Some open problems in deciding bisimulation equivalence

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Outline

- bisimulation equivalence on labelled transition systems (LTSs)

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  - pushdown automata (pushdown processes)
  - first-order grammars, or FO-grammars (also pushdown processes)

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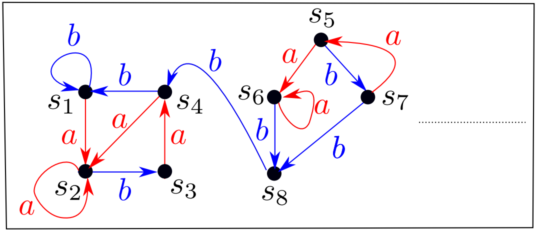
  - branching bisimilarity (van Glabbeek, Weijland, JACM 1996); recent interesting twists by Y. Fu (ICALP'13) and others: BPA, PDA

Labelled transition systems; bisimulation equivalence



$$\mathcal{L} = (\mathcal{S}, \mathcal{A}, (\xrightarrow{a})_{a \in \mathcal{A}})$$

$$\mathcal{S} = \{s_1, s_2, s_3, \ldots\}$$

$$\mathcal{A} = \{a, b\} \qquad \xrightarrow{a} \subseteq \mathcal{S} \times \mathcal{S} \qquad \xrightarrow{b} \subseteq \mathcal{S} \times \mathcal{S}$$

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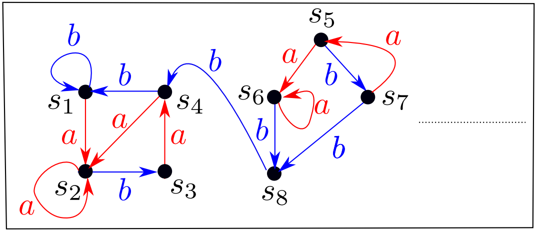

$s_1 \xrightarrow{ab} s_3 \xrightarrow{a}$

$s_5 \xrightarrow{ab} s_8 \stackrel{a}{\not\rightarrow}$

$s_1 \underset{\sim_2}{\not\sim_3} s_5$

$EL(s_1, s_5) = 2$

$s \sim_0 t$ (for all s, t)

$s \sim_{k+1} t$:

$\forall (s \xrightarrow{a} s') \; \exists (t \xrightarrow{a} t') : s' \sim_k t'$

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$s \sim_\omega t$ ... $\forall k \in \mathbf{N} : s \sim_k t$

$EL(s,t) = \max\{k \mid s \sim_k t\}$

Bisimulation equivalence as a game

Assume LTS $\mathcal{L} = (\mathcal{S}, \mathcal{A}, (\xrightarrow{a})_{a \in \mathcal{A}})$.

In a round starting with a position (s, t),

1. Attacker chooses either some $s \xrightarrow{a} s'$ or some $t \xrightarrow{a} t'$.
2. Defender responses by some $t \xrightarrow{a} t'$ or some $s \xrightarrow{a} s'$, respectively.

The new position is (s', t').

The rounds are repeated. If a player is stuck, then (s)he loses.
An infinite play is a win of Defender.

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We have $s \sim t$ iff Defender has a winning strategy from position (s, t),
and $s \sim_k t$ iff Defender can survive k rounds.

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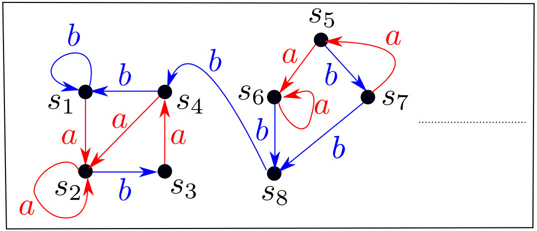
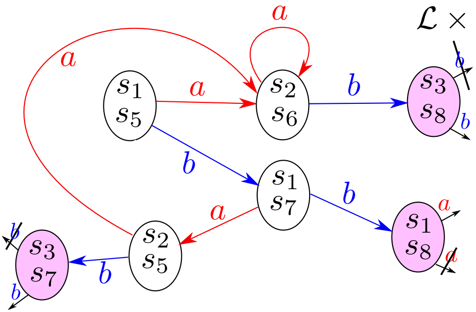
Observation. For deterministic LTSs, bisimulation equivalence coincides with trace equivalence.

Labelled transition systems; bisimulation equivalence





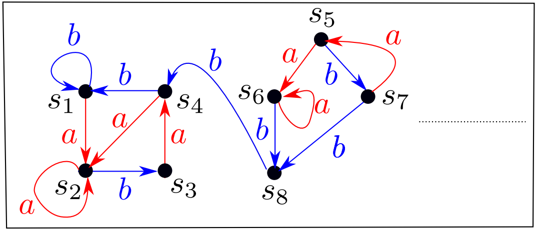
$$\mathcal{L}\times\mathcal{L} = (\mathcal{S}\times\mathcal{S}, \mathcal{A}, (\xrightarrow{a})_{a\in\mathcal{A}})$$

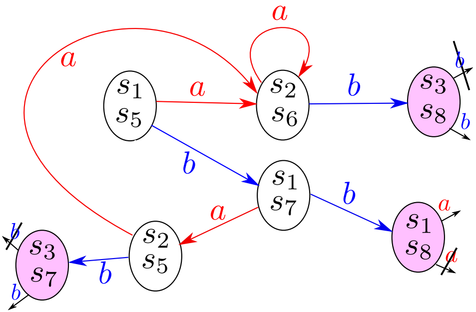
$$(r_1, r_2) \xrightarrow{a} (r_1', r_2')$$

$$\Updownarrow$$

$$r_1 \xrightarrow{a} r_1' \;\wedge\; r_2 \xrightarrow{a} r_2'$$

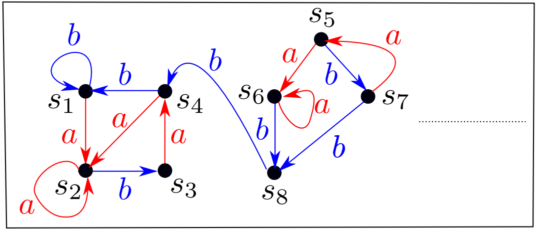
Labelled transition systems; bisimulation equivalence

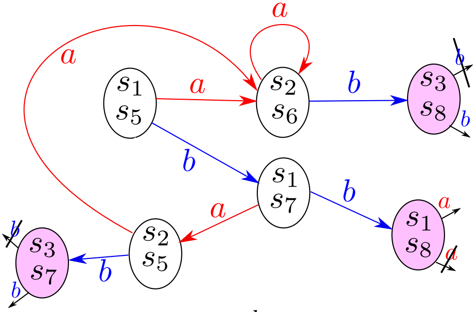




$(s_1, s_5) \xrightarrow{a} (s_2, s_6) \xrightarrow{b} (s_3, s_8)$ ... an optimal play
(EL drops by 1 in each step)

Labelled transition systems; bisimulation equivalence





Observation:

$$r \sim_{k+1} \boxed{s \begin{smallmatrix}\not\sim_{k+1}\\ \sim_k\end{smallmatrix} t}$$

$$\Downarrow$$

$$\boxed{r \begin{smallmatrix}\not\sim_{k+1}\\ \sim_k\end{smallmatrix} t}$$

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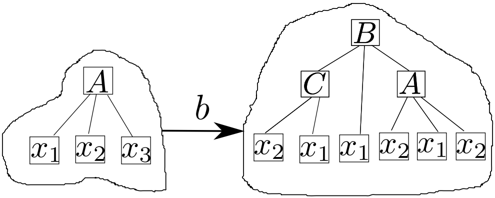
FO-grammar $\mathcal{G} = (\mathcal{N}, \mathcal{A}, \mathcal{R})$ ... rules $A(x_1, \ldots, x_m) \xrightarrow{a} E$

$$A(x_1, x_2, x_3) \xrightarrow{b} B(C(x_2, x_1), x_1, A(x_2, x_1, x_2))$$

$$B(x_1, x_2, x_3) \xrightarrow{a} x_2$$



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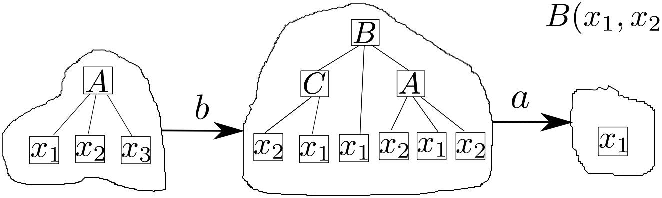

