Analysis of Distributed Probabilistic Systems: Limitations and Possibilities

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Overview

- Motivation
- Distributed Schedulers
- Strongly Distributed Schedulers
- Distributed Schedulers under secrecy
- (Un)decidability results
- Concluding remarks

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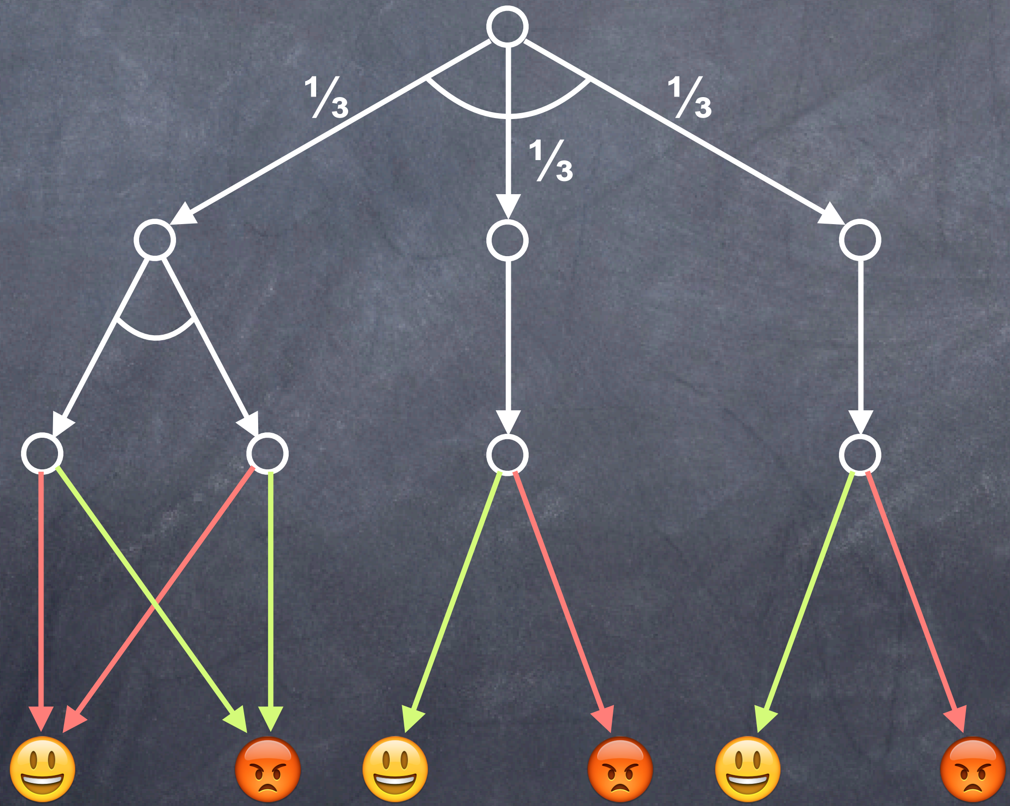
Model Checking
Probabilistic Concurrent Systems

- Nondeterminism resolved through schedulers



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Model Checking Probabilistic Concurrent Systems

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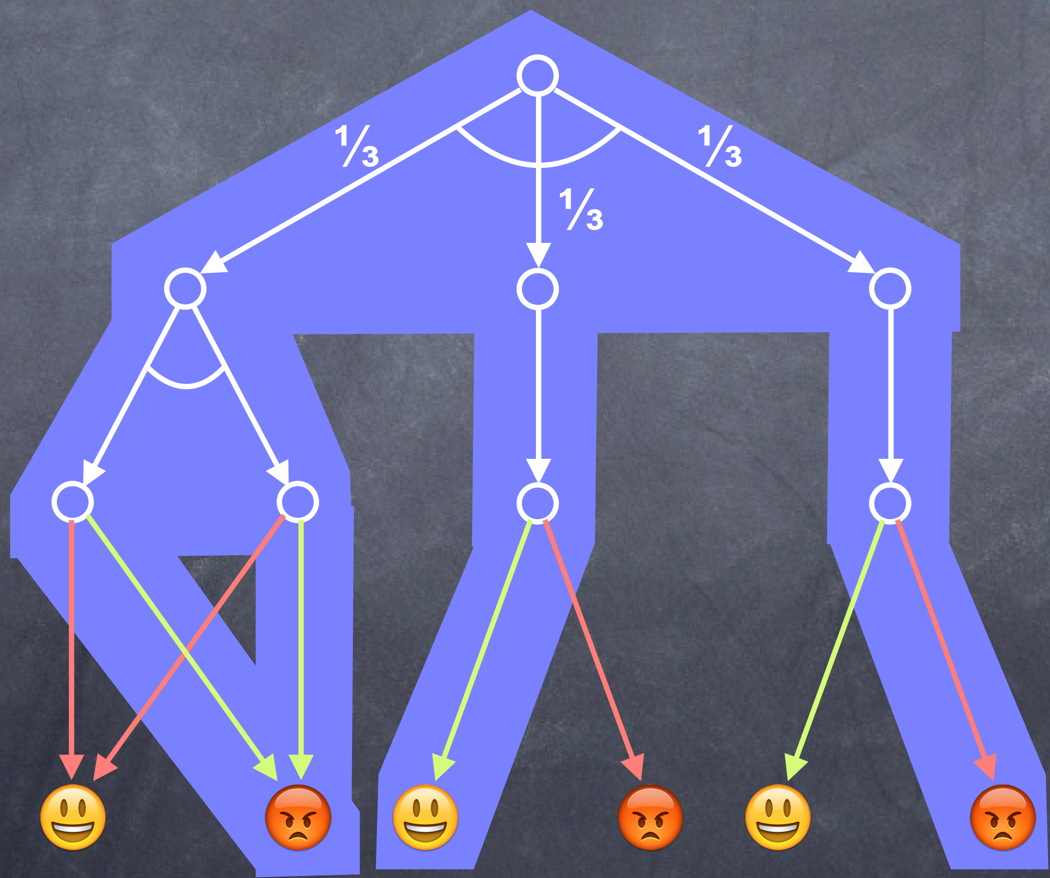
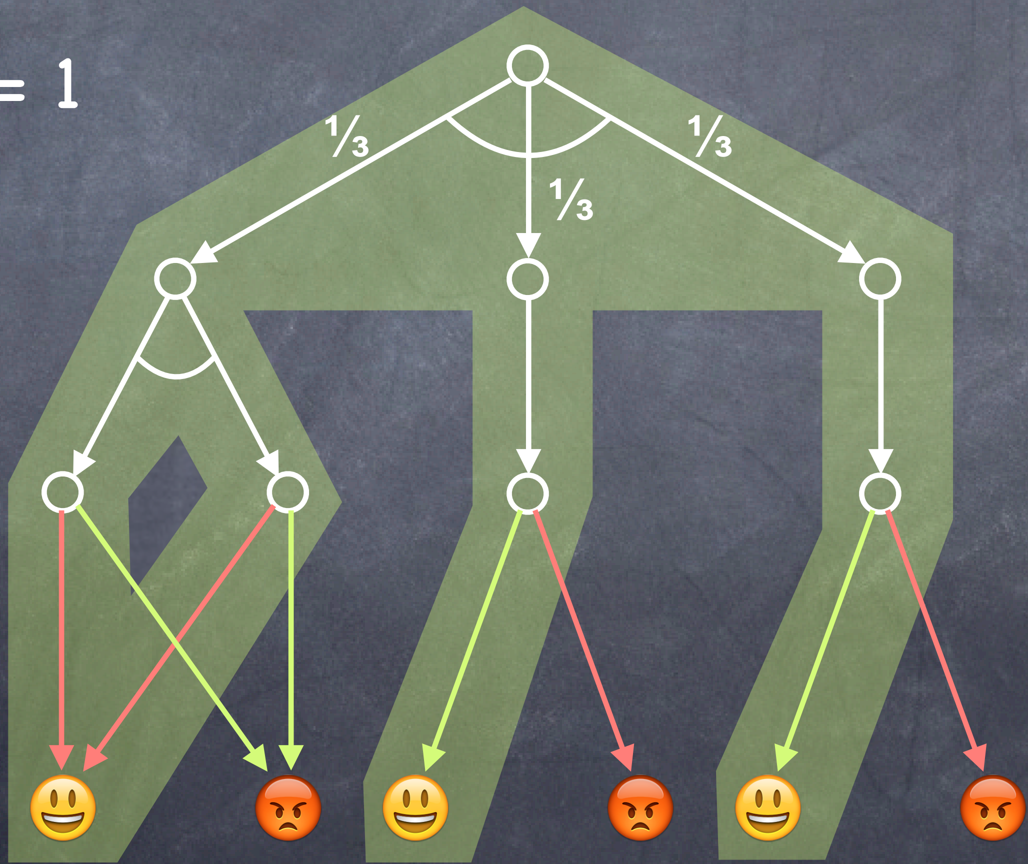


Model Checking
Probabilistic Concurrent Systems

- Nondeterminism resolved through schedulers
- Quantifies over all possible schedulers

sup P(F😃) = 1



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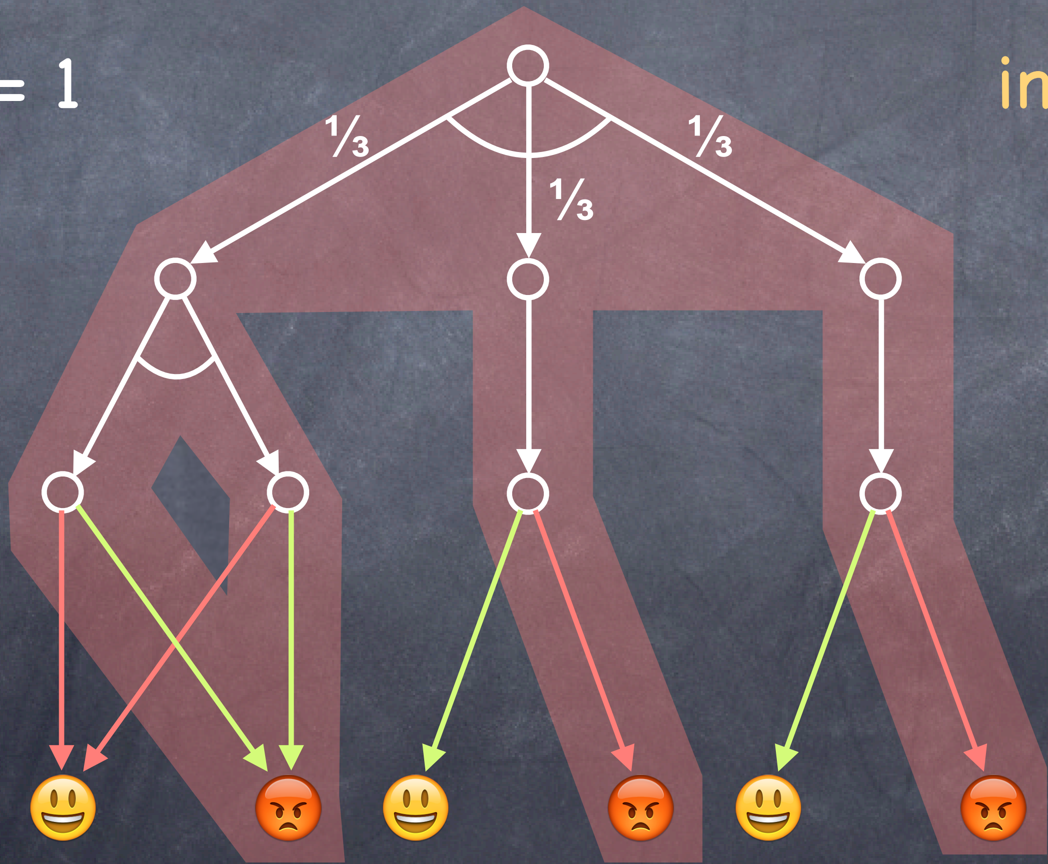
Model Checking Probabilistic Concurrent Systems

- Nondeterminism resolved through schedulers
- Quantifies over all possible schedulers

sup P(F😃) = 1

inf P(F😃) = 0



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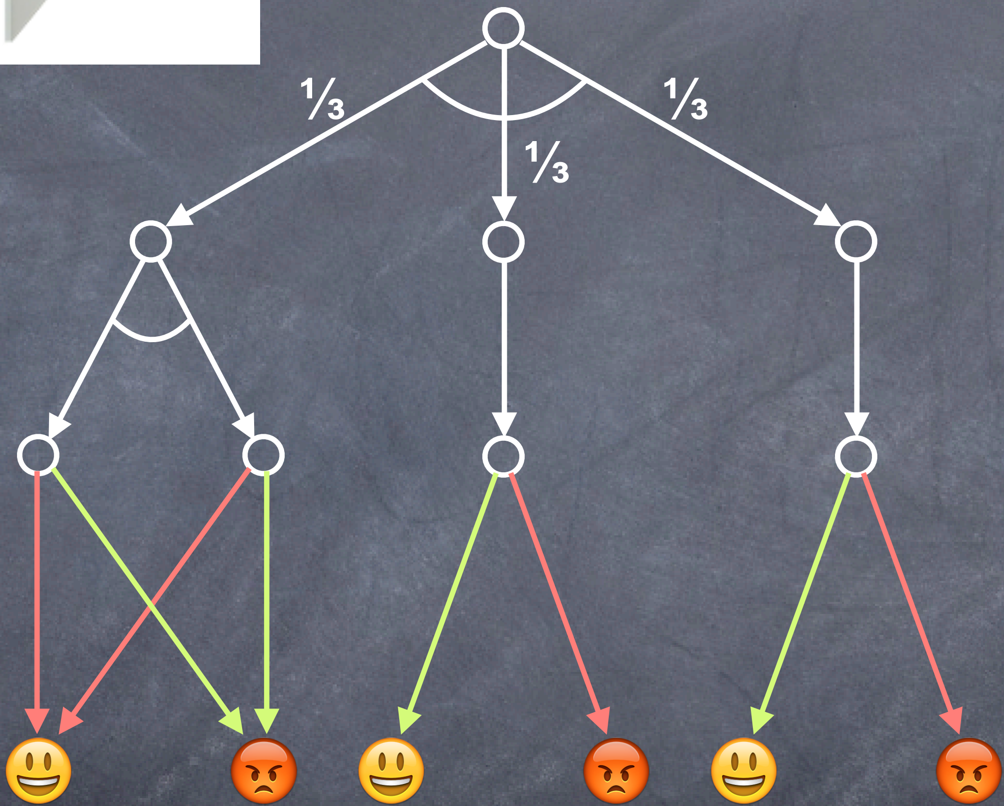
Monty Hall problem



choose door

open door

keep door

switch door

sup P(F😃) = 2/3 inf P(F😃) = 1/3

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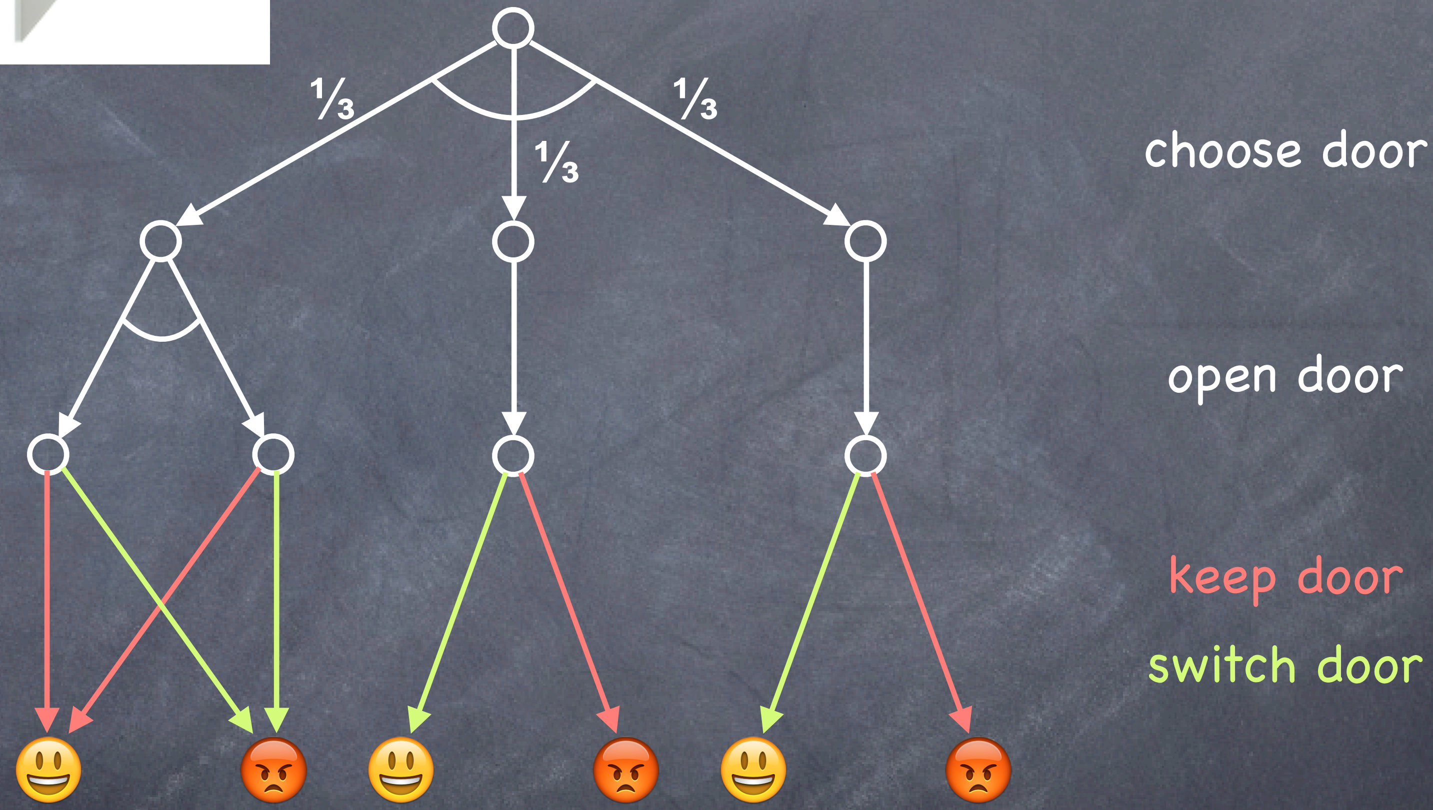
Probabilistic model checking provides a safe over-approximation of the actual probability value

…l Checking …
… Concurrent S…

All schedulers are too many!

