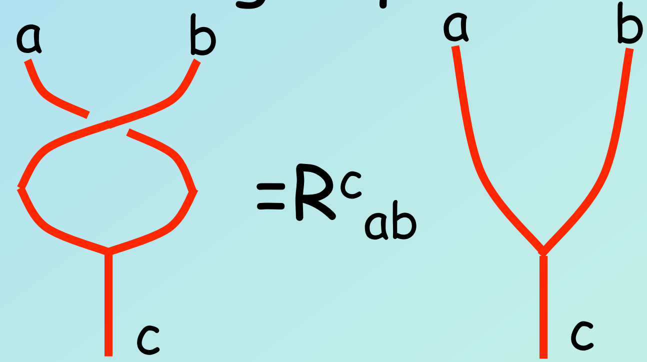


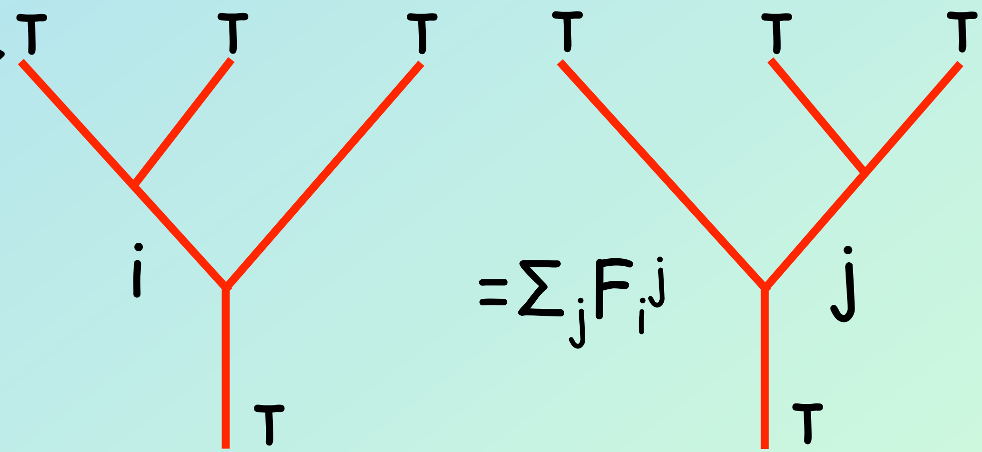
State |1>

$$\left|\tau,\tau \rightarrow 1\right\rangle = \left|0\right\rangle$$

$$\left|\tau,\tau \rightarrow \tau\right\rangle = \left|1\right\rangle$$

Evolving a qubit:





Unitaries B and F are dense in SU(2).
[Freedman, Larsen, Wang, CMP 228, 177 (2002)]

Fibonacci Anyons and QC

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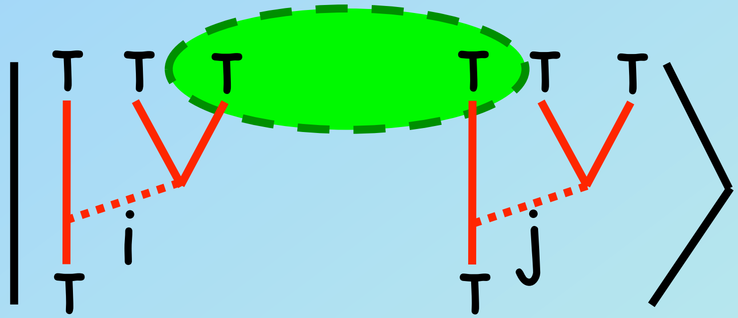

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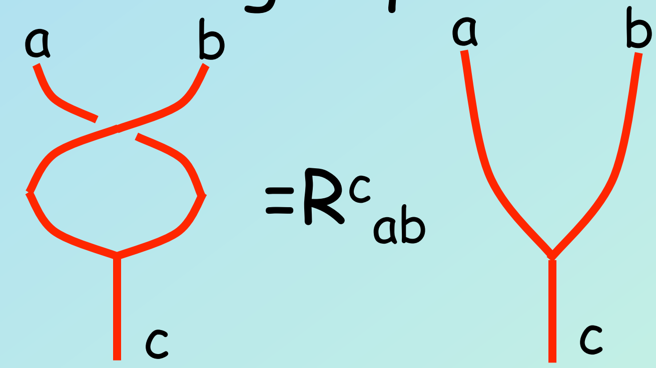