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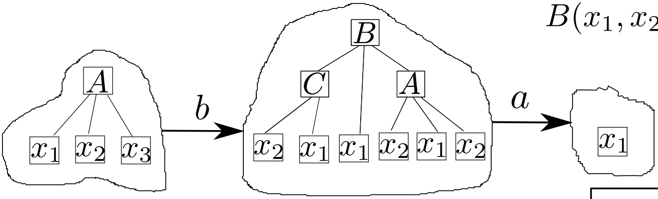
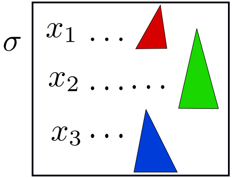


$F \xrightarrow{a} G$ implies $F\sigma \xrightarrow{a} G\sigma$



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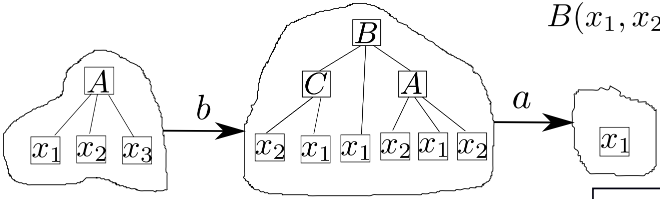

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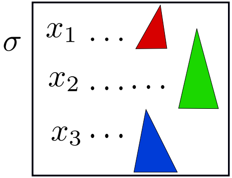
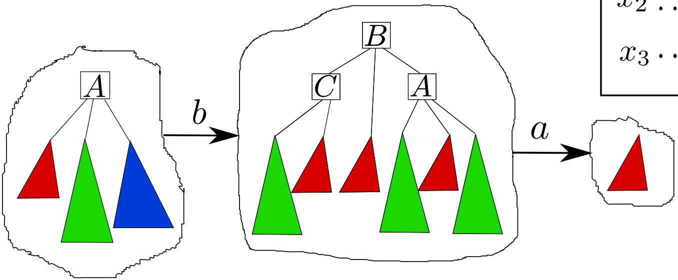
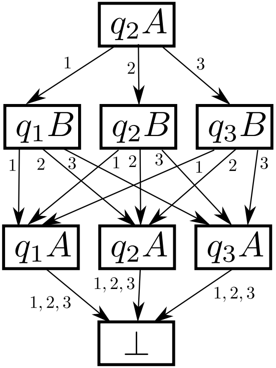


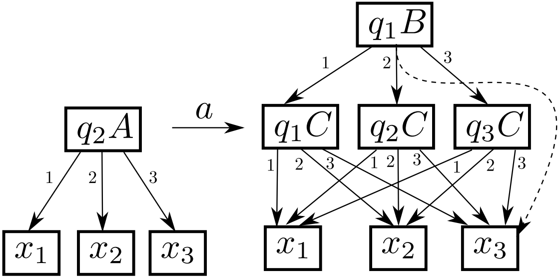


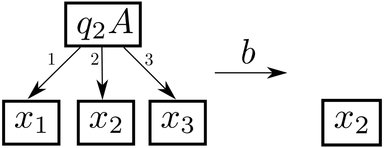
(D)pda from a first-order term perspective

$Q = \{q_1, q_2, q_3\}$
configuration q_2ABA

(pushing) rule $q_2A \xrightarrow{a} q_1BC$

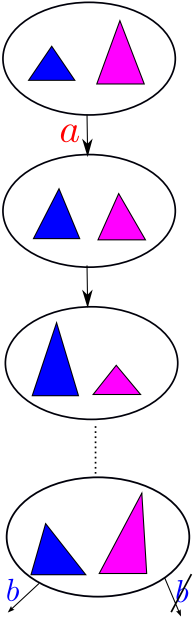






(popping) rule $q_2A \xrightarrow{b} q_2$ $\qquad$ $q_2C \xrightarrow{\varepsilon} q_3$

Bounding lengths of witnesses (where EL keeps dropping)



Theorem.

> There is an elementary function g such that for any
> det-FO grammar $\mathcal{G} = (\mathcal{N}, \mathcal{A}, \mathcal{R})$ and $T \not\sim U$ of size n we have
>
> $$EL(T, U) \leq tower(g(n)).$$

$tower(0) = 1$
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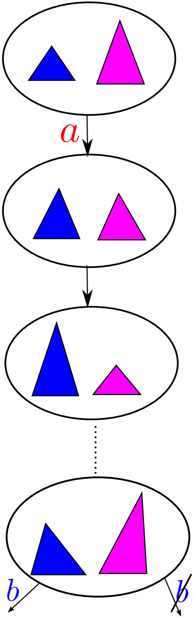

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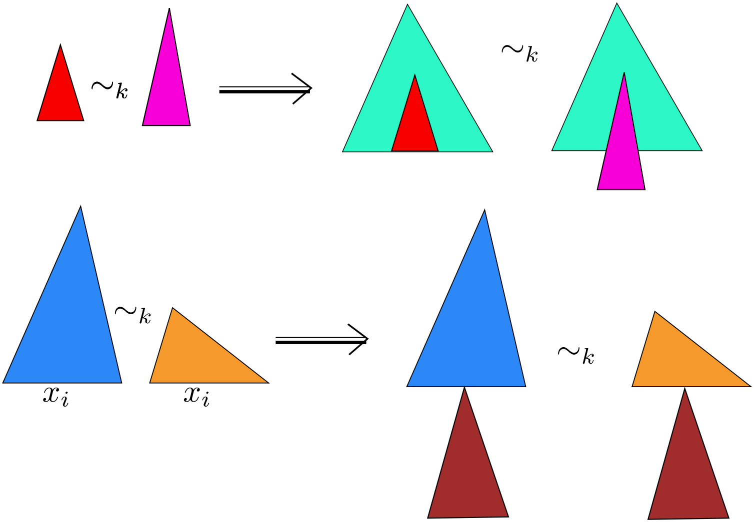
Proof is based on two ideas:

1. "Synchronize" the growth of lhs-terms and rhs-terms *while not changing the respective eq-levels*. (Hence no repeat.)
2. Derive a tower-bound on the size of terms in the (modified) sequence.

Congruence properties of $\sim_k$ and $\sim$



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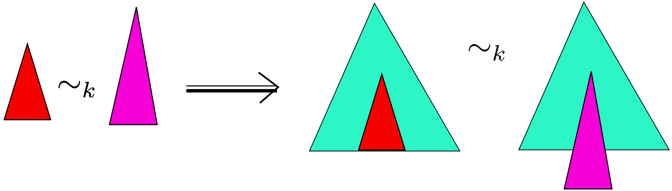
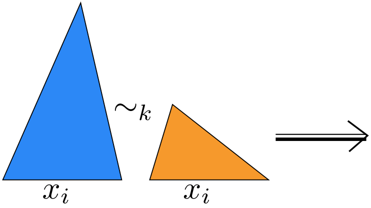
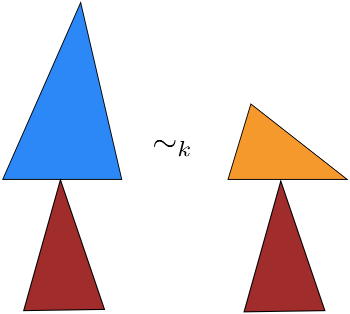
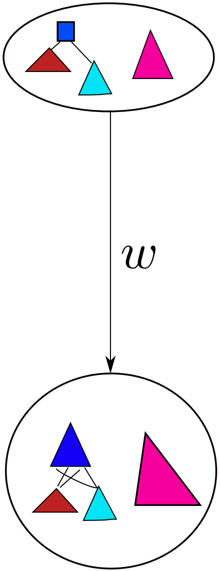






$$x_i \overset{\not\sim_1}{\sim_0} H \quad (\text{for } H \neq x_i)$$

Balancing (the crucial tool for “synchronizing”)