Monty Hall problem



sup P(F😃) = 2/3

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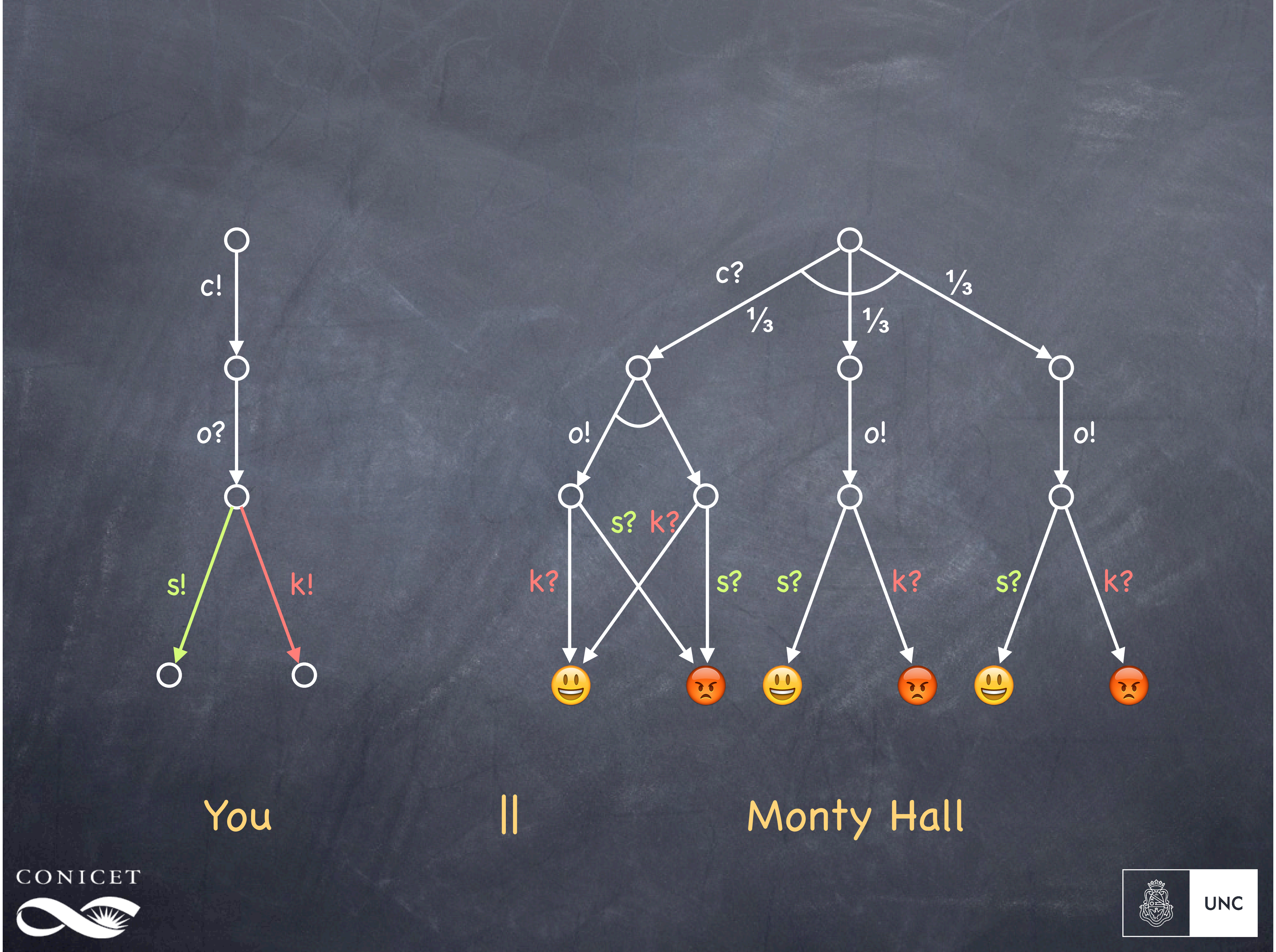
c!
o?
s!
k!
c?
1/3
1/3
1/3
o!
o!
o!
s? k?
k?
s?
s?
k?
s?
k?
You
||
Monty Hall
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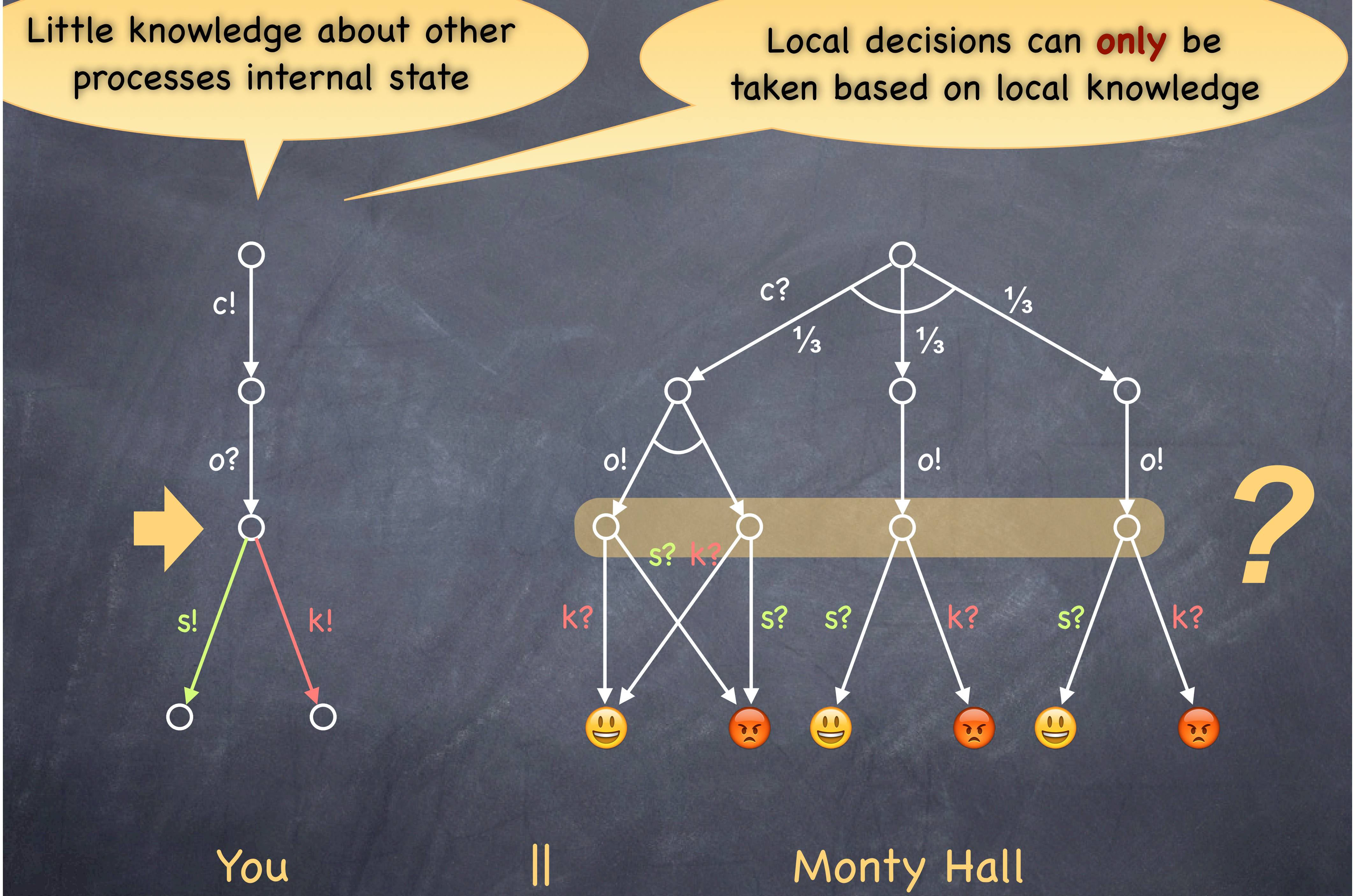
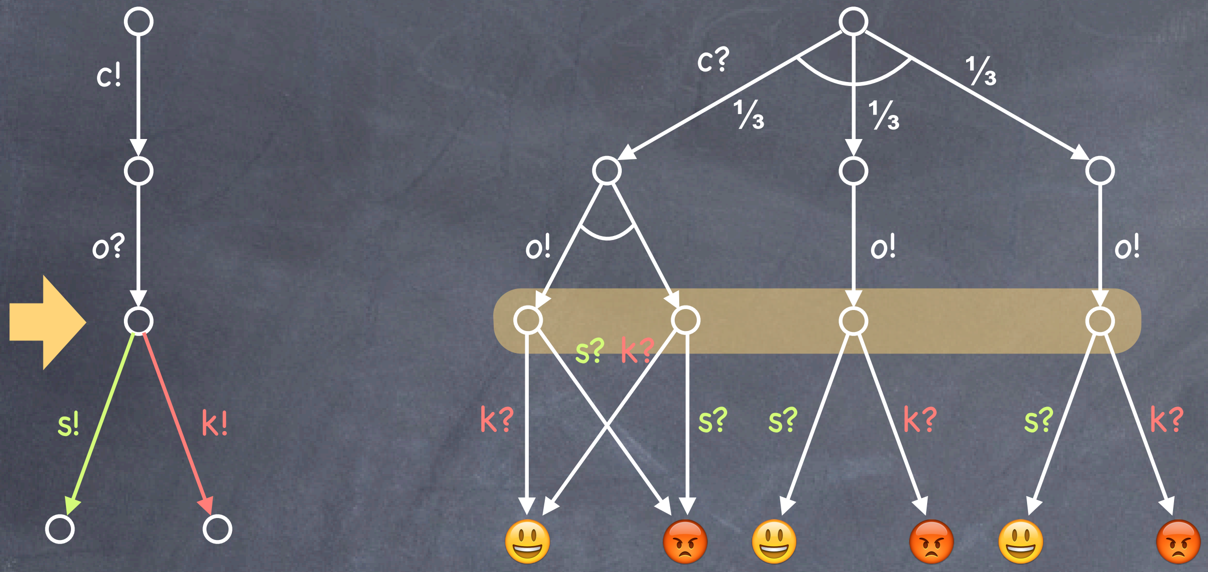
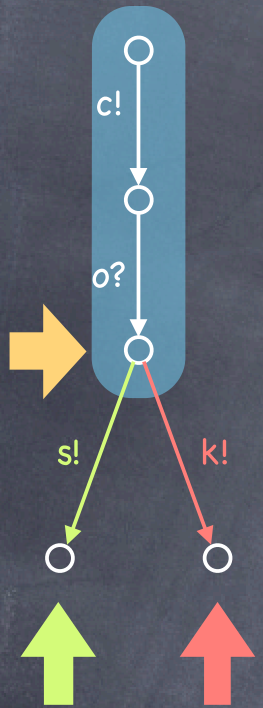
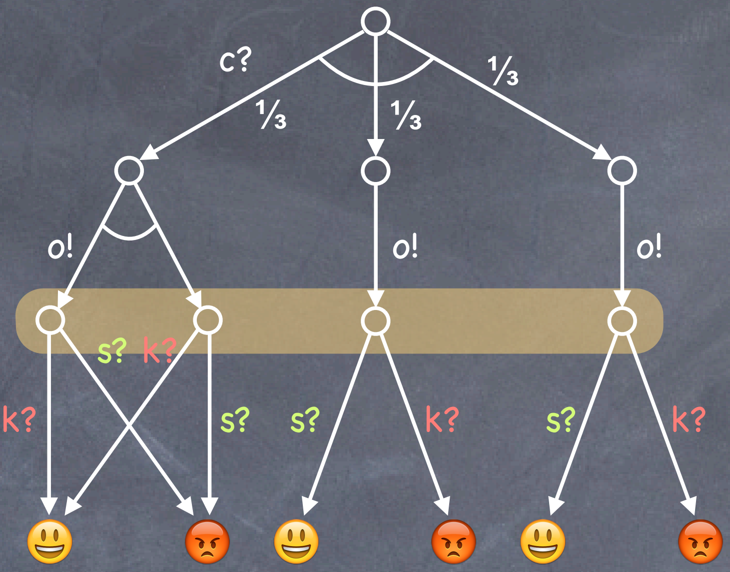


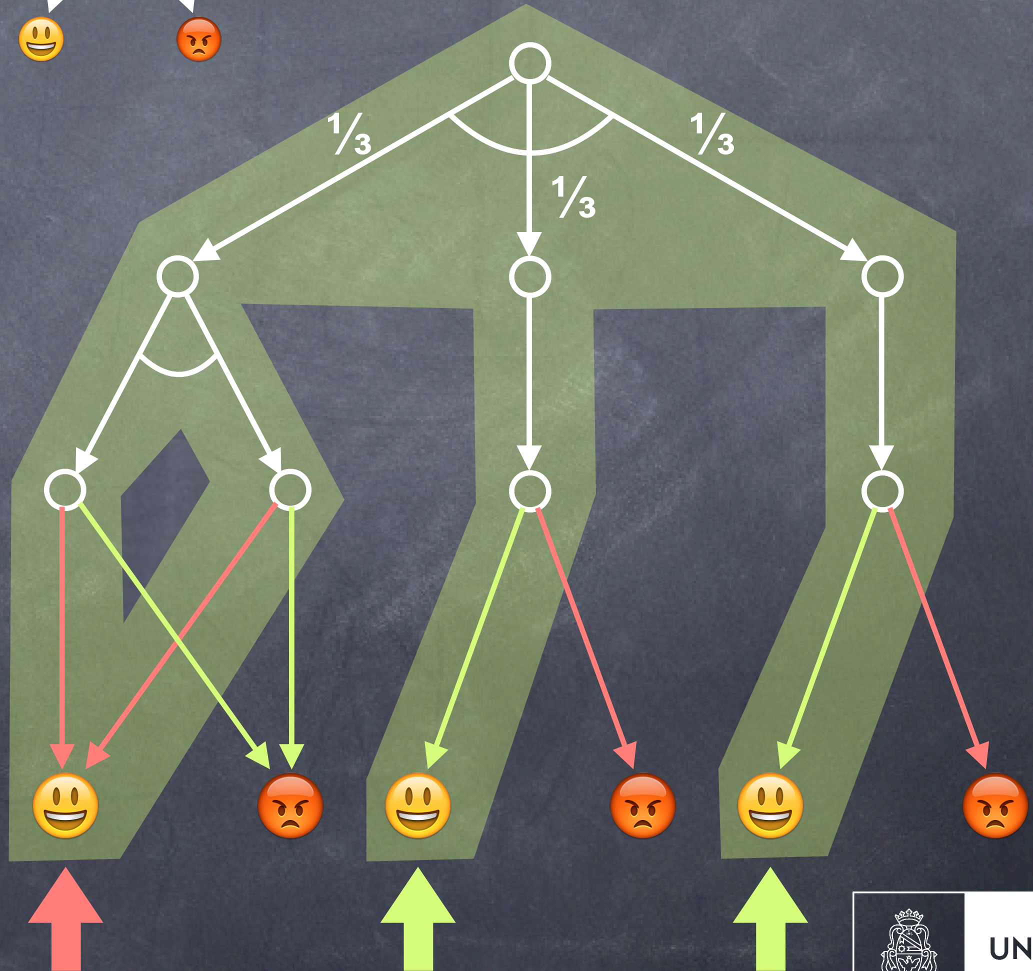
Little knowledge about other processes internal state
Local decisions can **only** be taken based on local knowledge
c!
o?
s!
k!
c?
1/3
1/3
1/3
o!
o!
o!
s?
k?
k?
s?
s?
k?
s?
k?
?
You
||
Monty Hall



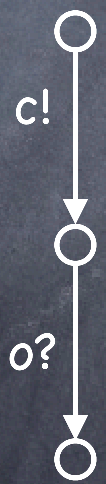
- A distributed scheduler is a scheduler that respects the local decisions of each component.
- Local decisions are only taken with the information available to each component.







Two different choices with the same local knowledge!!



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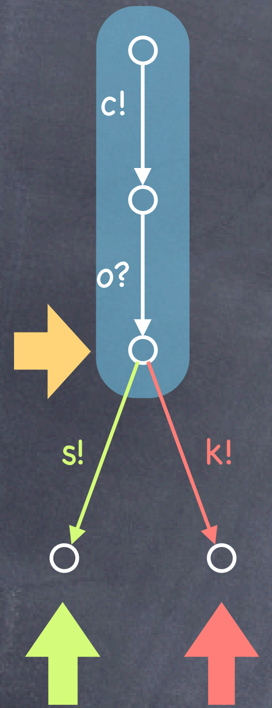
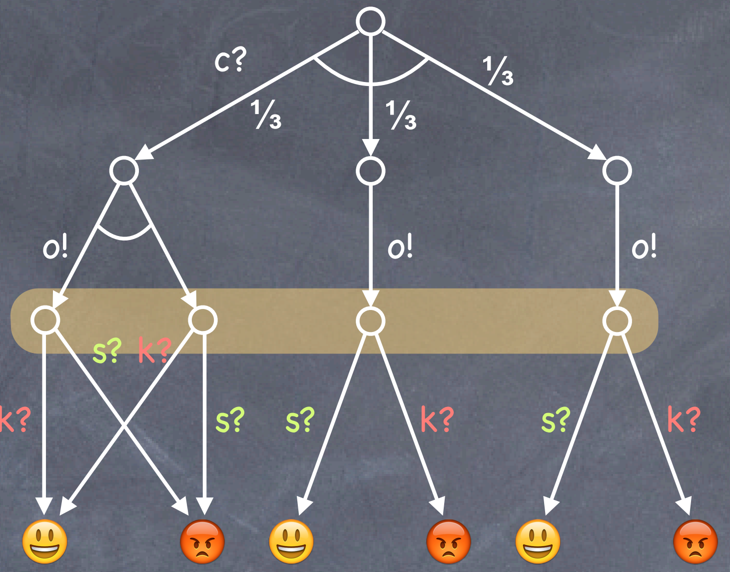
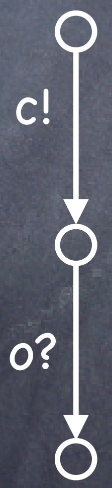
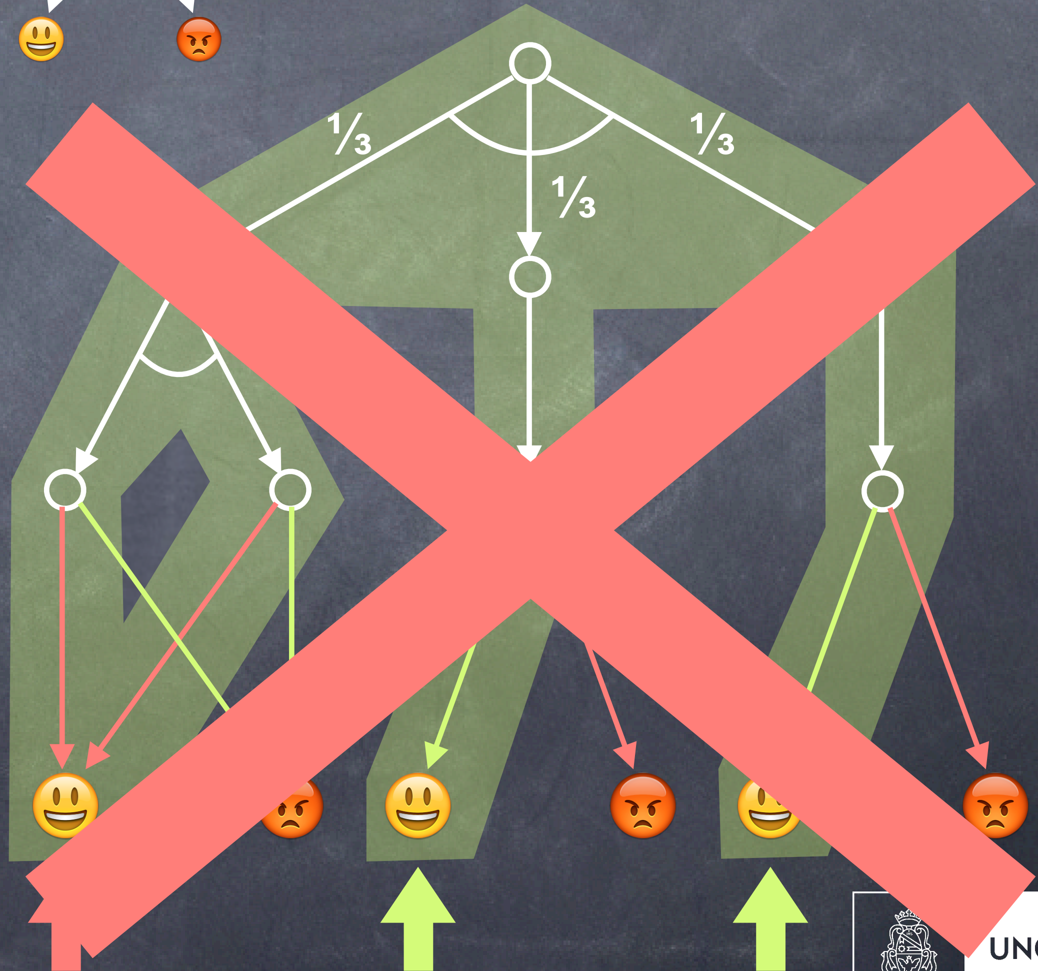




Any acceptable scheduler can do either "keep" or "switch" but not both

Two different choices with the same local knowledge!!