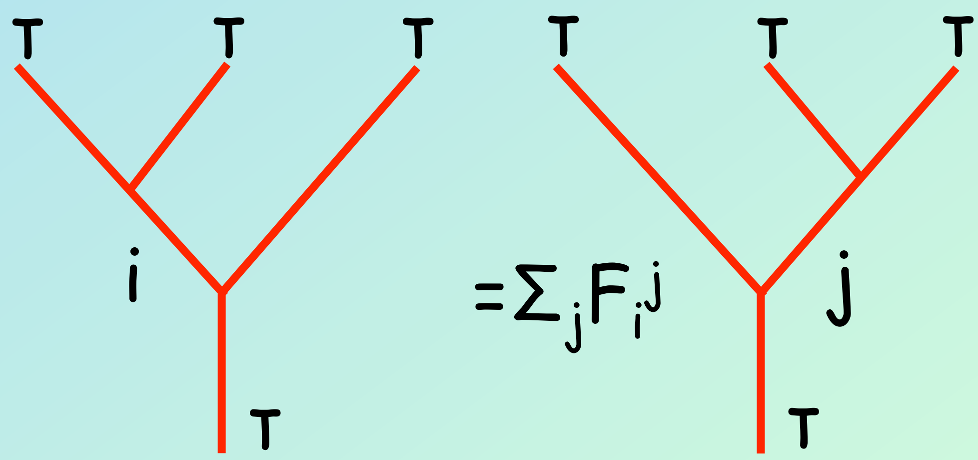

Unitaries B and F are dense in SU(2).

Extends to SU(d_n) when n anyons are employed.

Fibonacci Anyons and QC

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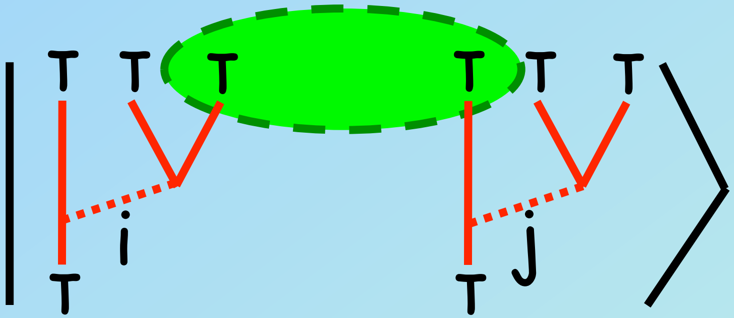

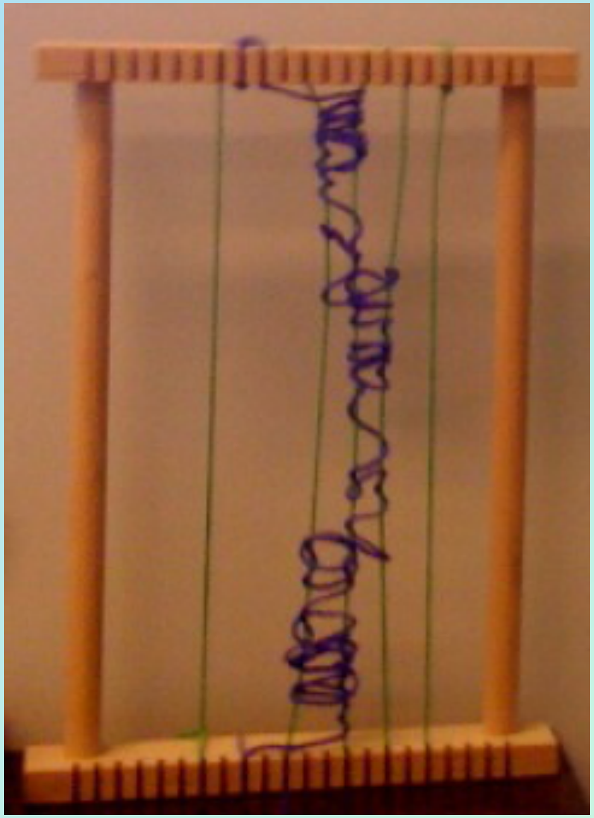

CNOT

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Extends to SU(d_n) when n anyons are employed.

Conclusions

- Topological Quantum Computation promises to **overcome** the problem of **decoherence** and errors in the most direct way.

- There is lots of work to be done to make anyons work for us.

- Is it worth it?

Aesthetics says YES!