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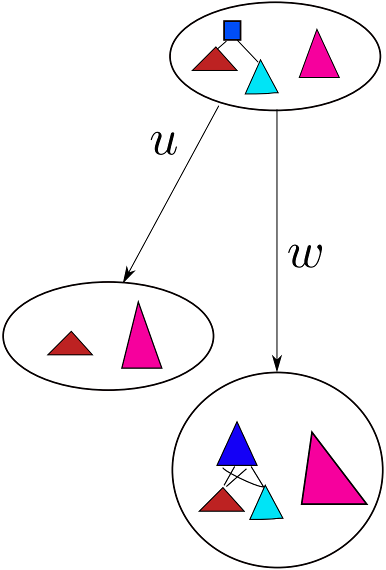

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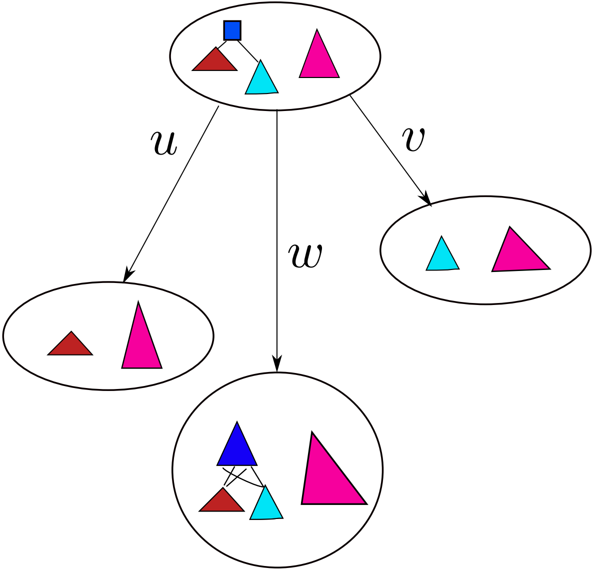

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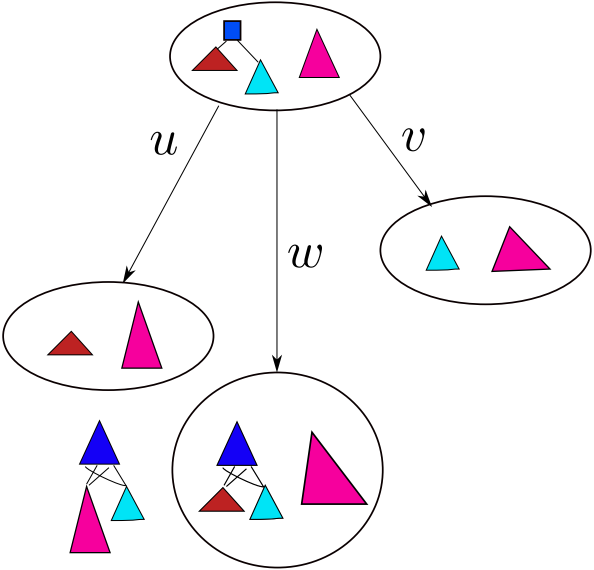

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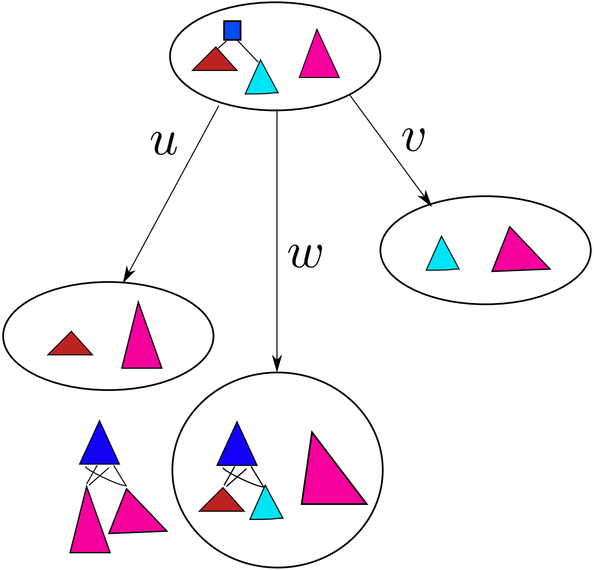

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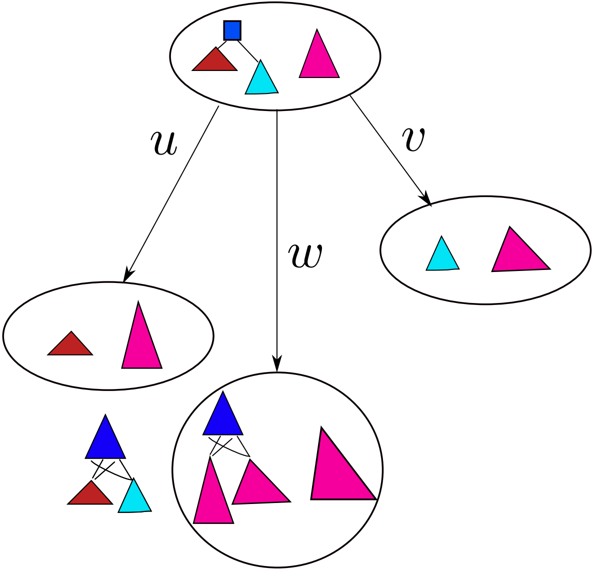

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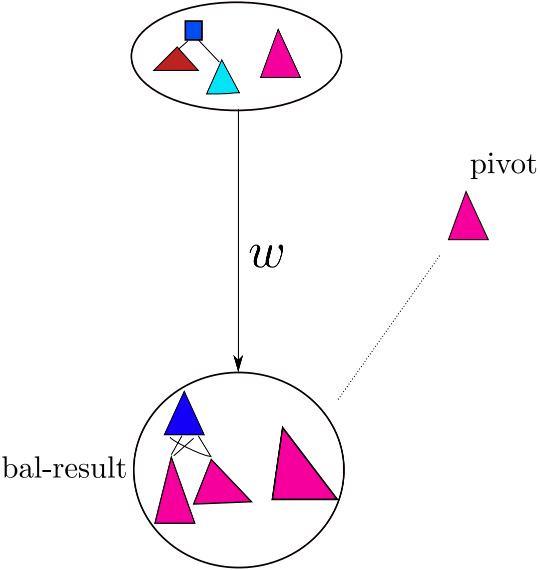

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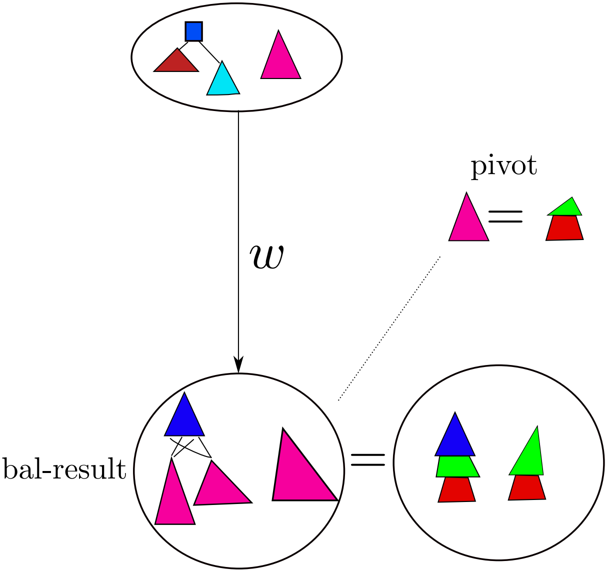
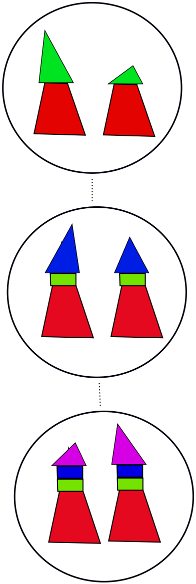
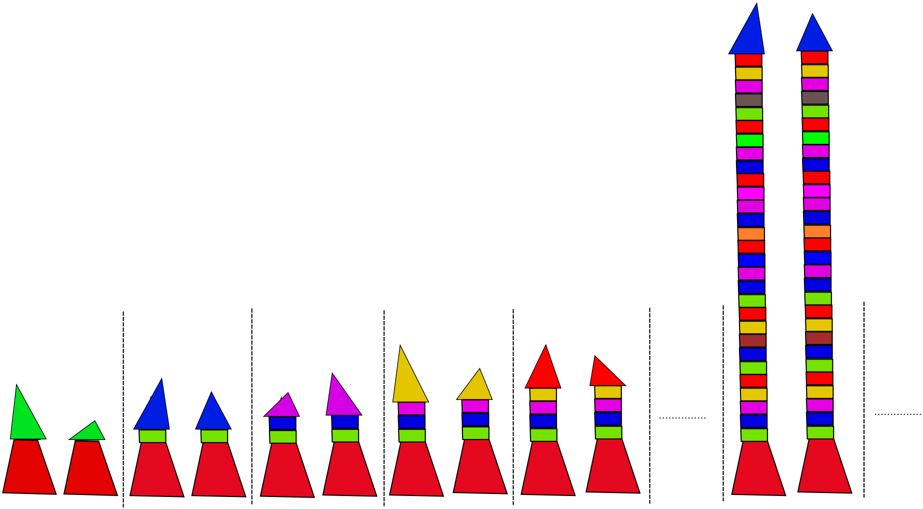