Probabilistic I/O automata

$(S, \bar{s}, L, \longrightarrow)$

- set of states
- initial state, $\bar{s} \in S$
- set of labels partitioned in inputs (I) and outputs (O)
- $\longrightarrow \in S \times L \times \text{Dist}(S)$ is the (probabilistic) transition relation

input enabled: $\forall a \in I : s \xrightarrow{a}$

label deterministic: $\forall a \in L : (s \xrightarrow{a} \mu' \wedge s \xrightarrow{a} \mu'') \to \mu' = \mu''$

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Composition of PIOA

- Two PIOA A_1, A_2 are compatible if $O_1 \cap O_2 = \varnothing$.
- Their parallel composition is defined by

$$A_1 \,||\, A_2 = (S_1 \times S_2, (\bar{s}_1, \bar{s}_2), L_1 \cup L_2, \to)$$

- with $O = O_1 \cup O_2$ and $I = (L_1 \cup L_2) \setminus O$, and

$$\frac{s_1 \xrightarrow{a} \mu_1}{(s_1, s_2) \xrightarrow{a} \mu_1 \times \delta_{s_2}} \quad a \in L_1 \setminus L_2$$

$$\frac{s_1 \xrightarrow{a} \mu_1 \qquad s_2 \xrightarrow{a} \mu_2}{(s_1, s_2) \xrightarrow{a} \mu_1 \times \mu_2} \quad a \in L_1 \cap L_2$$

Because of compatibility, at most one component produces an output in the composed transition

Extends to multiple components as expected

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Execution of PIOA

An execution fragment of a PIOA is a sequence

$$s_0 a_0 \mu_0 s_1 a_1 \mu_1 s_2 \ldots s_{m-1} a_{m-1} \mu_{m-1} s_m$$

such that $s_i \xrightarrow{a_i} \mu_i$ and $\mu_i(s_{i+1}) > 0$

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Schedulers

- A scheduler is a mapping from execution fragments to distributions on transitions enabled in the current state.
- Two steps to construct distributed schedulers:
  1. choose the active component A_i (i.e. the one that will produce an output),
  2. let A_i choose one output transition according to the local knowledge (suppose its label is a).
- All other A_j matching a (as an input) will do so in a parallel composition (ensured by input enabledness and determinism)

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Schedules output transitions provided this component is chosen to execute.

- For each component A_i we consider an output scheduler $\Theta_i : \mathsf{Frag}_i \to \mathsf{Dist}(O_i)$, s.t.

$$\Theta_i(\sigma)(a) > 0 \quad \text{implies} \quad \mathsf{last}(\sigma) \xrightarrow{a}_i$$

- For the system $A_1 \,||\cdots||\, A_n$ we define the interleaving scheduler $\mathcal{I} : \mathsf{Frag} \to \mathsf{Dist}(\{1,\ldots,n\})$, s.t.

$$\mathcal{I}(\sigma)(i) > 0 \quad \text{implies} \quad \exists a \in O_i : \mathsf{last}(\sigma) \xrightarrow{a}$$

Selects randomly the component that will execute an output

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Projection of an execution

The projection on a compnent A_i of an execution fragment σ of a system $A_1 \,||\cdots||\, A_n$ is defined inductively by

$$[(\bar{s}_1, \ldots, \bar{s}_n)]_i \;=\; \bar{s}_i$$

$$[\sigma\, a\, (\mu_1 \times \cdots \times \mu_n)\, (s_1, \ldots, s_n)]_i \;= \;\begin{cases} [\sigma]_i\, a\, \mu_i\, s_i & \text{if } a \in L_i \\ [\sigma]_i & \text{if } a \notin L_i \end{cases}$$

It defines the idea of "local knowledge"

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Distributed Scheduler

A distributed schedulers is a mapping

$$\eta : \mathsf{Frag} \to \mathsf{Dist}(O)$$

s.t. there is a family of output schedulers $\{\Theta_i\}_i$ and an interleaving scheduler $\mathcal{I}$ so that for all $\sigma \in \mathsf{Frag}$:

$$\eta(\sigma)(a) = \textstyle\sum_{i=1}^{n} \mathcal{I}(\sigma)(i) \cdot \Theta_i(\underline{[\sigma]_i})(a)$$

$$= \mathcal{I}(\sigma)(j) \cdot \Theta_j(\underline{[\sigma]_j})(a) \qquad \text{provided } a \in O_j$$

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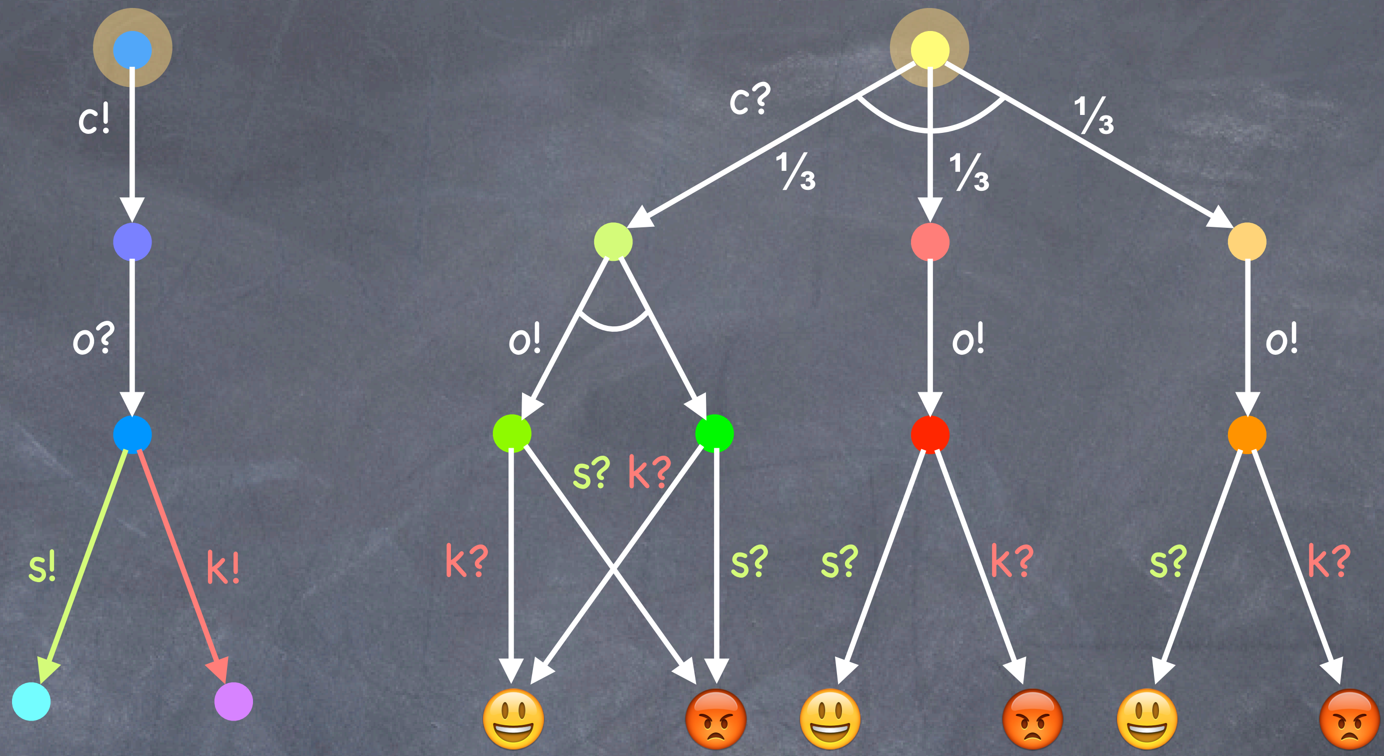
Example revisited



$\mathcal{I}$ ((•,•)) = You

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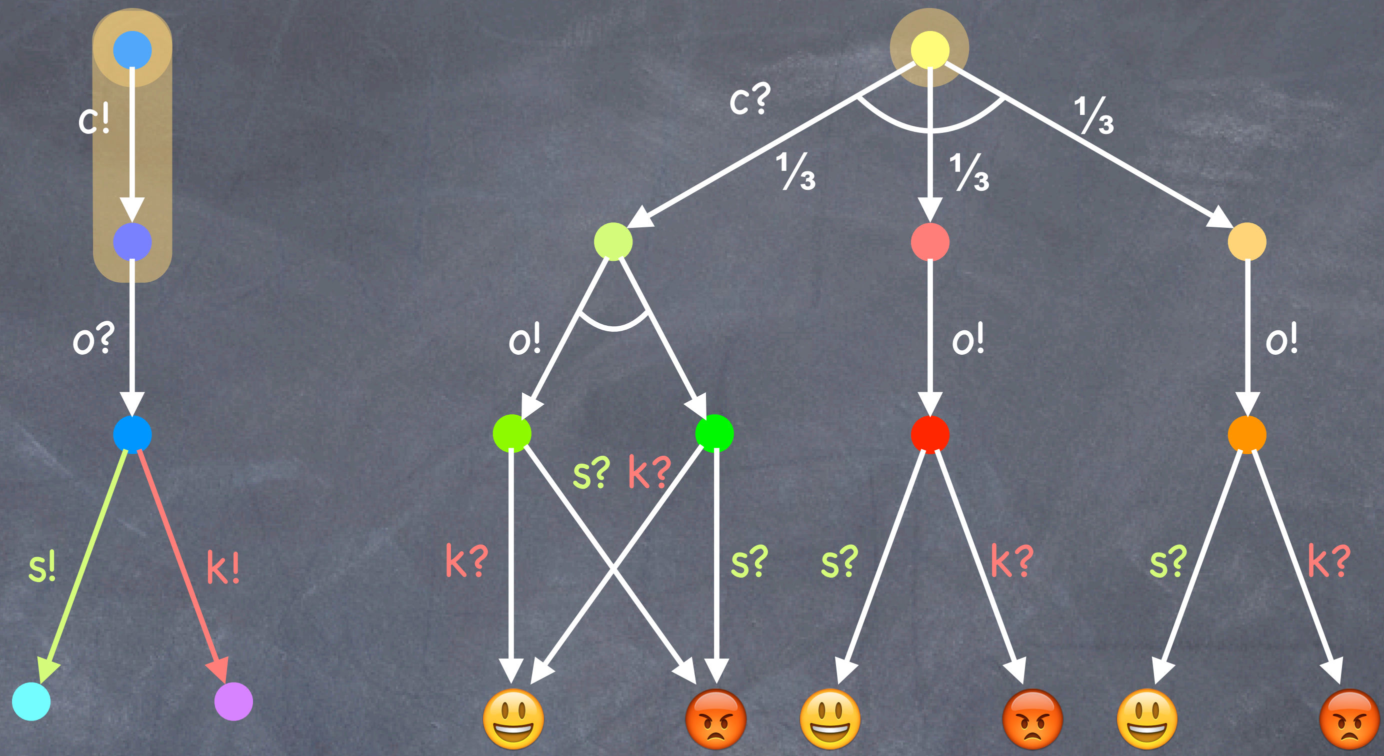
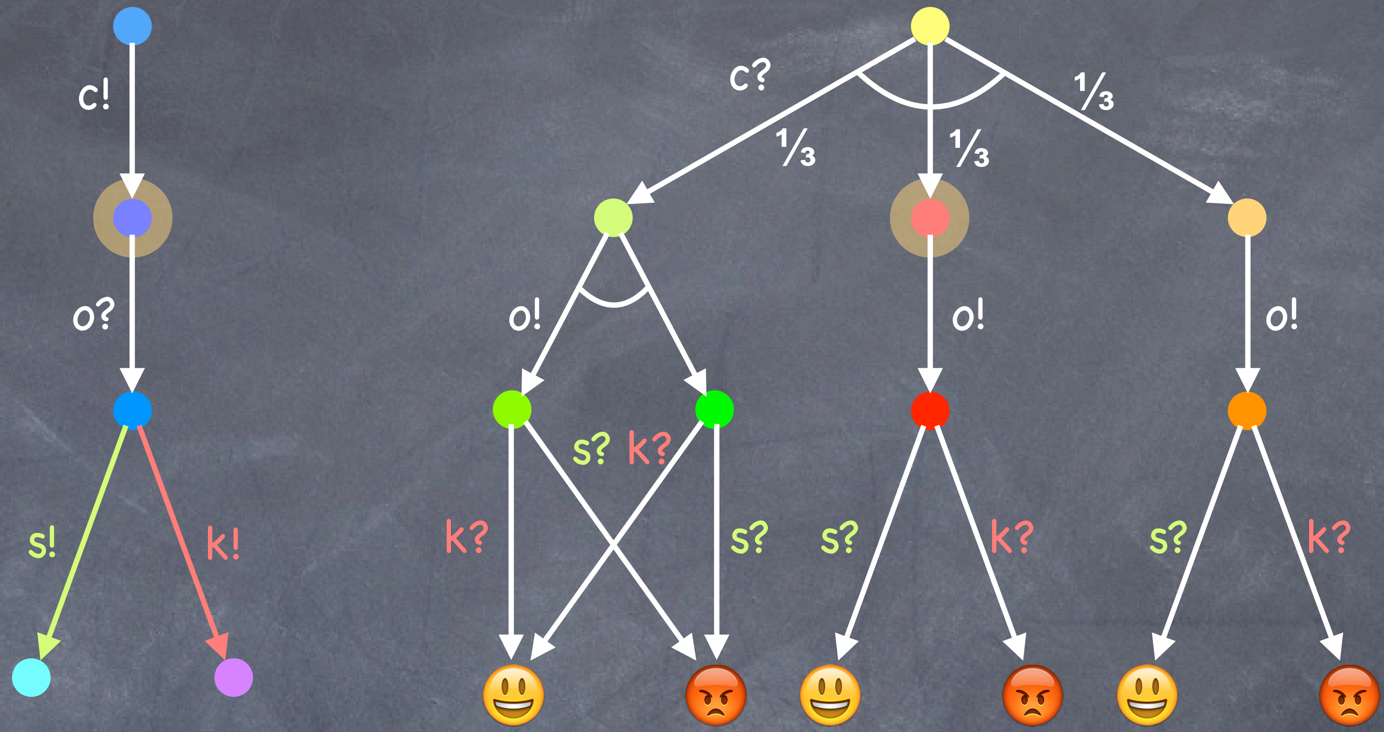
Example revisited



$\mathcal{I}$ ((•,•)) = You

Θ_Y ([(•,•)]$_Y$) = Θ_Y (•) = c!