“Stair subsequence” of pairs (on balanced witness path)



Stair subsequence of pairs (written horizontally)



(ℓ, n)-(sub)sequences, with 2^ℓ pairs

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$(1, n)$-sequence

2^1 pairs

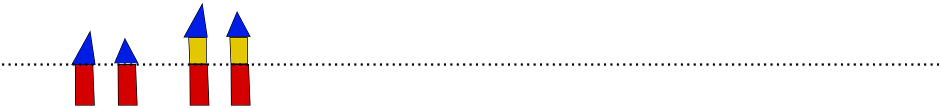
n ... thickness

There is no EL-decreasing $(1,0)$-sequence.

$(1,n)$-sequence

2^1 pairs

n ... thickness



(ℓ, n)-(sub)sequences, with 2^ℓ pairs

There is no EL-decreasing $(1, 0)$-sequence.

$(1, n)$-sequence

q ... cardinality of "alphabet"

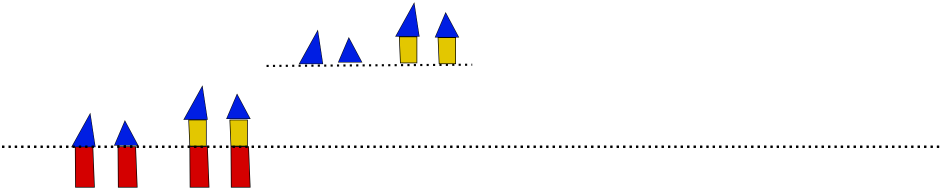
2^1 pairs

In $\boxed{h(1) = 1 + q}$ pairs (of thickness n)

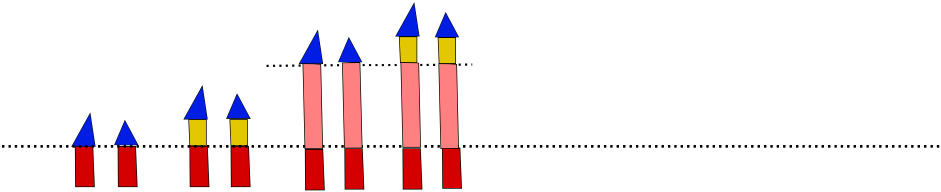
n ... thickness

there is some $(1, n)$-sequence.

(ℓ, n)-(sub)sequences, with 2^ℓ pairs



(ℓ, n)-(sub)sequences, with 2^ℓ pairs

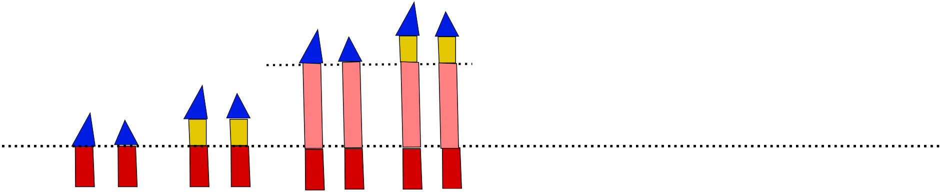


(ℓ, n)-(sub)sequences, with 2^ℓ pairs

$(2, n)$-sequence

$2^2 = 4$ pairs

n ... thickness



(ℓ, n)-(sub)sequences, with 2^ℓ pairs

q ... cardinality of "alphabet"

$(2,n)$-sequence

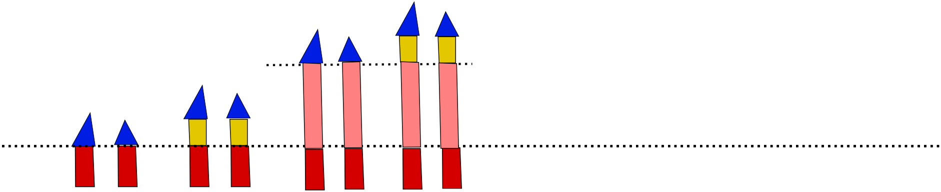
$h(1) = 1 + q$ ... $(1,n)$-sequence

$2^2 = 4$ pairs

In $\boxed{h(2) = h(1) \cdot (1 + q^{h(1)})}$ pairs

n ... thickness

there is some $(2,n)$-sequence.



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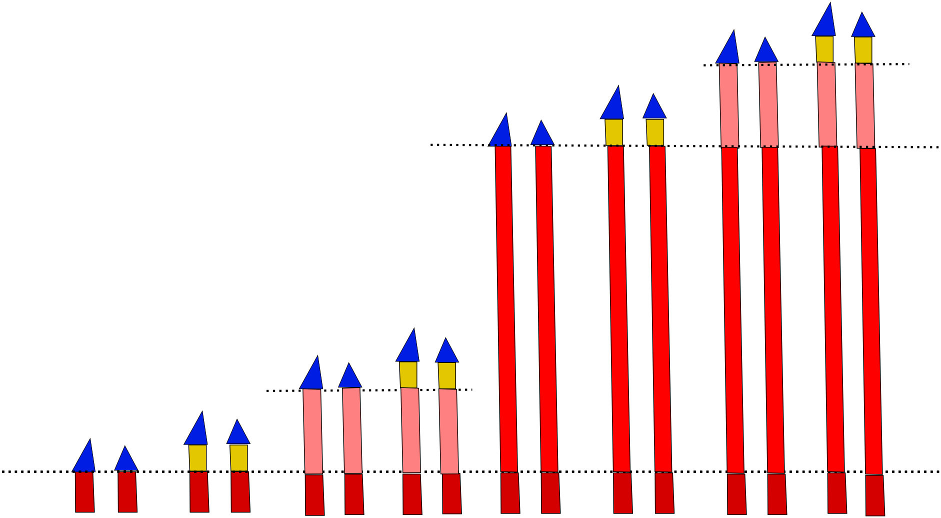
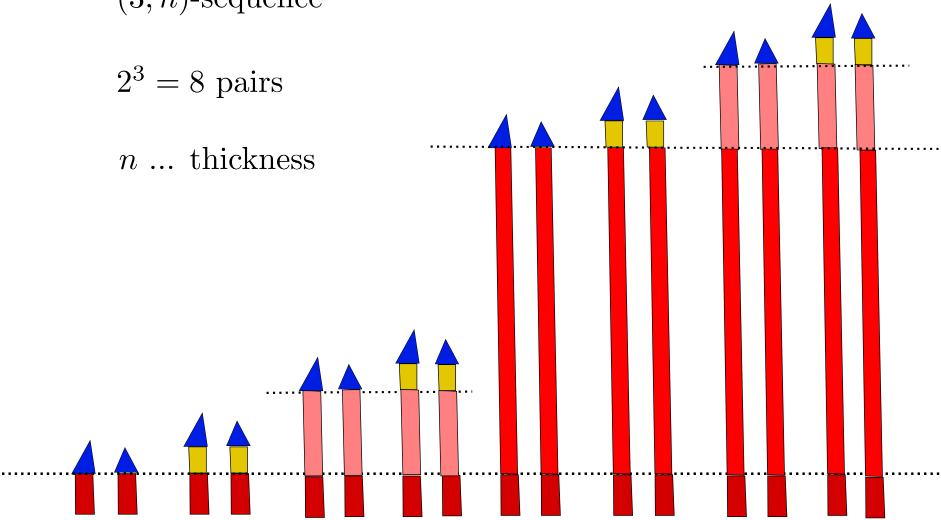


(ℓ, n)-(sub)sequences, with 2^ℓ pairs

$(3, n)$-sequence

$2^3 = 8$ pairs

n ... thickness



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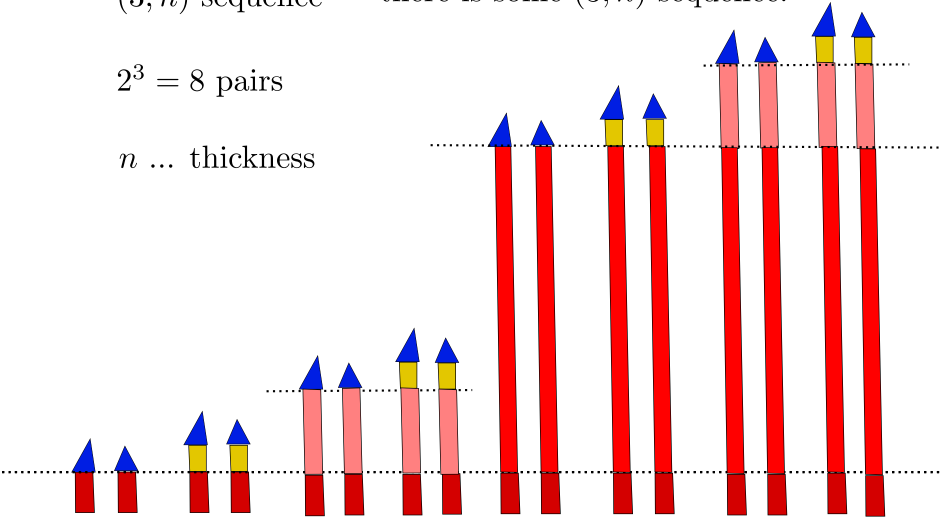

Final (conditional) step of the "TOWER-proof"

Recall: There is no EL-decreasing $(1, 0)$-sequence.

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Recall that

$$h(1) = 1 + q,$$
$$h(j{+}1) = h(j) \cdot (1 + q^{h(j)})$$

and that $h(j)$ "stairs" gives rise to (j, n)-sequence (n being the "small" thickness).

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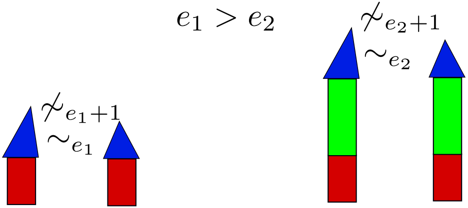
Corollary. There are less than $h(n{+}1)$ stairs, and $h(n{+}1) \leq \mathit{tower}(g(n))$.

Repeating heads yield an "equation"

$e_1 > e_2$



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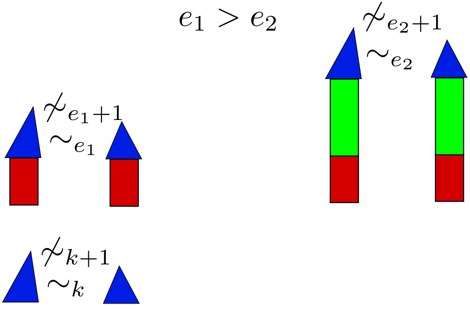
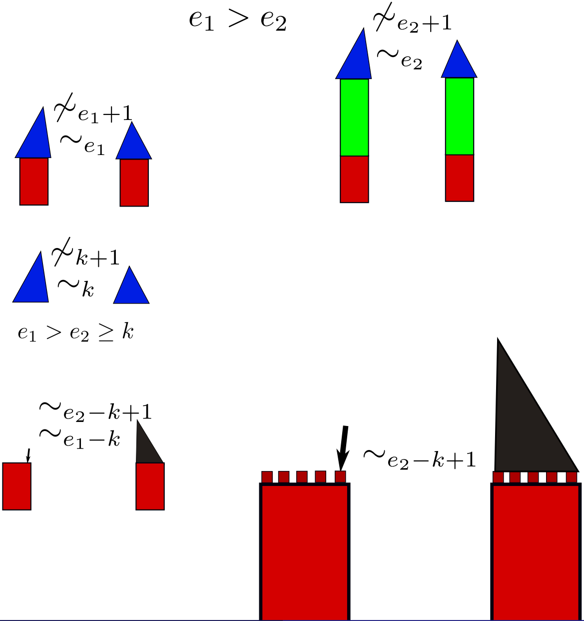


$e_1 > e_2 \geq k$

Repeating heads yield an "equation"



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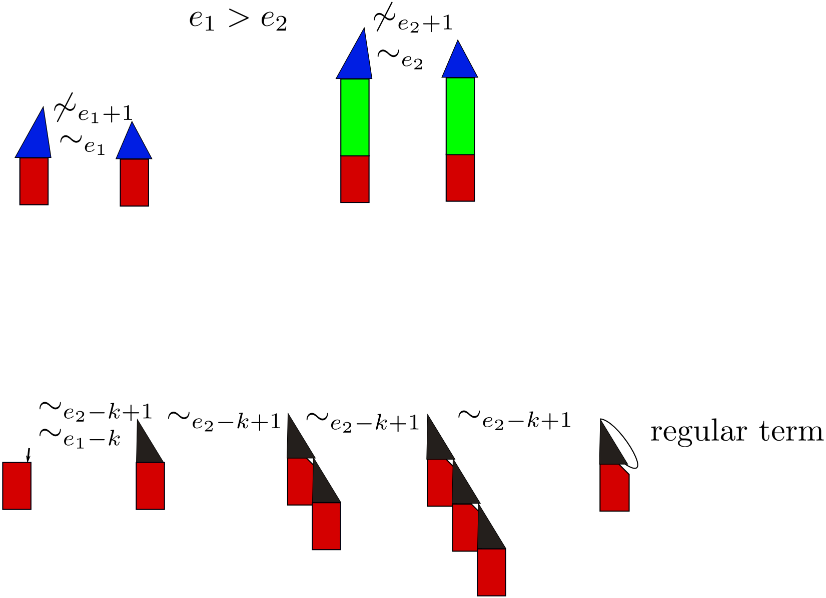

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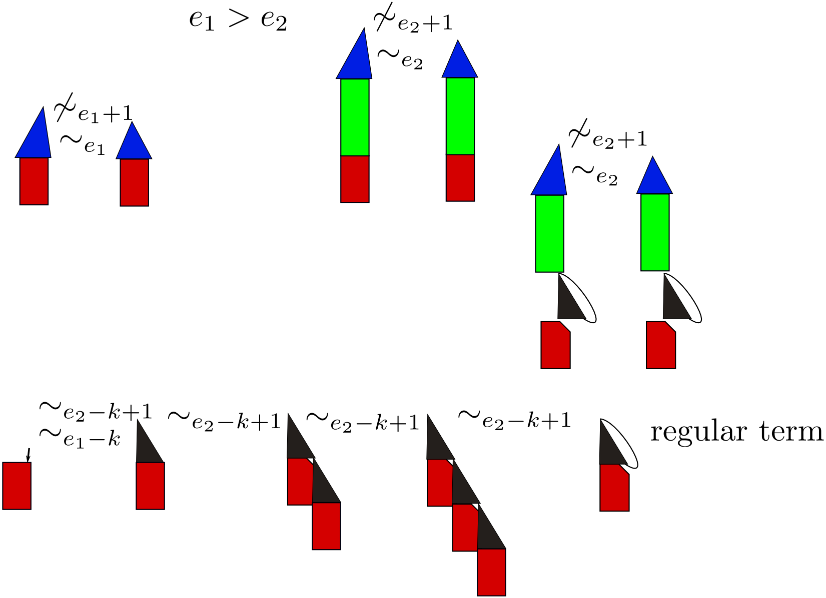
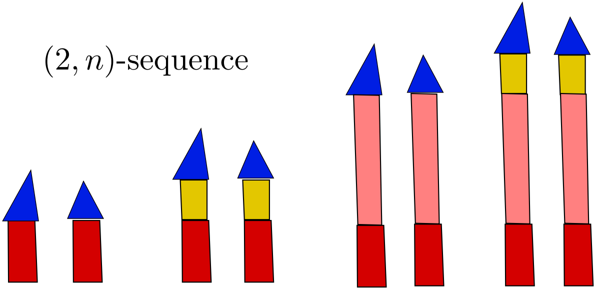