Example revisited



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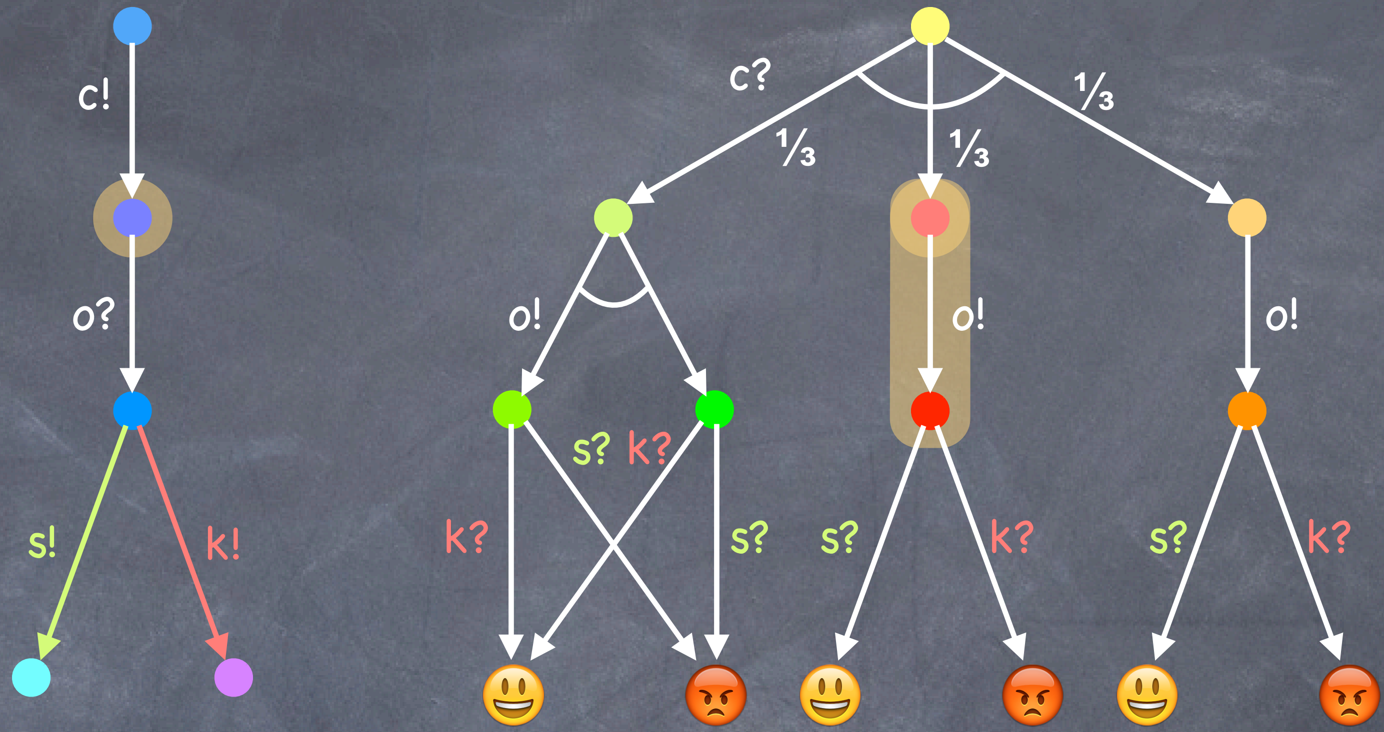
$\mathcal{I}$ ((•,•)c(•,•)) = MH

Example revisited



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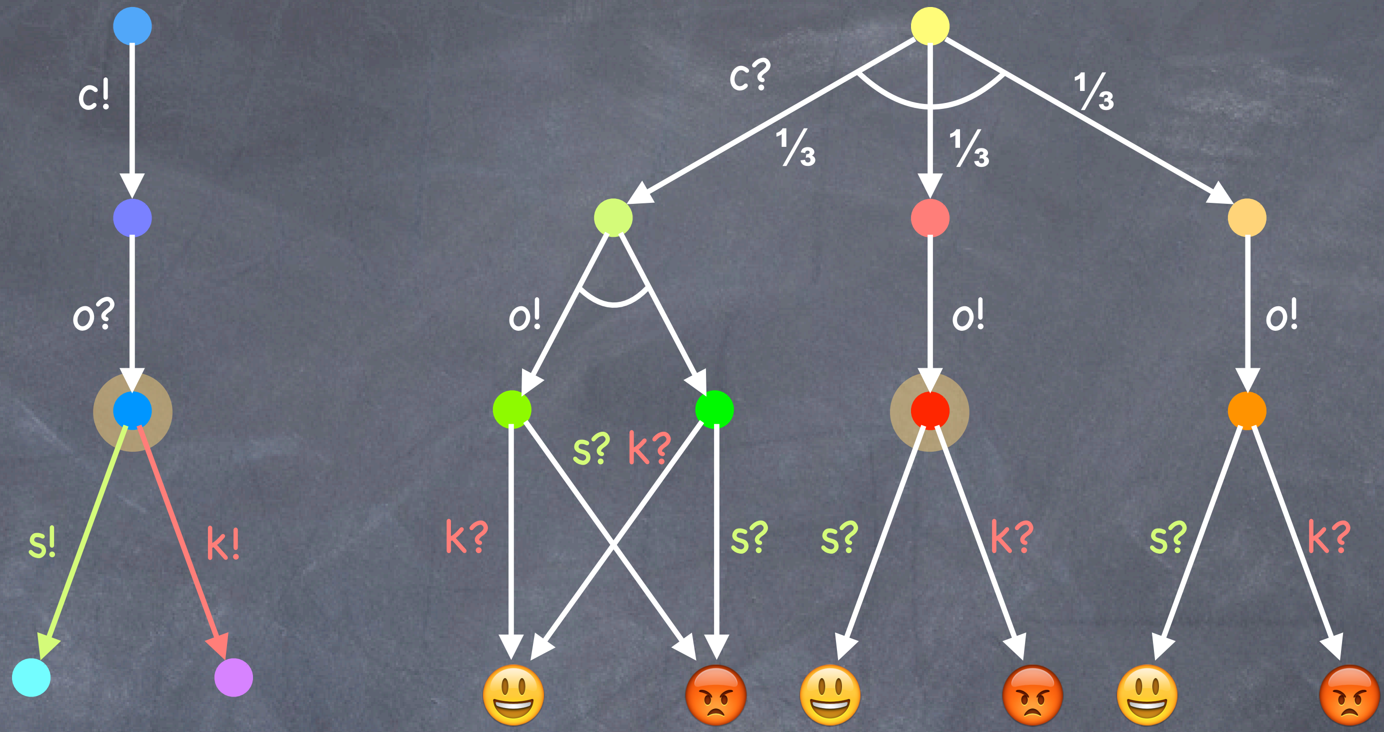
Example revisited



$\mathcal{I}(\,(\bullet,\bullet)\,) = \text{You}$

$\Theta_Y(\,[(\bullet,\bullet)]_Y\,) = \Theta_Y(\bullet) = c!$

$\mathcal{I}(\,(\bullet,\bullet)c(\bullet,\bullet)) = \text{MH}$

$\Theta_{MH}(\,[(\bullet,\bullet)c(\bullet,\bullet)]_{MH}\,) = \Theta_{MH}(\bullet c \bullet) = o!$

$\mathcal{I}(\,(\bullet,\bullet)c(\bullet,\bullet)o(\bullet,\bullet)\,) = \text{You}$

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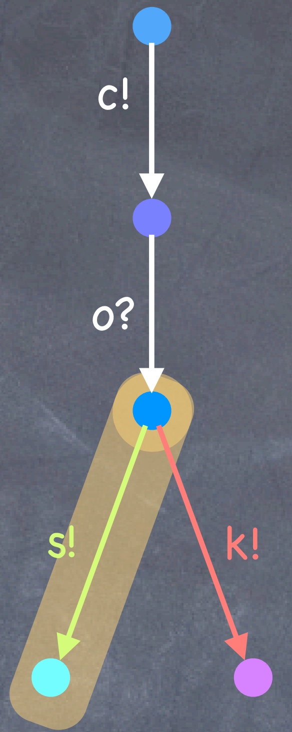
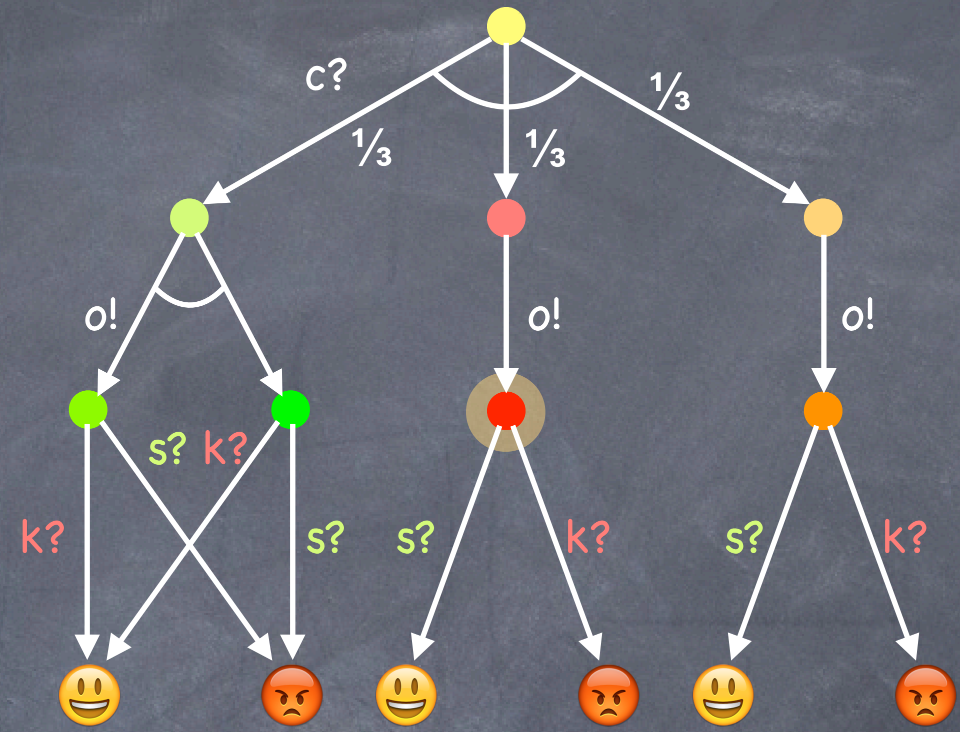
Example revisited





$\mathcal{I}\,(\,(\bullet,\bullet)\,) = \text{You}$

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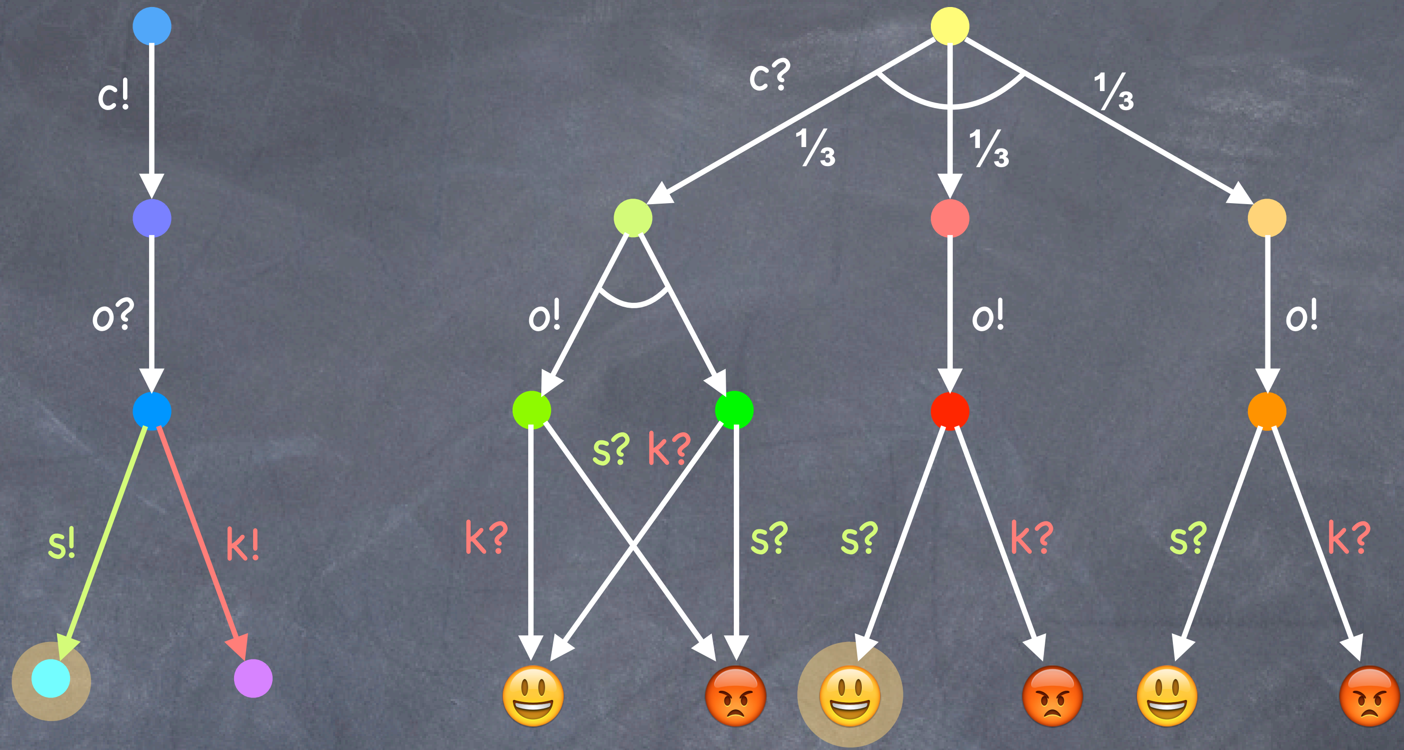
$\Theta_{Y}\,(\,[(\bullet,\bullet)c(\bullet,\bullet)o(\bullet,\bullet)]_{Y}\,) = \Theta_{Y}\,(\bullet c \bullet o \bullet) = s!$

Example revisited



$\mathcal{I}\,(\,(\bullet,\bullet)\,) = \mathrm{You}$

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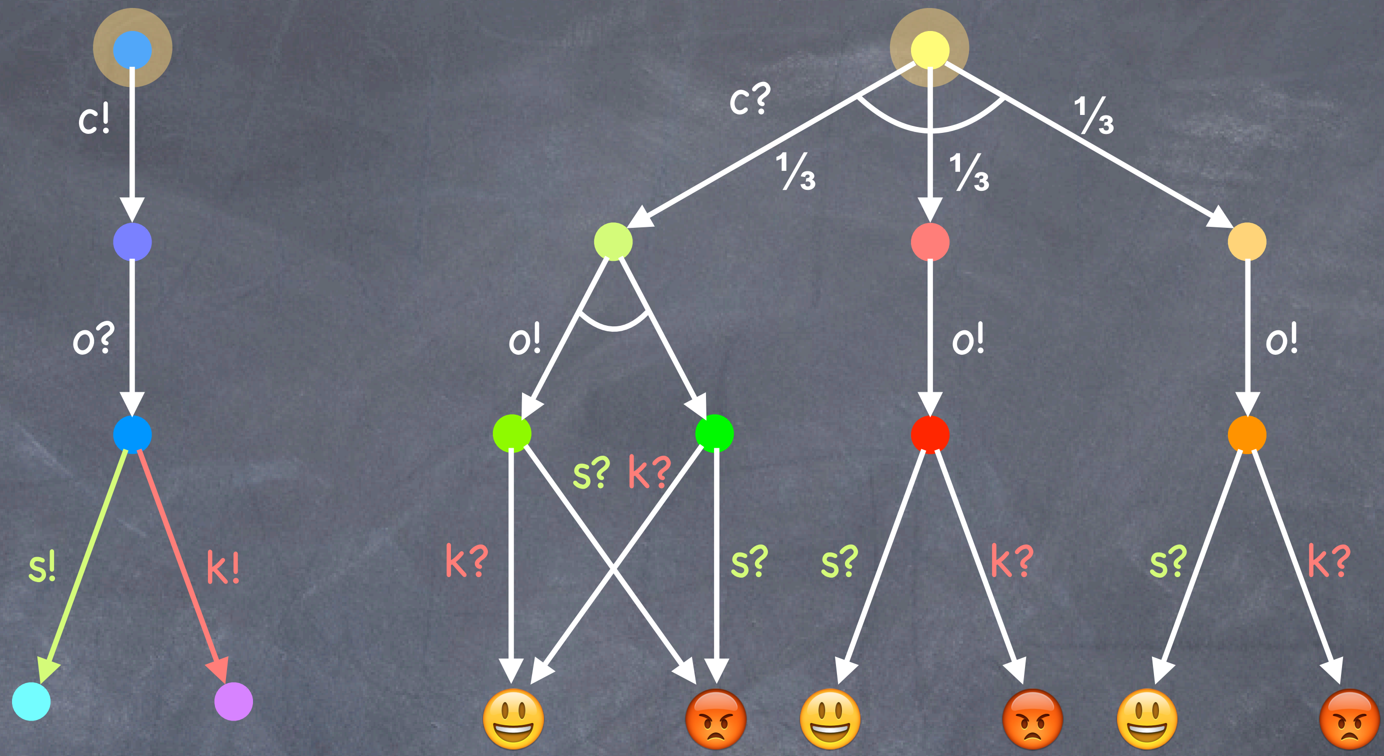
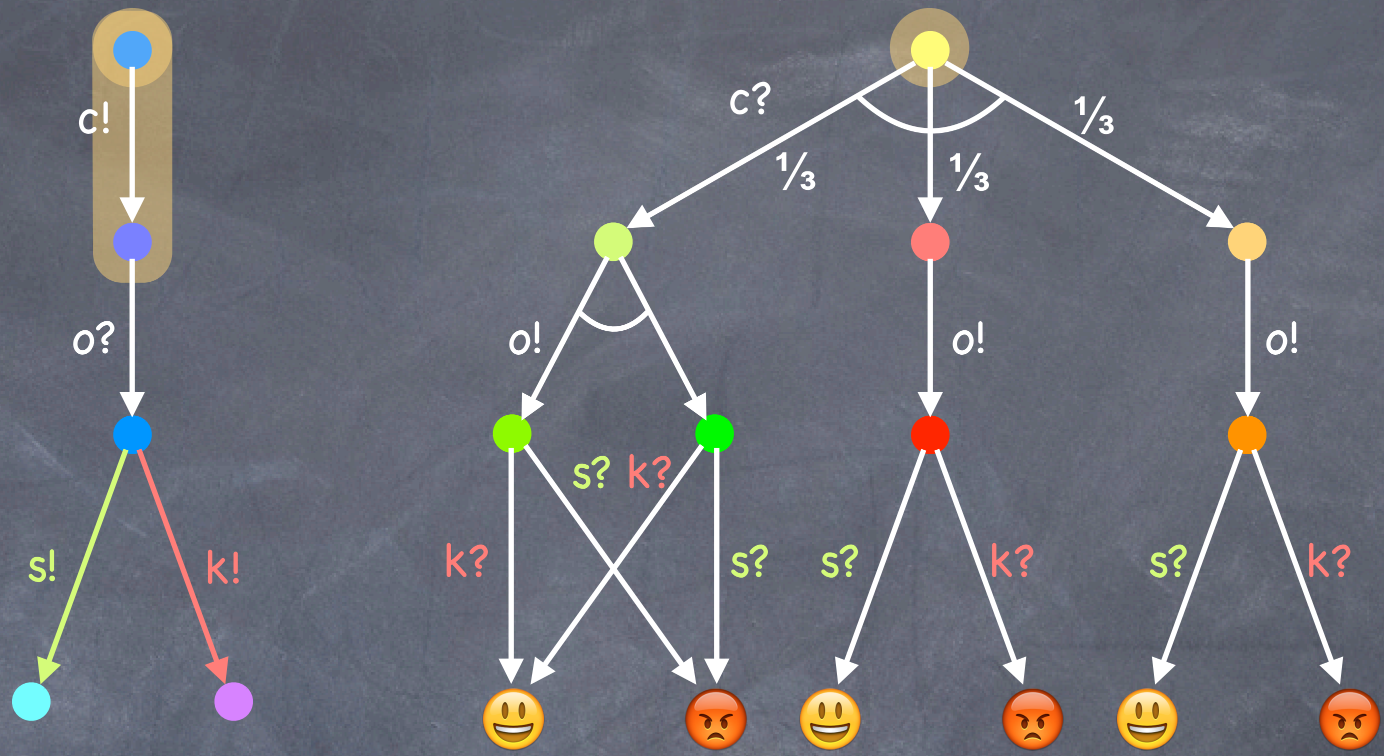
Example revisited



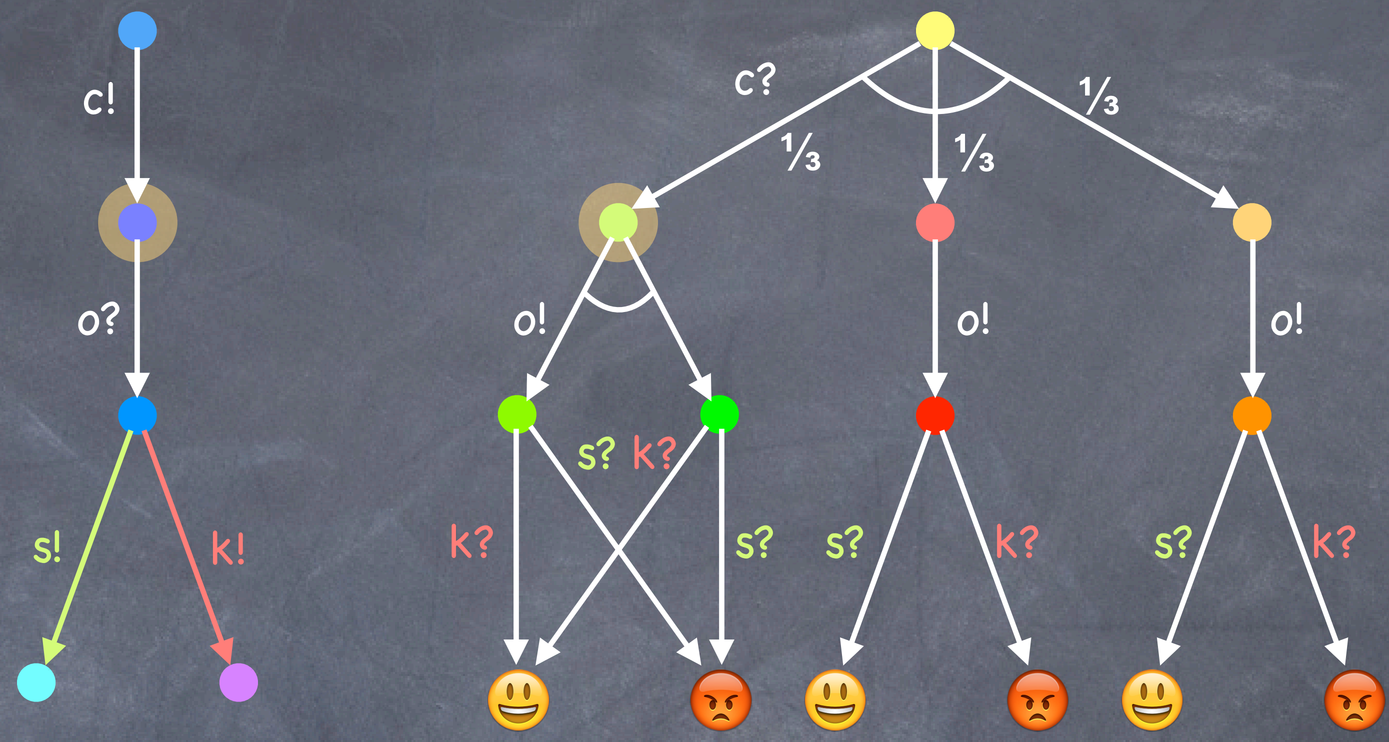
Θ_Y ([(•,•)c(•,•)o(•,•)]$_Y$) = Θ_Y (•c•o•) = s!

Example revisited



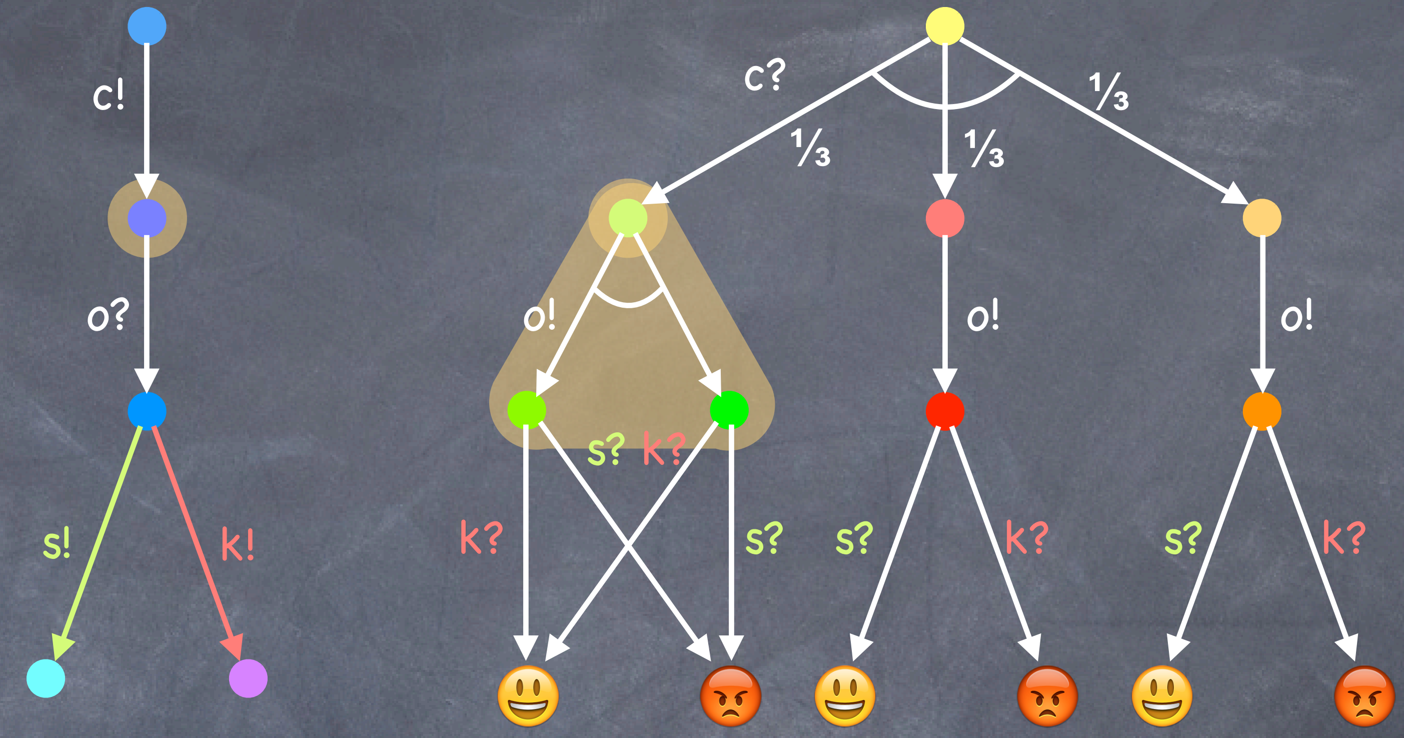
Θ_Y ($[(\bullet,\bullet)c(\bullet,\bullet)o(\bullet,\bullet)]_Y$) = Θ_Y ($\bullet c \bullet o \bullet$) = s!

Example revisited



$$\Theta_Y \left([(\bullet,\bullet)c(\bullet,\bullet)o(\bullet,\bullet)]_Y \right) = \Theta_Y \left(\bullet c \bullet o \bullet\right) = s!$$

Example revisited



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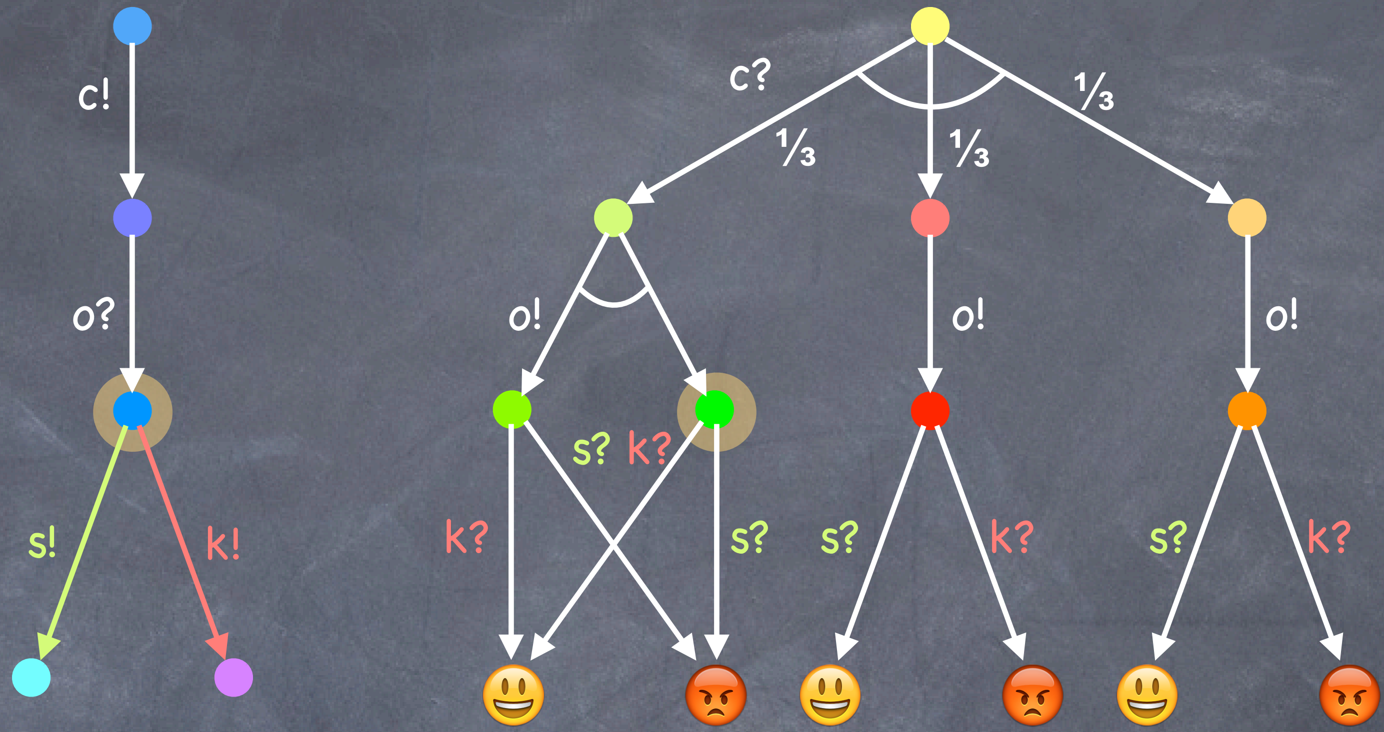
Example revisited



Θ_Y ([(•,•)c(•,•)o(•,•)]$_Y$) = $\boxed{\Theta_Y \text{ (•c•o•)}}$ = s!

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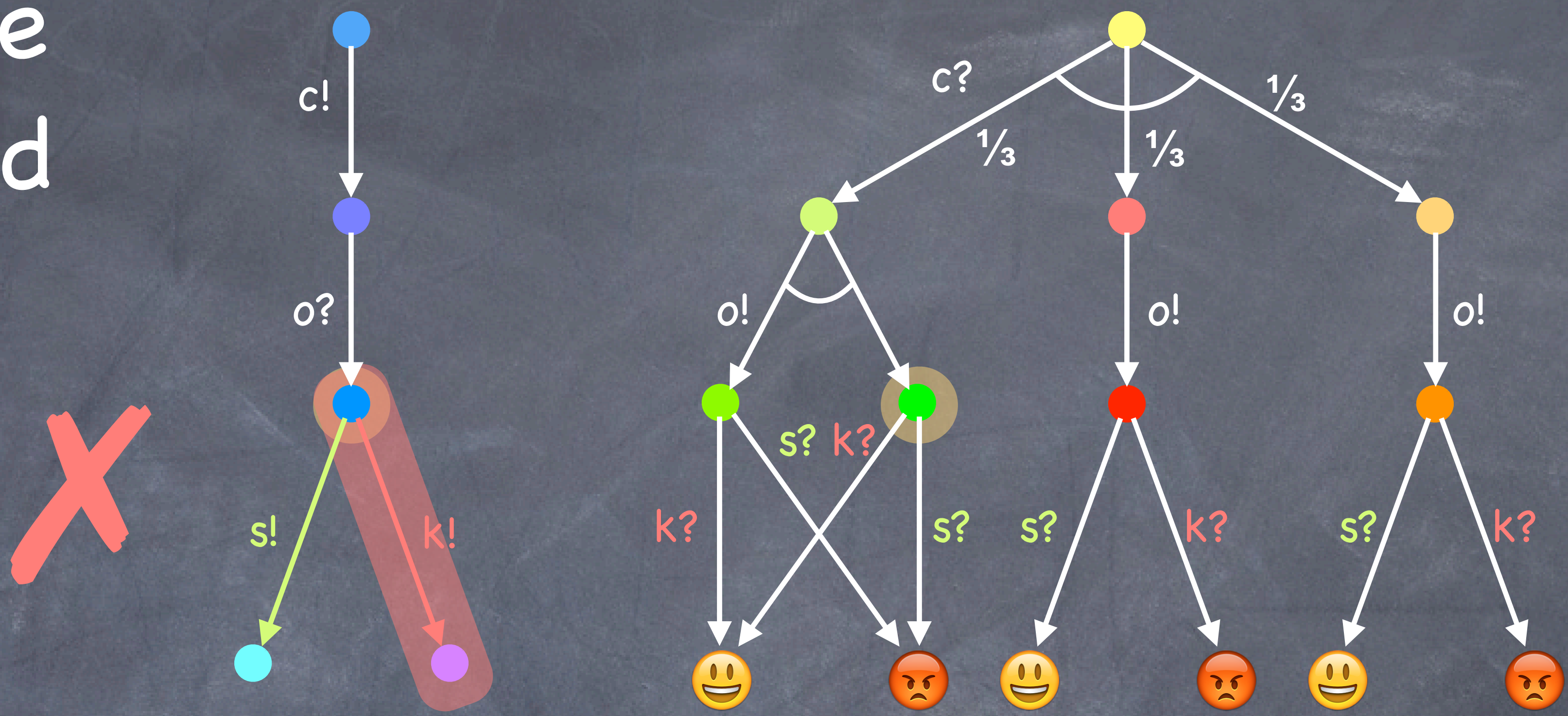
Example revisited



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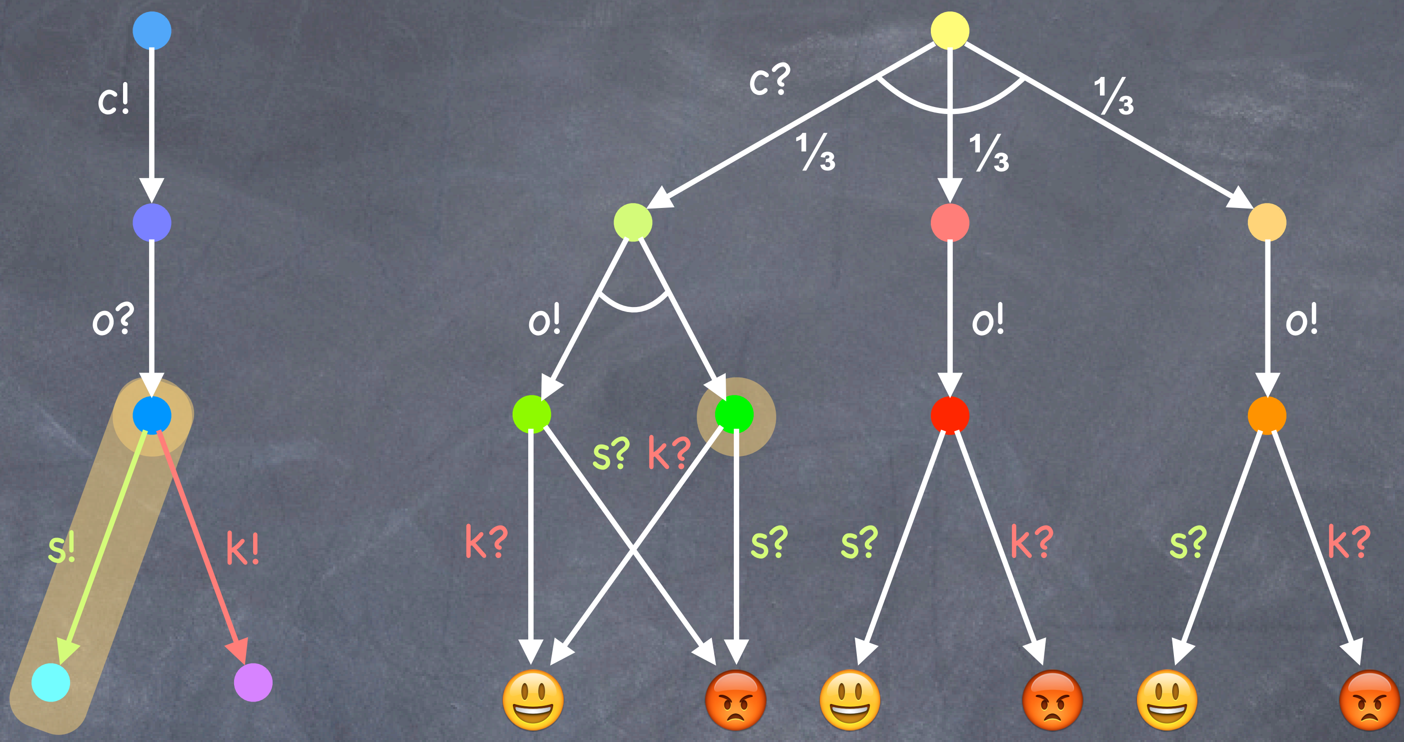

$$\Theta_Y\left(\,[(\bullet,\bullet)c(\bullet,\bullet)o(\bullet,\bullet)]_Y\,\right) = \boxed{\Theta_Y\left(\bullet c \bullet o \bullet\right)} = s!$$