From (ℓ, n) to $(\ell-1, n-1)$ ... decreasing thickness

$(2, n)$-sequence



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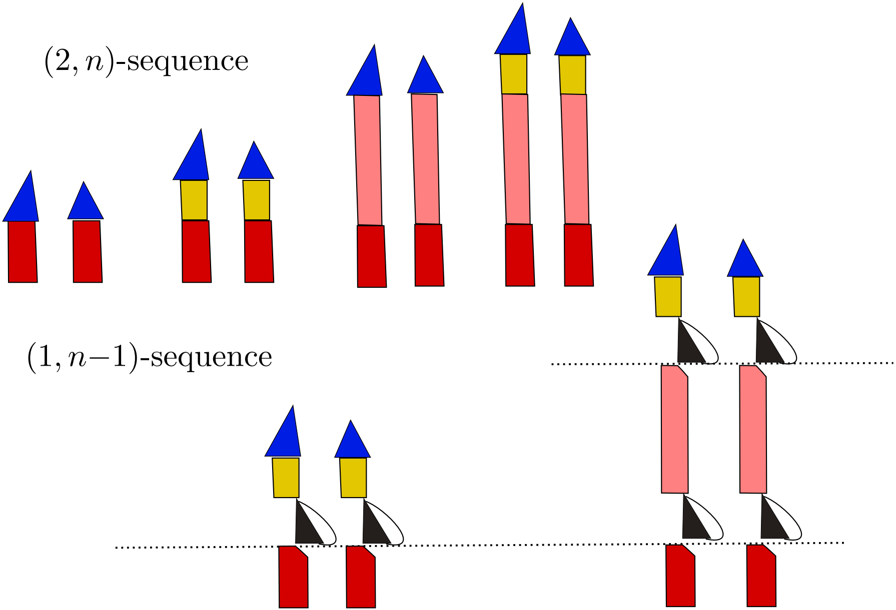

From (ℓ, n) to $(\ell-1, n-1)$ ... decreasing thickness

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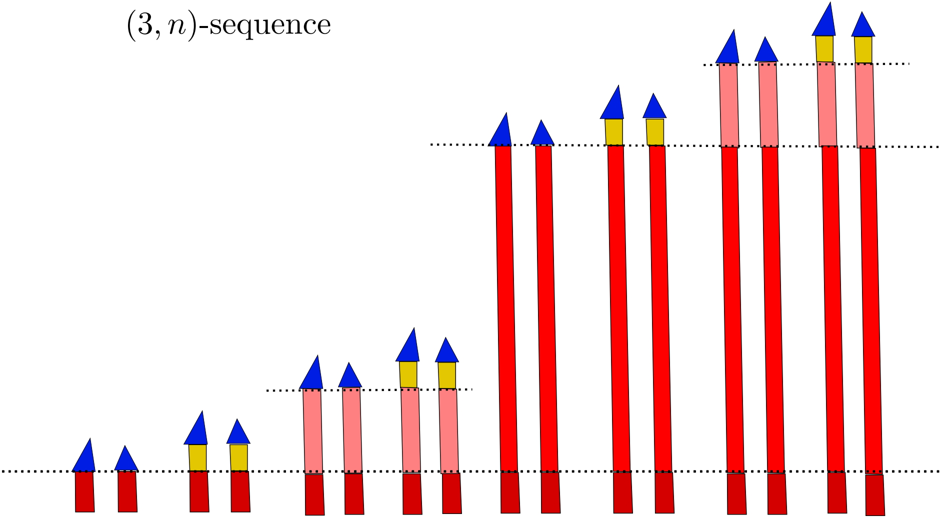

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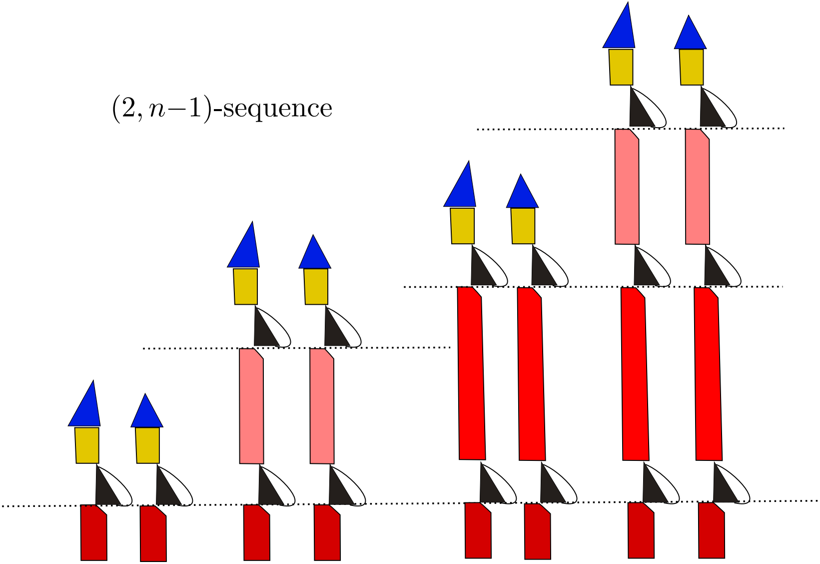
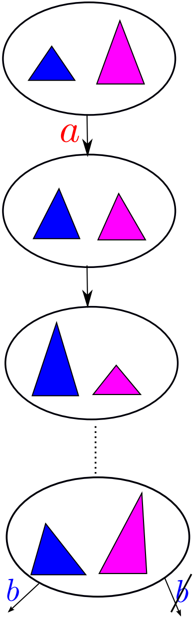


Bounding lengths of witnesses in the deterministic case



Theorem.

There is an elementary function g such that for any det-FO grammar $\mathcal{G} = (\mathcal{N}, \mathcal{A}, \mathcal{R})$ and $T \not\sim U$ of size n we have

$$EL(T,U) \leq tower(g(n)).$$

Proof is based on two ideas:

1. "Synchronize" the growth of lhs-terms and rhs-terms *while not changing the respective eq-levels*. (Hence no repeat.)
2. Derive a tower-bound on the size of terms in the (modified) sequence.

A lower bound

Bisimulation equivalence for FO-grammars is Ackermann-hard.

Note:
Benedikt M., Göller S., Kiefer S., Murawski A.S.:
Bisimilarity of Pushdown Automata is Nonelementary. LICS 2013
(no ε-transitions)

Ackermann function, class ACK, ACK-completeness

Family $f_0, f_1, f_2, \ldots$ of functions:

$$f_0(n) = n{+}1$$
$$f_{k+1}(n) = f_k(f_k(\ldots f_k(n)\ldots)) = f_k^{(n+1)}(n)$$

Ackermann function f_A: $f_A(n) = f_n(n)$.

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where g is a primitive recursive function.

Ackermann-budget halting problem (AB-HP):

Instance: Minsky counter machine M.
Question: does M halt from the zero initial configuration within $f_A(size(M))$ steps ?

Fact. AB-HP is ACK-complete.

Control state reachability in reset counter machines

Reset counter machines (RCMs).

nonnegative counters $c_1, c_2, \ldots, c_d$,

control states $1, 2, \ldots, r$,

configuration $(\ell, (n_1, n_2, \ldots, n_d))$, initial conf. $(1, (0, 0, \ldots, 0))$,

(nondeterministic) instructions of the types

$\ell \xrightarrow{inc(c_i)} \ell'$ *(increment c_i),*

$\ell \xrightarrow{dec(c_i)} \ell'$ *(decrement c_i, if $c_i > 0$),*

$\ell \xrightarrow{reset(c_i)} \ell'$ *(reset c_i, i.e., put $c_i = 0$).*

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CS-reach problem for RCM:

Instance: an RCM M, a control state ℓ_{FIN} .

Question: is $(1, (0, 0, \ldots, 0)) \longrightarrow^* (\ell_{\text{FIN}}, (\ldots))$?

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Question: is $(1, (0, 0, \ldots, 0)) \longrightarrow^* (\ell_{\mathrm{FIN}}, (\ldots))$?

Fact. CS-reach problem for RCM is ACK-complete.
(See [Schnoebelen, MFCS 2010].)