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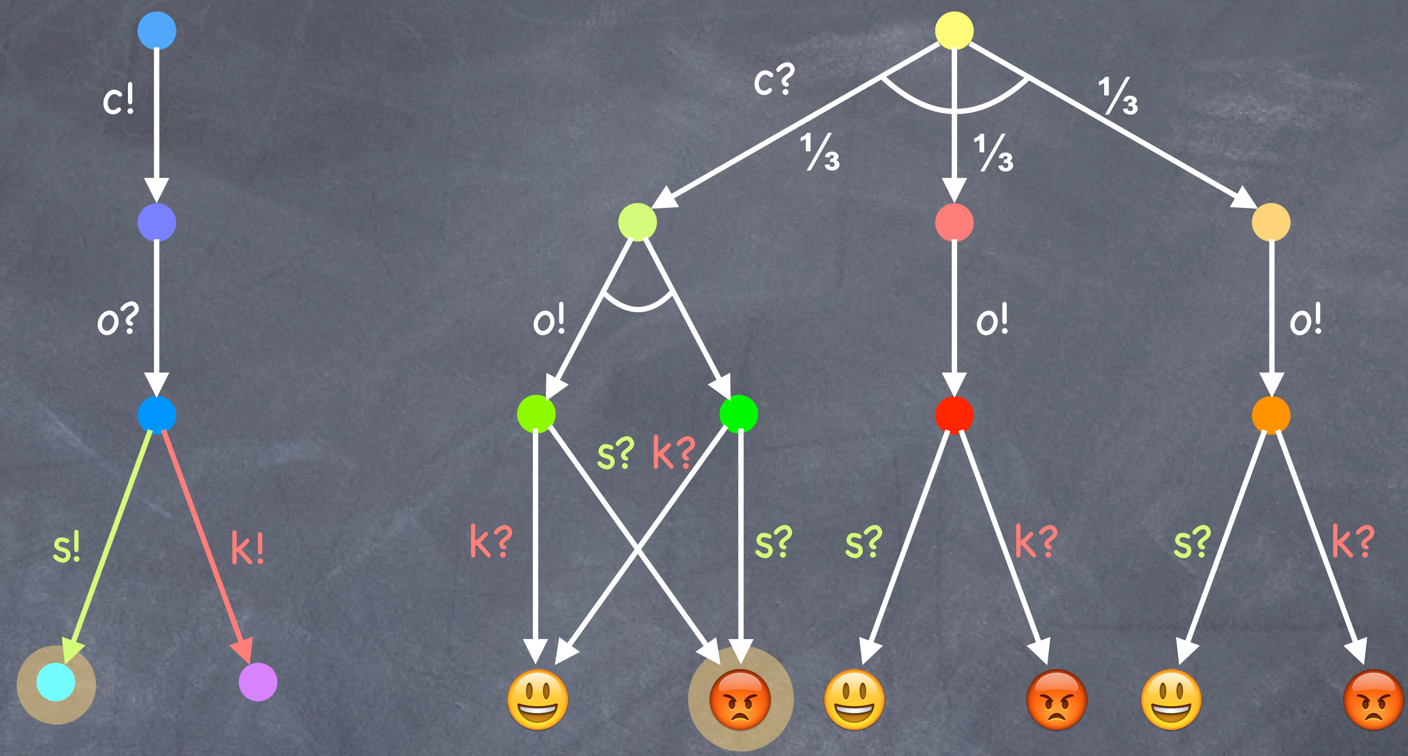
Example revisited



$$\Theta_Y\ (\ [(\bullet,\bullet)c(\bullet,\bullet)o(\bullet,\bullet)]_Y\ ) = \Theta_Y\ (\bullet c \bullet o \bullet) = s!$$

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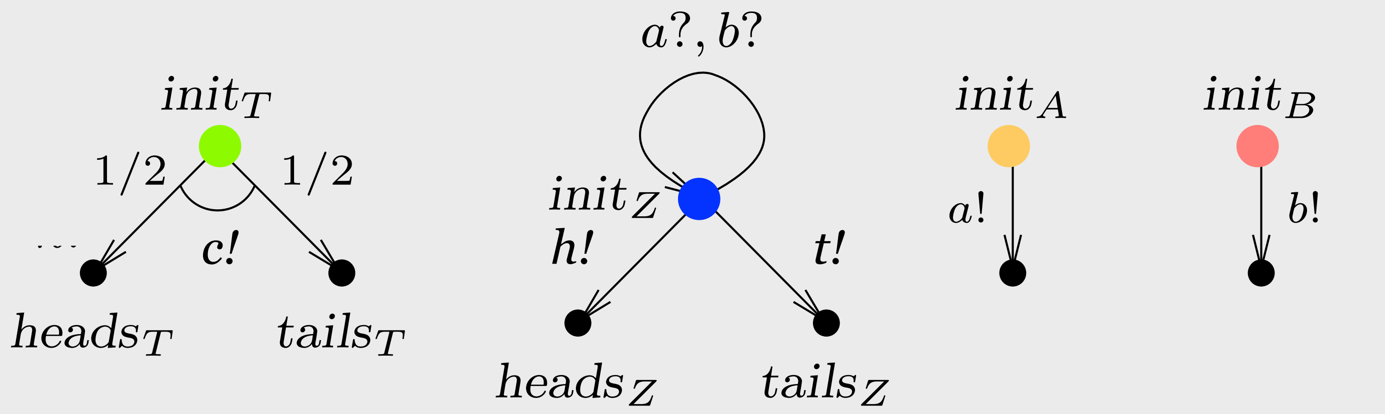
Are distributed schedulers what we need?



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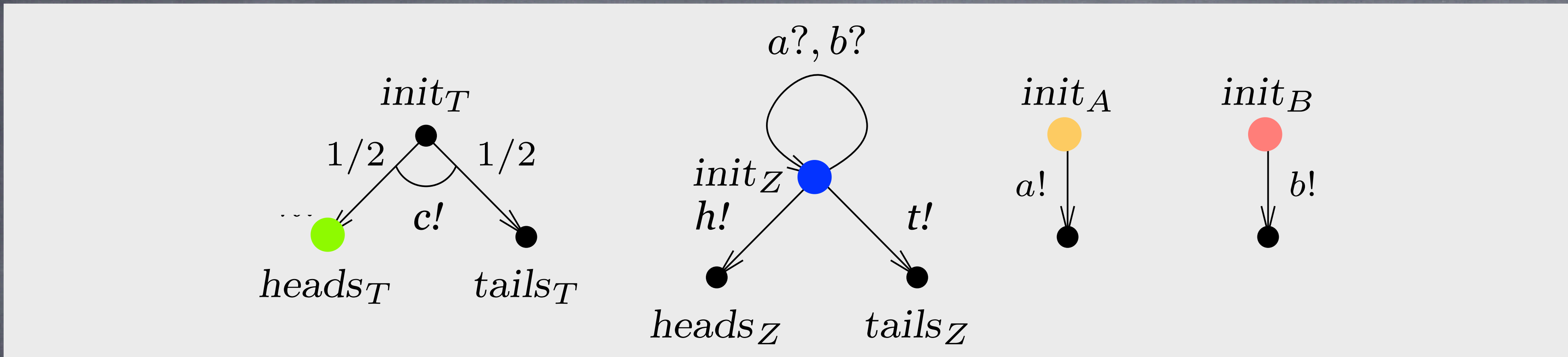
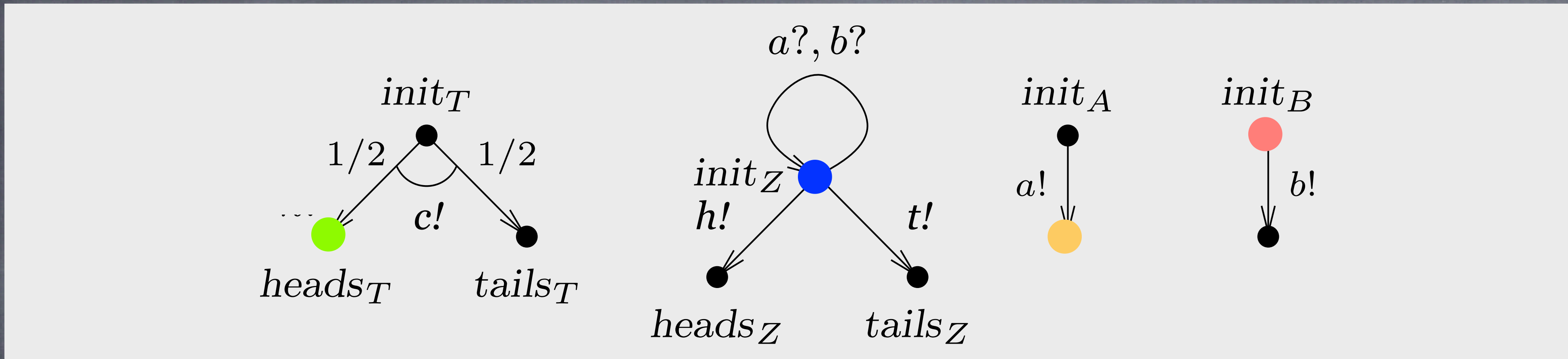


$\mathcal{I}\ (\ (i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ ) = 3$

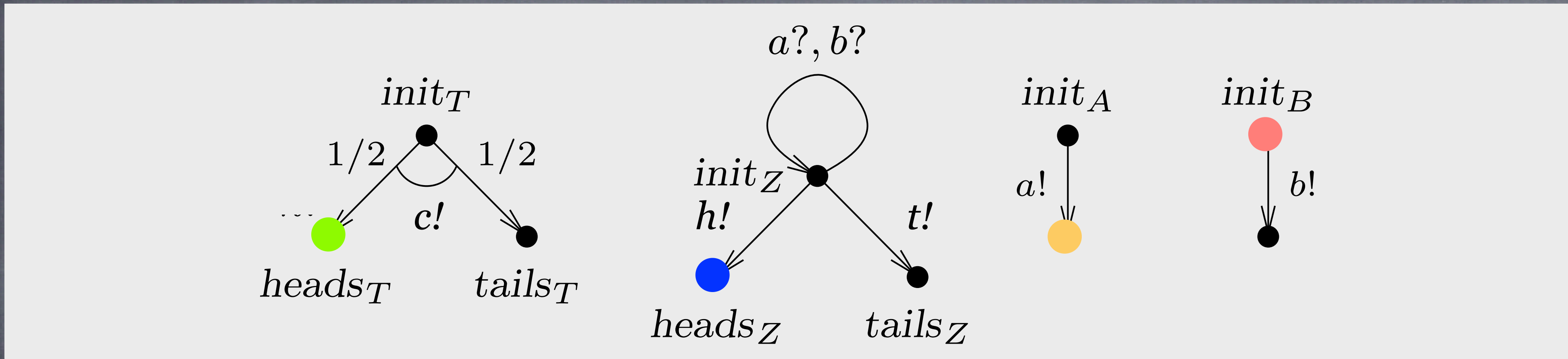
Are distributed schedulers what we need?



$\mathcal{I}$ ((i_T, i_Z, i_A, i_B) $c!$ (h_T, i_Z, i_A, i_B)) = 3

$\mathcal{I}$ ((i_T, i_Z, i_A, i_B) $c!$ (h_T, i_Z, i_A, i_B) $a!$ (h_T, i_Z, e_A, i_B)) = 2

Are distributed schedulers what we need?

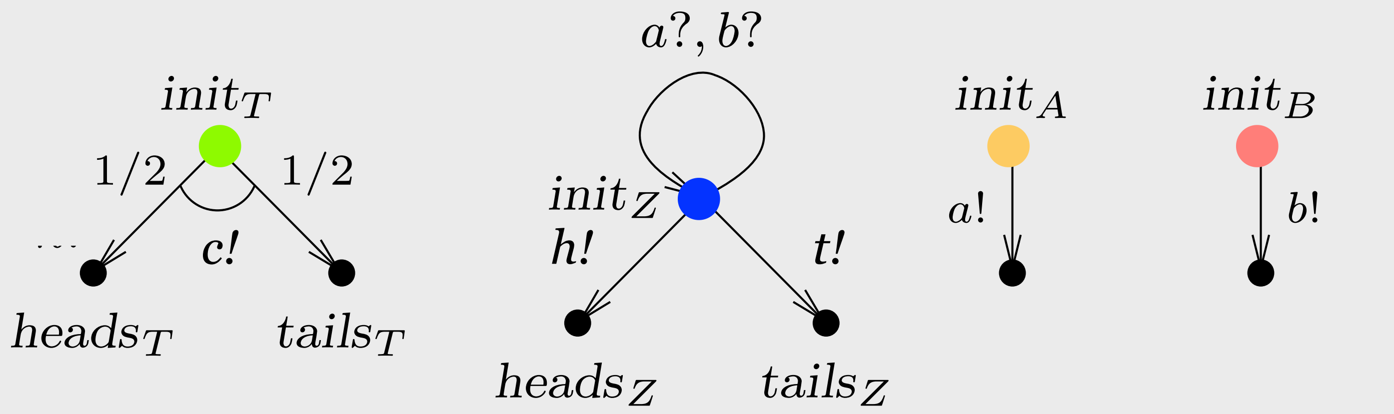


$\mathcal{I}\left((i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B) \right) = 3$

$\mathcal{I}\left((i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ a!\ (h_T, i_Z, e_A, i_B) \right) = 2$

$\Theta_2\left([(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ a!\ (h_T, i_Z, e_A, i_B)]_2 \right) = \Theta_2\left(i_Z\ a!\ i_Z \right) = h!$

Are distributed schedulers what we need?



$\mathcal{I}$ ((i_T, i_Z, i_A, i_B) $c!$ (h_T, i_Z, i_A, i_B)) = 3

$\mathcal{I}$ ((i_T, i_Z, i_A, i_B) $c!$ (h_T, i_Z, i_A, i_B) $a!$ (h_T, i_Z, e_A, i_B)) = 2

Θ_2 ($[(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ a!\ (h_T, i_Z, e_A, i_B)]_2$) = Θ_2 ($i_Z\ a!\ i_Z$) = $h!$

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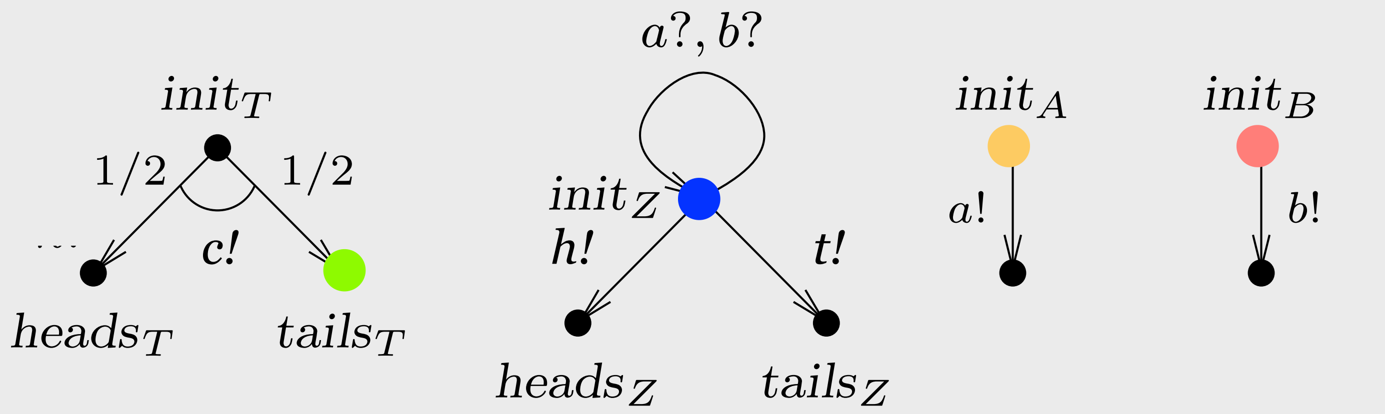
Are distributed schedulers what we need?



$\mathcal{I}$ ((i_T, i_Z, i_A, i_B) $c!$ (h_T, i_Z, i_A, i_B)) = 3

$\mathcal{I}$ ((i_T, i_Z, i_A, i_B) $c!$ (h_T, i_Z, i_A, i_B) $a!$ (h_T, i_Z, e_A, i_B)) = 2

Θ_2 ($[(i_T, i_Z, i_A, i_B)$ $c!$ (h_T, i_Z, i_A, i_B) $a!$ $(h_T, i_Z, e_A, i_B)]_2$) = Θ_2 (i_Z $a!$ i_Z) = $h!$

$\mathcal{I}$ ((i_T, i_Z, i_A, i_B) $c!$ (t_T, i_Z, i_A, i_B)) = 4

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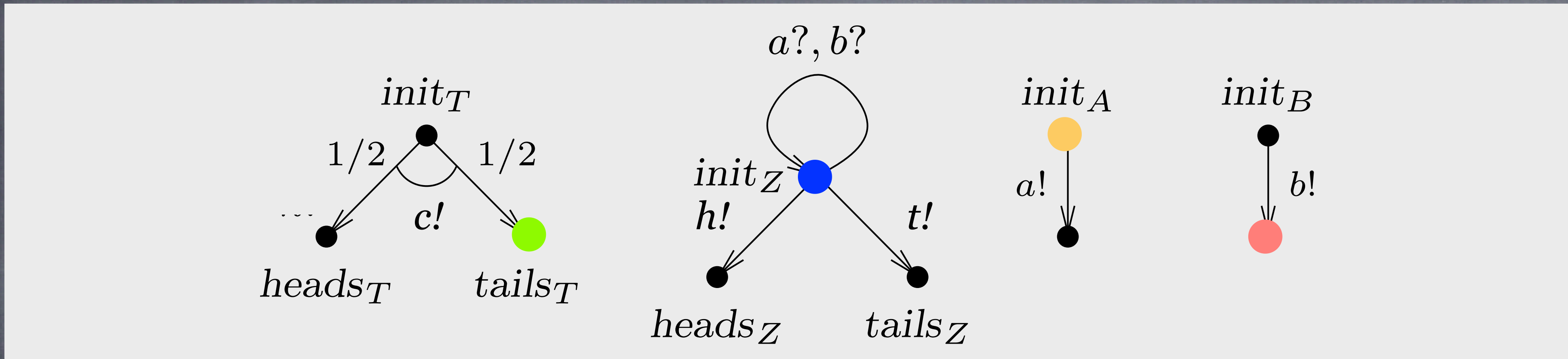
Are distributed schedulers what we need?



$\mathcal{I}\ (\ (i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ ) = 3$

$\mathcal{I}\ (\ (i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ a!\ (h_T, i_Z, e_A, i_B)\ ) = 2$

$\Theta_2\ (\ [(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ a!\ (h_T, i_Z, e_A, i_B)]_2\ ) = \Theta_2\ (\ i_Z\ a!\ i_Z\ ) = h!$

$\mathcal{I}\ (\ (i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)\ ) = 4$

$\mathcal{I}\ (\ (i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)\ b!\ (t_T, i_Z, i_A, e_B)\ ) = 2$

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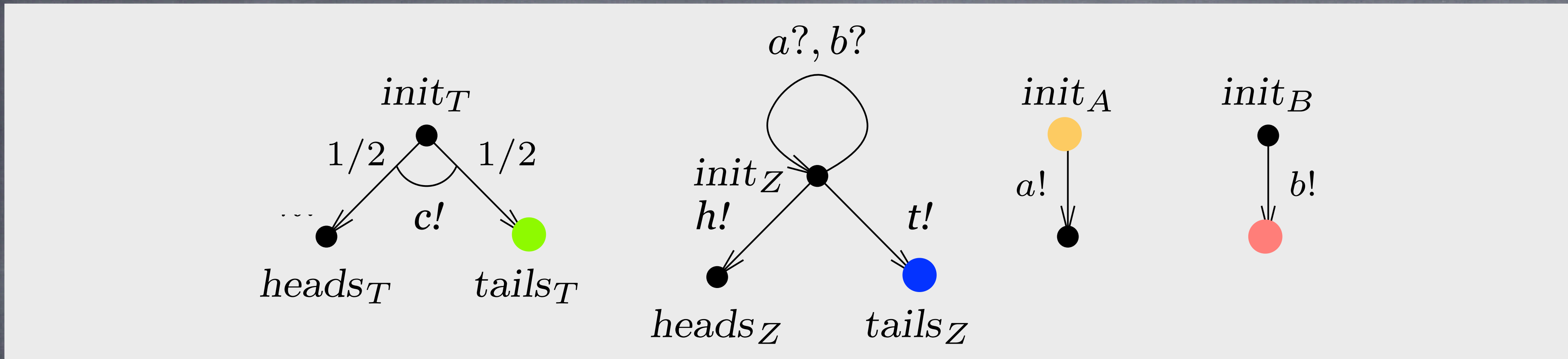
Are distributed schedulers what we need?



$\mathcal{I}\,(\,(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\,) = 3$

$\mathcal{I}\,(\,(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ a!\ (h_T, i_Z, e_A, i_B)\,) = 2$

$\Theta_2\,(\,[(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\ a!\ (h_T, i_Z, e_A, i_B)]_2\,) = \Theta_2\,(\,\underbrace{i_Z\ a!\ i_Z}\,) = h!$

$\neq$

$\mathcal{I}\,(\,(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)\,) = 4$

$\mathcal{I}\,(\,(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)\ b!\ (t_T, i_Z, i_A, e_B)\,) = 2$

$\Theta_2\,(\,[(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)\ b!\ (t_T, i_Z, i_A, e_B)]_2\,) = \Theta_2\,(\,\overbrace{i_Z\ b!\ i_Z}\,) = t!$

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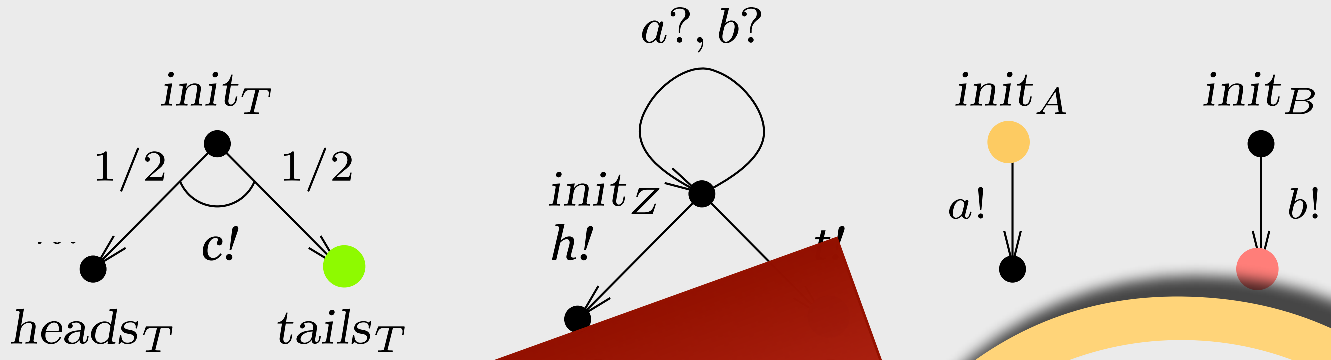
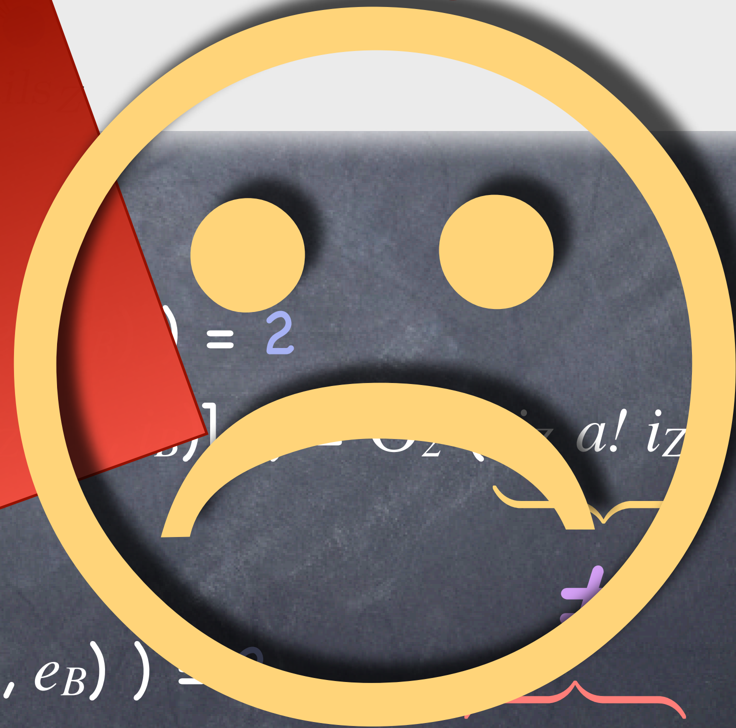
Are distributed schedulers what we need?



MAXIMUM PROBABILIY IS 1



$\mathcal{I}$ ((i_T, i_Z, i_A ...

$\mathcal{I}$ ((...) = 2

Θ_2 ([(...]) = Θ_2 (... a! i_Z) = h!

$\mathcal{I}$ ((i_T, i_Z ... 4

$\mathcal{I}$ ((i_T, i_Z, ... i_A, i_B) b! (t_T, i_Z, i_A, e_B)) = ...

Θ_2 ([(i_T, i_Z ... i_A, i_B) c! (t_T, i_Z, i_A, i_B) b! (t_T, i_Z, i_A, e_B)$]_2$) = Θ_2 (i_Z b! i_Z) = t!

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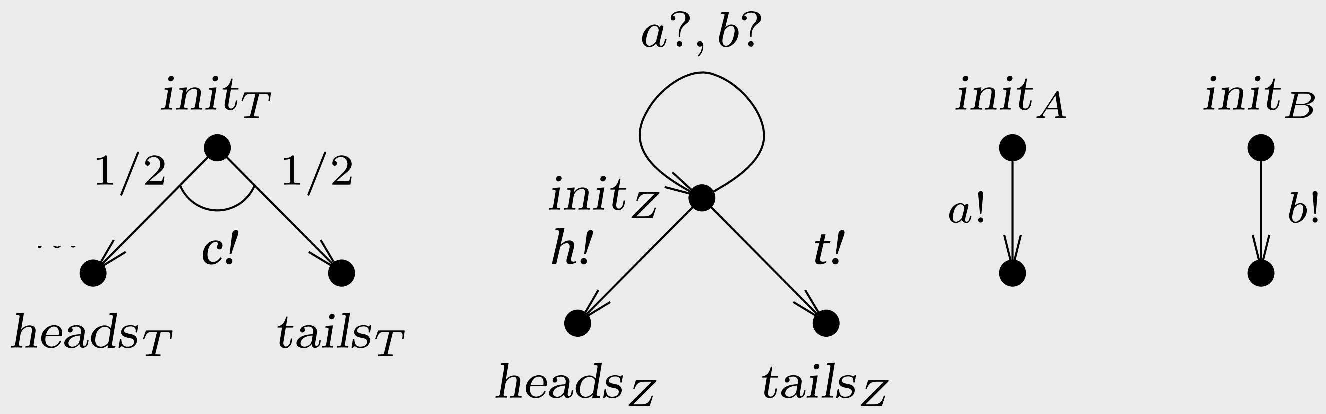
Are distributed schedulers what we need?



$\mathcal{I}\,(\,(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)\,) = 3$

$\mathcal{I}\,(\,(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)\,) = 4$

$[(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)]_3 \;=\; i_A \;=\; [(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)]_3$

$[(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)]_4 \;=\; i_B \;=\; [(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)]_4$

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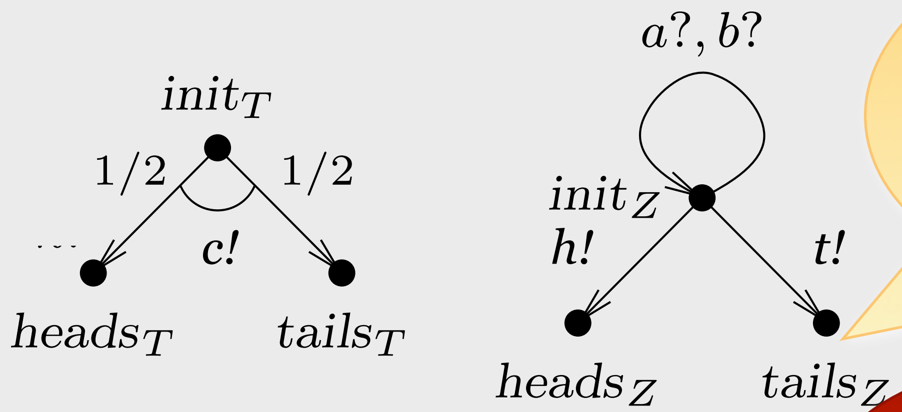
Are distributed schedulers what we need?



None of components 3 and 4 can distinguish the system after these two executions

… and yet they are consider diferently

$$\mathcal{I}\left((i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B) \right) = 3$$

$$\mathcal{I}\left((i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B) \right) = 4$$

$$[(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)]_3 = i_A = [(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)]_3$$

$$[(i_T, i_Z, i_A, i_B)\ c!\ (h_T, i_Z, i_A, i_B)]_4 = i_B = [(i_T, i_Z, i_A, i_B)\ c!\ (t_T, i_Z, i_A, i_B)]_4$$

Strongly distributed schedulers

A strongly distributed scheduler is a distributed scheduler where I (the interleaving scheduler) meets the following condition:

for all $\sigma, \sigma' \in \mathsf{Frag}$ and components A_i, A_j,

such that $[\sigma]_i = [\sigma']_i, [\sigma]_j = [\sigma']_j$, it holds that

$$\frac{\mathcal{I}(\sigma)(i)}{\mathcal{I}(\sigma)(i) + \mathcal{I}(\sigma)(j)} = \frac{\mathcal{I}(\sigma')(i)}{\mathcal{I}(\sigma')(i) + \mathcal{I}(\sigma')(j)}$$

provided $\mathcal{I}(\sigma)(i) + \mathcal{I}(\sigma)(j) \neq 0 \neq \mathcal{I}(\sigma')(i) + \mathcal{I}(\sigma')(j)$

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Strongly distrib

If two components cannot distinguish two executions, their relative probabilities after such executions must be the same

A strongly distributed sched
scheduler where I (the interleavin eets
the following condition:

for all $\sigma, \sigma' \in \mathsf{Frag}$ and components A_i, A_j,

such that $[\sigma]_i = [\sigma']_i, [\sigma]_j = [\sigma']_j$, it holds that

$$\frac{\mathcal{I}(\sigma)(i)}{\mathcal{I}(\sigma)(i) + \mathcal{I}(\sigma)(j)} = \frac{\mathcal{I}(\sigma')(i)}{\mathcal{I}(\sigma')(i) + \mathcal{I}(\sigma')(j)}$$

provided $\mathcal{I}(\sigma)(i) + \mathcal{I}(\sigma)(j) \neq 0 \neq \mathcal{I}(\sigma')(i) + \mathcal{I}(\sigma')(j)$

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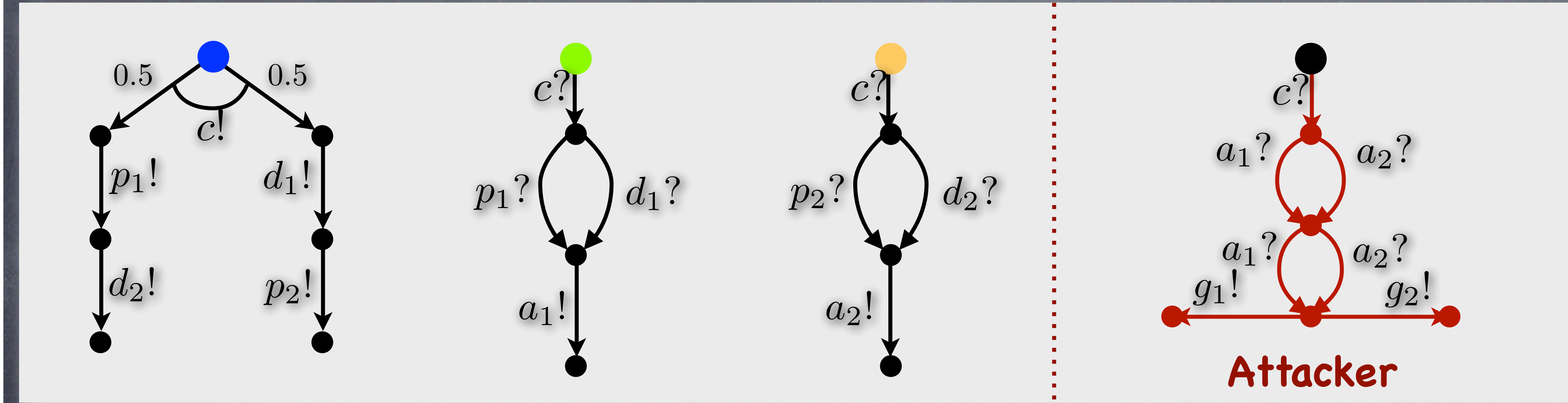
What about security?



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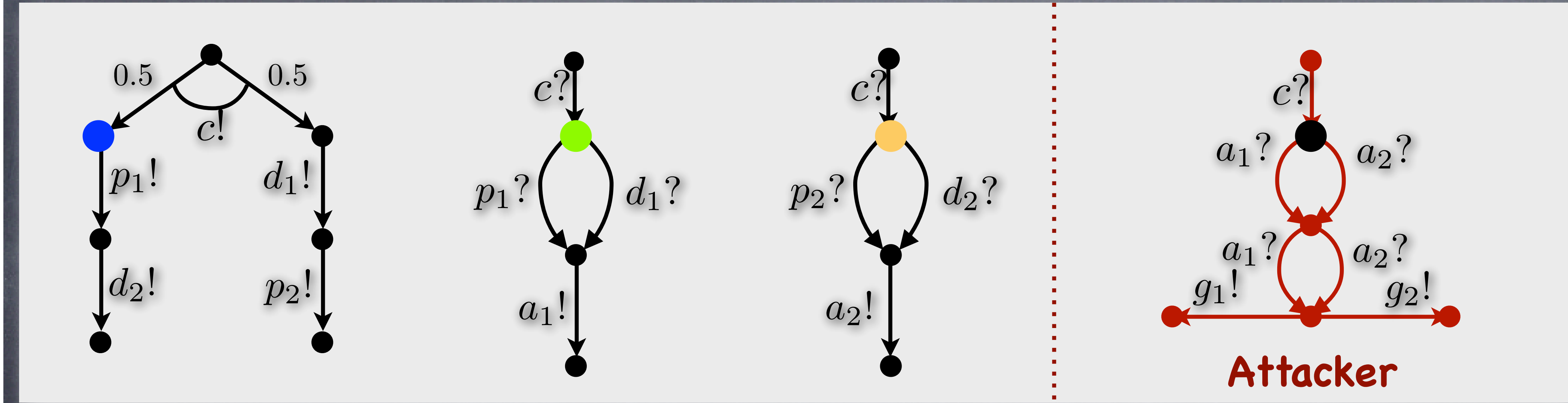
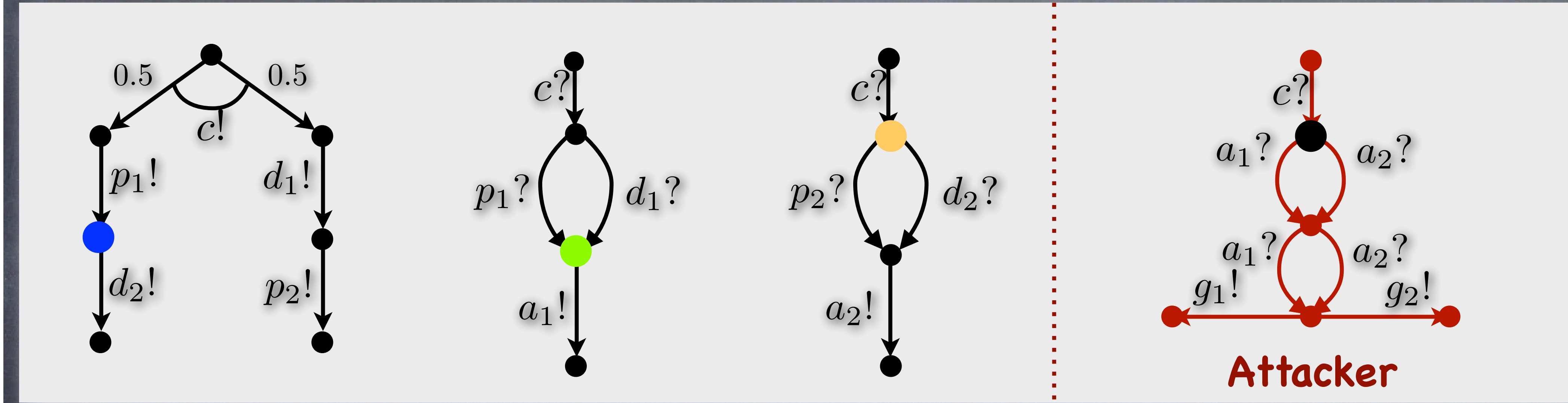
What about security?



What about security?