Reduction of CS-reach for RCM to FO-bisimilarity

Given an RCM M, i.e.,

counters $c_1, c_2, \ldots, c_d$,
control states $1, 2, \ldots, r$,

and instructions of the types

$\ell \xrightarrow{inc(c_i)} \ell'$ *(increment* c_i*)*,
$\ell \xrightarrow{dec(c_i)} \ell'$ *(decrement* c_i*, if* $c_i > 0$*)*,
$\ell \xrightarrow{reset(c_i)} \ell'$ *(reset* c_i*, i.e., put* $c_i = 0$*)*,

and ℓ_{FIN},

we construct $\mathcal{G} = (\mathcal{N}, \mathcal{A}, \mathcal{R})$ and E_0, F_0 so that

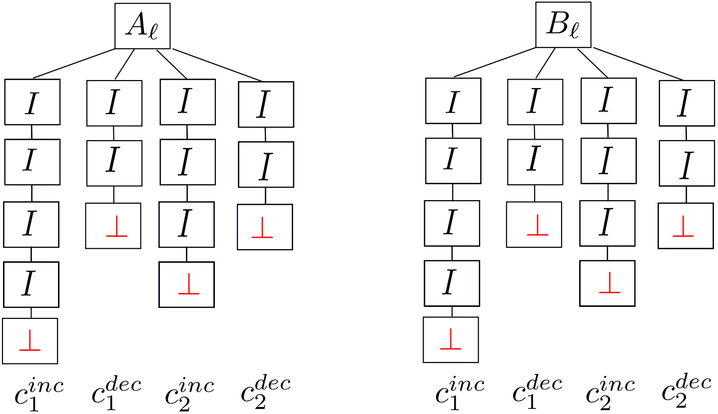
$$(1, (0, 0, \ldots, 0)) \longrightarrow^* (\ell_{\mathrm{FIN}}, (\ldots)) \quad \textit{iff} \quad E_0 \not\sim F_0.$$

CS-reachability as bisimulation game

Example with counters c_1, c_2; we start with the pair

$$(A_1(\perp, \perp, \perp, \perp,), B_1(\perp, \perp, \perp, \perp)).$$

The pair after mimicking $(1, (0, 0)) \longrightarrow^* \boxed{(\ell, (2, 1))}$ might be



Attacker's win

Attacker wins in

$$(A_{\ell_{\mathrm{FIN}}}(\dots), B_{\ell_{\mathrm{FIN}}}(\dots))$$

due to the rule $A_{\ell_{\mathrm{FIN}}}(x_1, x_2, x_3, x_4) \xrightarrow{a} \dots$ (while there is no rule for $B_{\ell_{\mathrm{FIN}}}$).

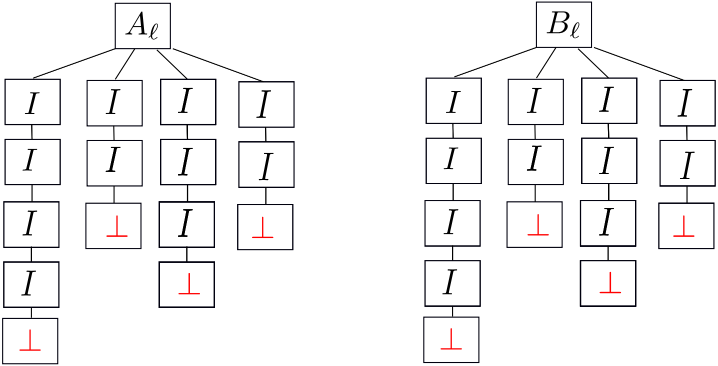
Counter increment

For $\boxed{ins = \ell \xrightarrow{inc(c_2)} \ell'}$

we have rules

$A_\ell(x_1, x_2, x_3, x_4) \xrightarrow{ins} A_{\ell'}(x_1, x_2, I(x_3), x_4)$,

$B_\ell(x_1, x_2, x_3, x_4) \xrightarrow{ins} B_{\ell'}(x_1, x_2, I(x_3), x_4)$,



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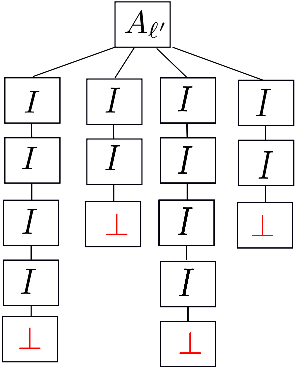

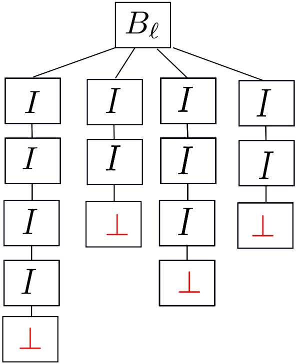

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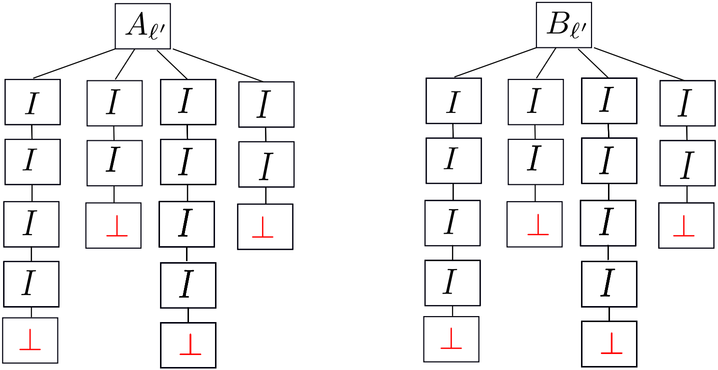

Counter reset

For $\boxed{ins = \ell \xrightarrow{reset(c_2)} \ell'}$

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$A_\ell(x_1, x_2, x_3, x_4) \xrightarrow{ins} A_{\ell'}(x_1, x_2, \perp, \perp)$,

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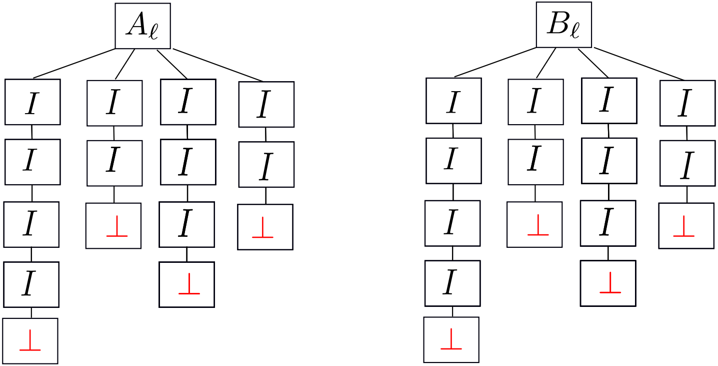

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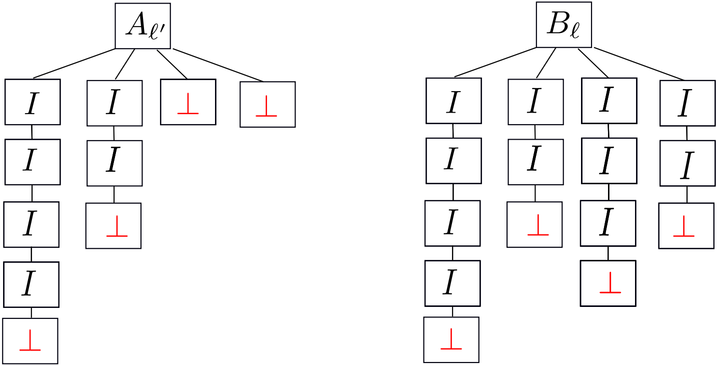

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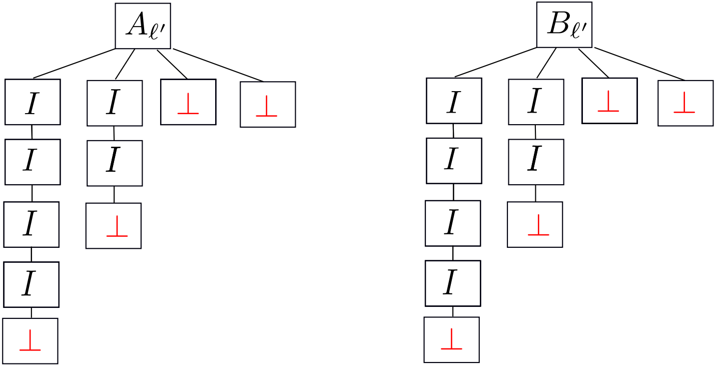

Counter decrement

For $\boxed{ins = \ell \xrightarrow{dec(c_2)} \ell'}$ we have two phases; the first-phase rules are

$$A_\ell \xrightarrow{ins} A_{(\ell',2)},\ A_\ell \xrightarrow{ins} B_{(\ell',2,a)},\ A_\ell \xrightarrow{ins} B_{(\ell',2,b)},$$

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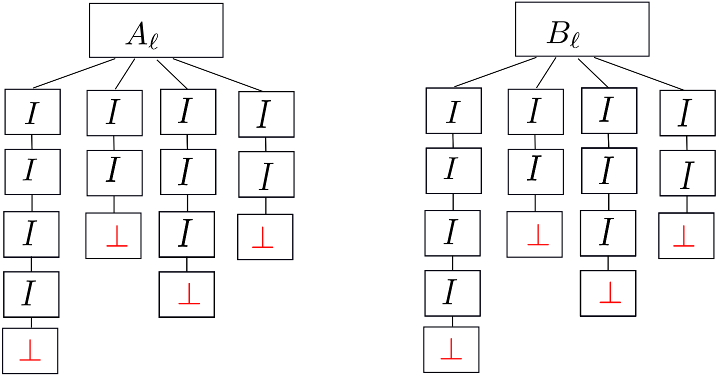

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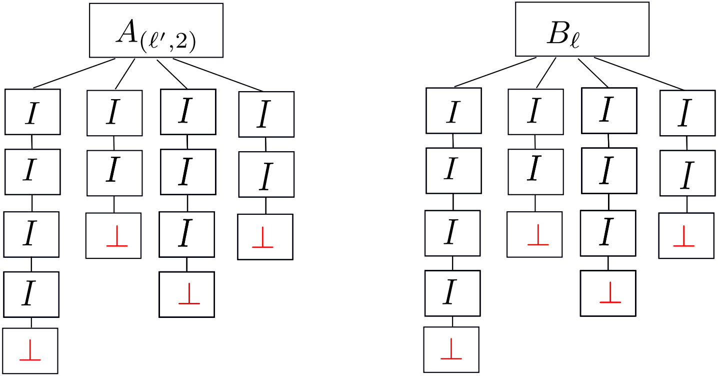

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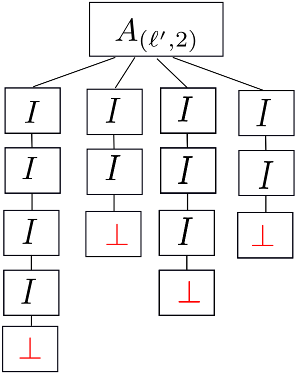
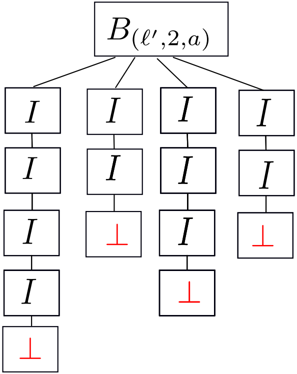
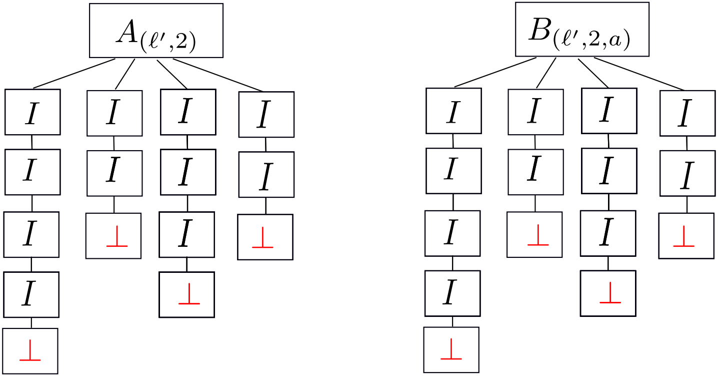




Counter decrement (option a)

$A_{(\ell',2)}(x_1,x_2,x_3,x_4) \xrightarrow{a} A_{\ell'}(x_1,x_2,x_3,I(x_4)),\ A_{\ell',2}(x_1,x_2,x_3,x_4) \xrightarrow{b} x_3,$
$B_{(\ell',2,a)}(x_1,x_2,x_3,x_4) \xrightarrow{a} B_{\ell'}(x_1,x_2,x_3,I(x_4)),$

$$B_{(\ell',2,a)}(x_1,x_2,x_3,x_4) \xrightarrow{b} x_3,$$



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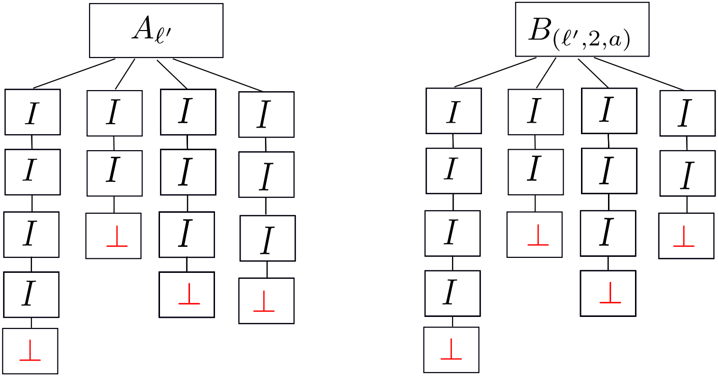

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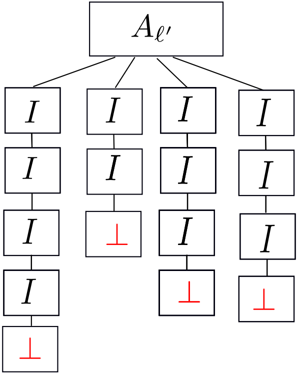
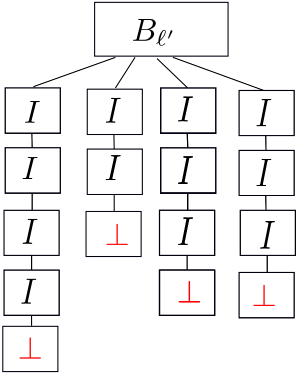
$$B_{(\ell',2,a)}(x_1,x_2,x_3,x_4) \xrightarrow{b} x_3,$$





Counter decrement (option b)

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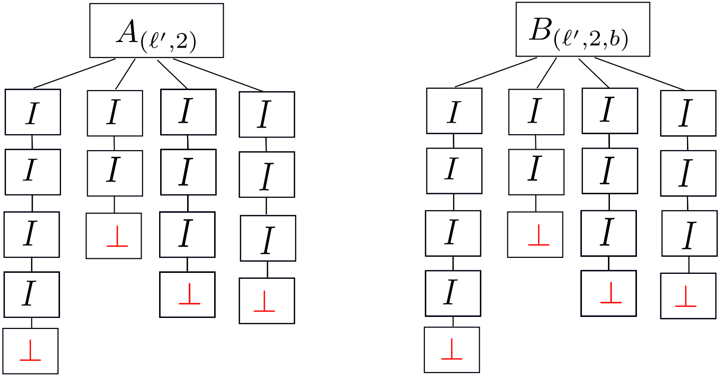

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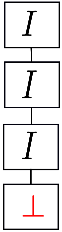

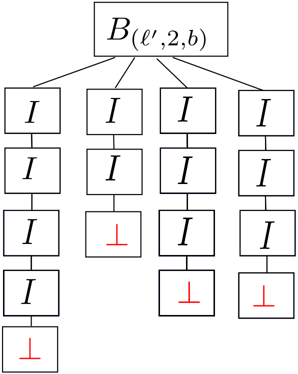

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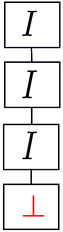

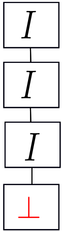

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  - J.: Bisimulation Equivalence of First-Order Grammars; arxiv.org/abs/1405.7923