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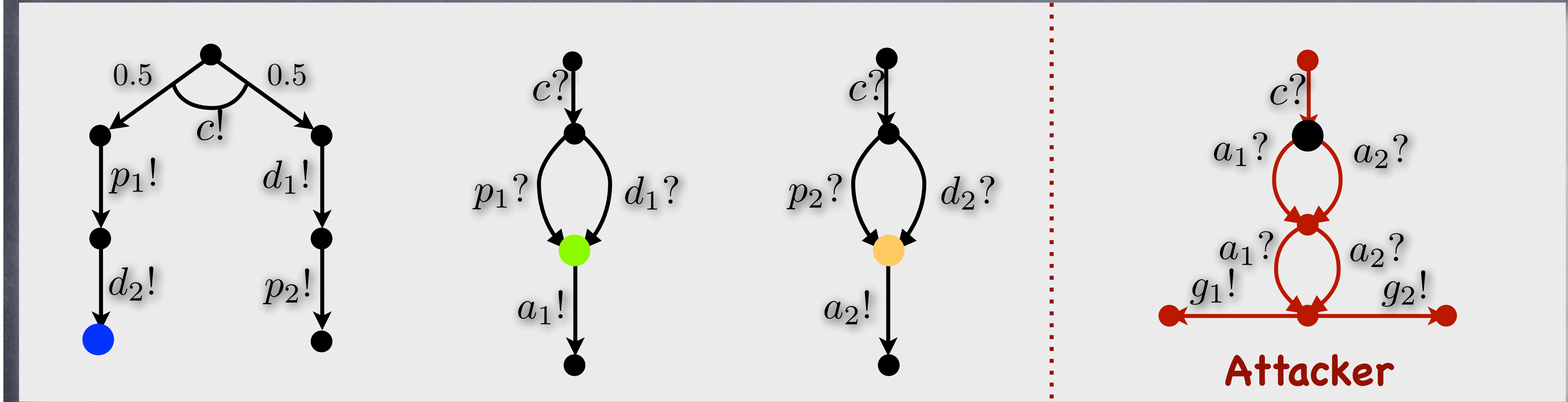
What about security?



$\mathcal{I}(c\ p_1\ d_2) = 1$

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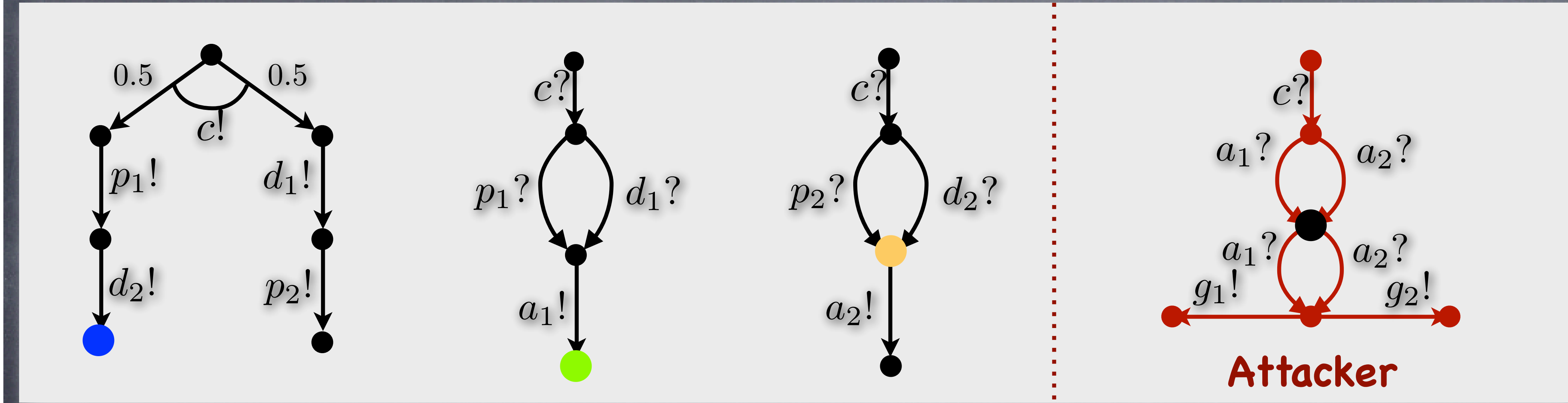
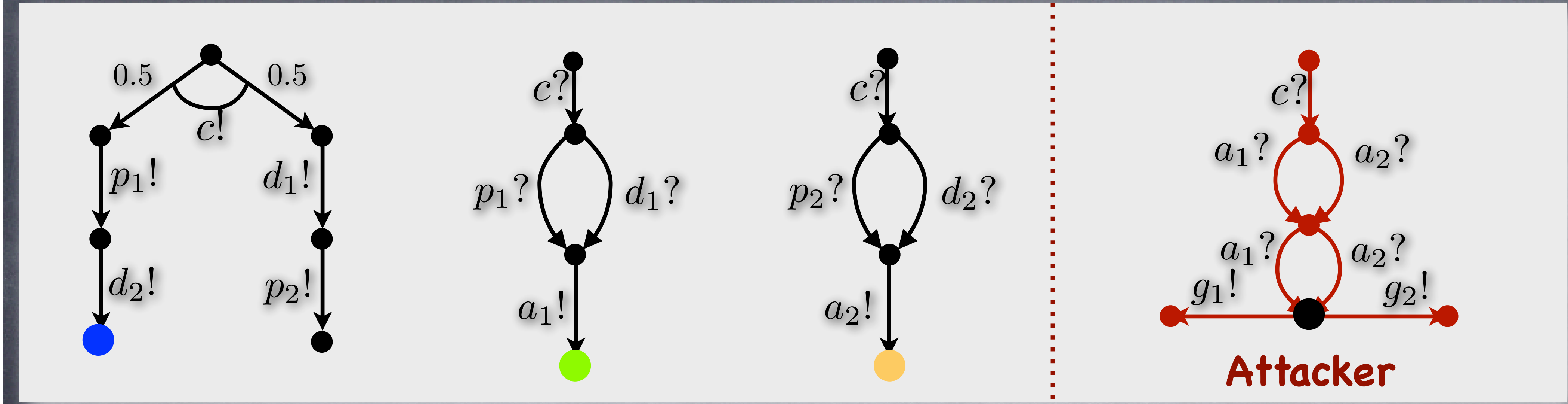
What about security?



$I(c\ p_1\ d_2) = 1$

$I(c\ p_1\ d_2\ a_1) = 2$

What about security?

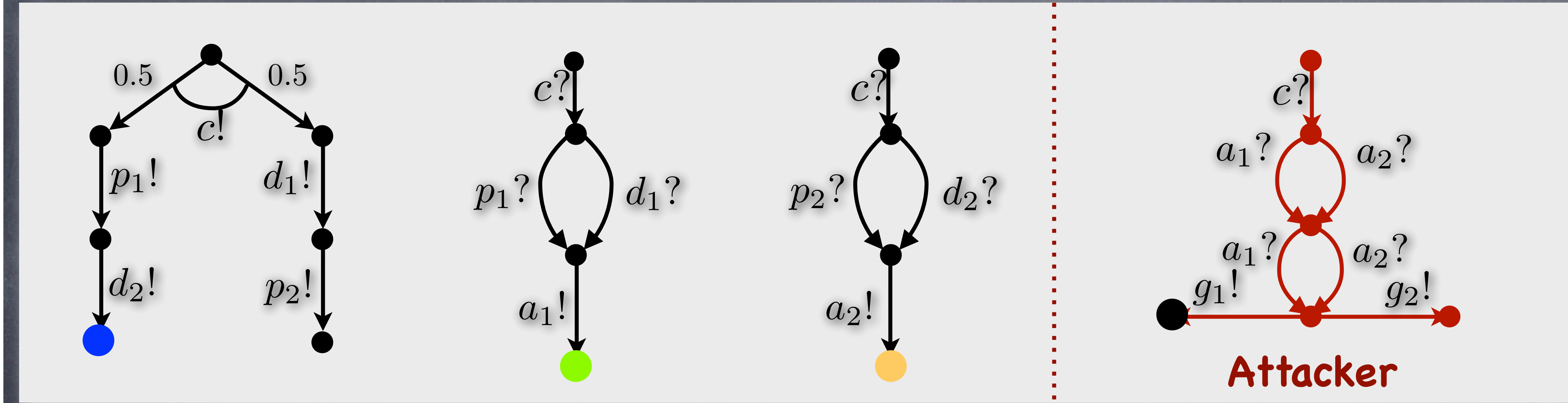


$I(c\ p_1\ d_2) = 1$

$I(c\ p_1\ d_2\ a_1) = 2$

$I(c\ p_1\ d_2\ a_1\ a_2) = \text{Atck}$

What about security?



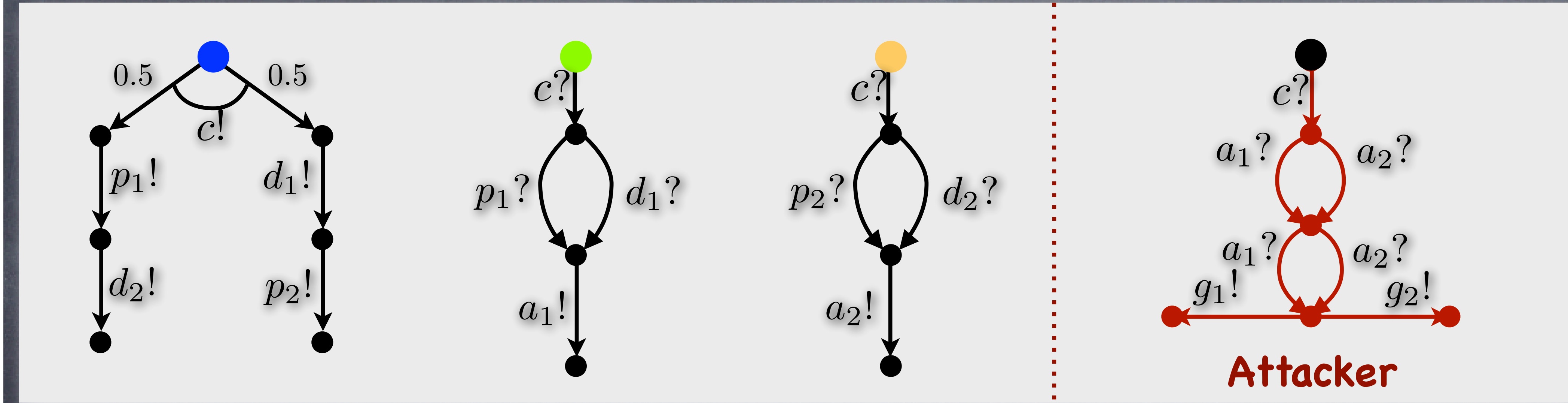
$\mathcal{I}(\, c \;\; p_1 \;\; d_2 \,) = 1$

$\mathcal{I}(\, c \;\; p_1 \;\; d_2 \;\; a_1 \,) = 2$

$\mathcal{I}(\, c \;\; p_1 \;\; d_2 \;\; a_1 \;\; a_2 \,) =$ Atck

Atacker guesses 1

What about security?



$I(c\ p_1\ d_2) = 1$

$I(c\ p_1\ d_2\ a_1) = 2$

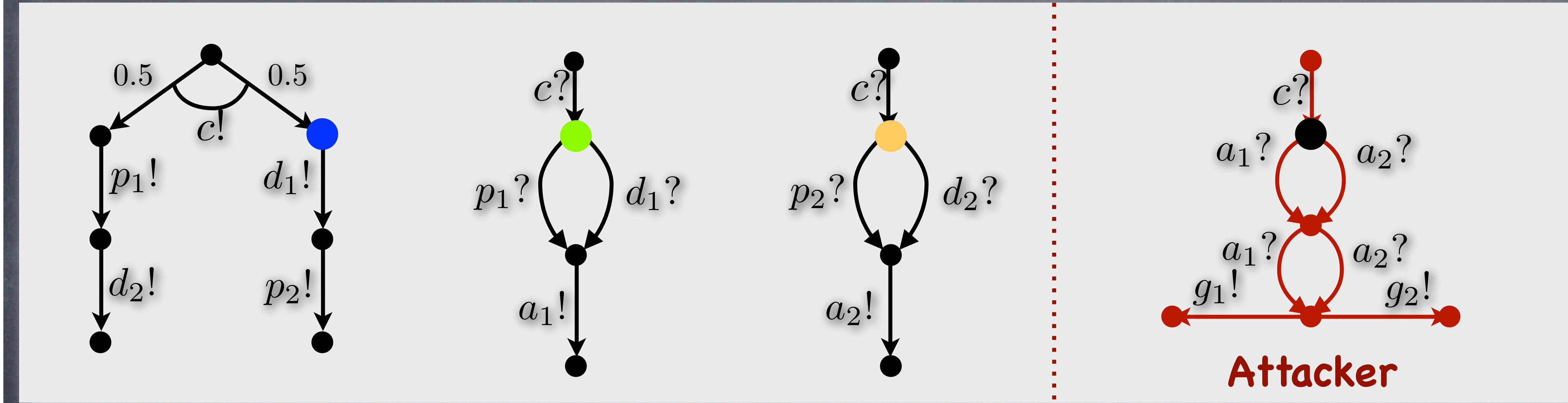
$I(c\ p_1\ d_2\ a_1\ a_2) =$ Atck

Atacker guesses 1

CONICET

UNC

What about security?



$\mathcal{I}(\ c\ \ p_1\ \ d_2\ ) = 1$

$\mathcal{I}(\ c\ \ p_1\ \ d_2\ \ a_1\ ) = 2$

$\mathcal{I}(\ c\ \ p_1\ \ d_2\ \ a_1\ \ a_2\ ) =$ Atck

Atacker guesses 1

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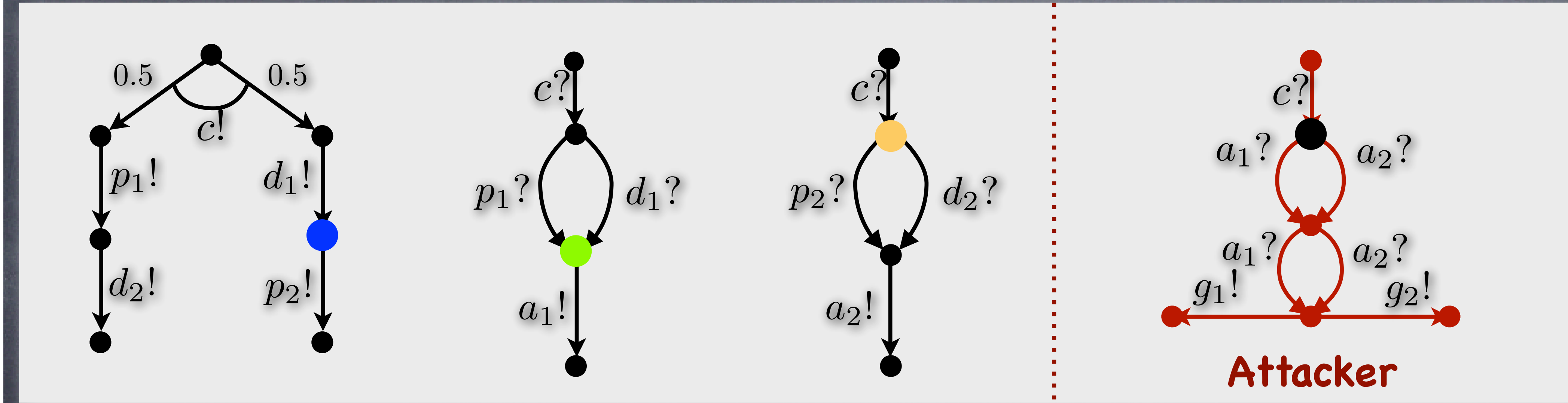
What about security?



$I(c\ p_1\ d_2) = 1$

$I(c\ p_1\ d_2\ a_1) = 2$

$I(c\ p_1\ d_2\ a_1\ a_2) = \text{Atck}$

Atacker guesses 1

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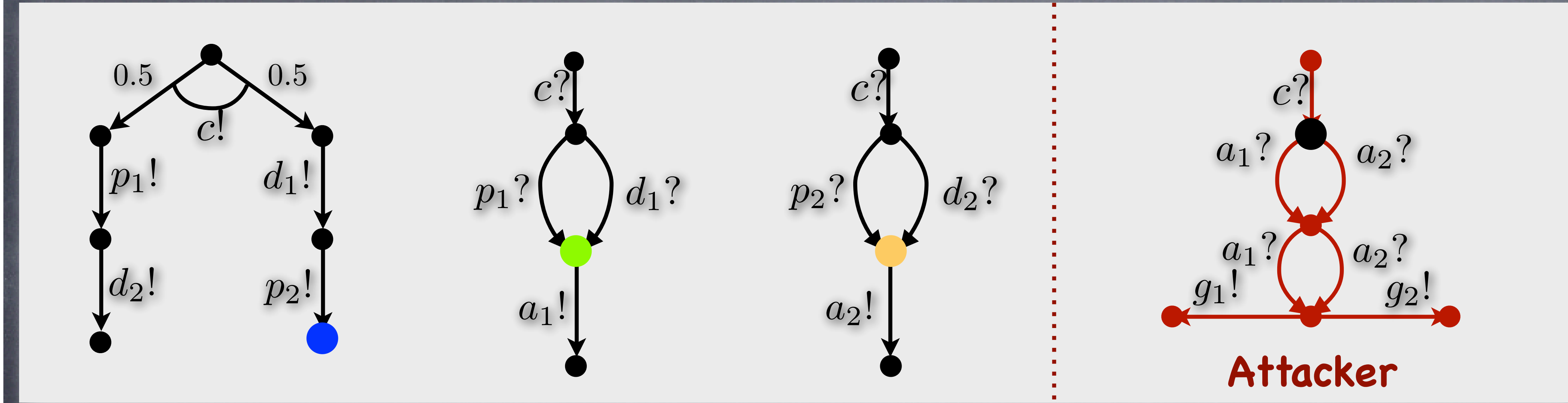
What about security?



$I(\underline{c\ p_1\ d_2}) = 1$

$I(c\ p_1\ d_2\ a_1) = 2$

$I(c\ p_1\ d_2\ a_1\ a_2) = \text{Atck}$

Atacker guesses 1

$I(\underline{c\ d_1\ p_2}) = 2$

$$[\,c\ p_1\ d_2\,]_1 = p_1 \neq d_1 = [\,c\ d_1\ p_2\,]_1$$

$$[\,c\ p_1\ d_2\,]_2 = p_2 \neq d_2 = [\,c\ d_1\ p_2\,]_2$$

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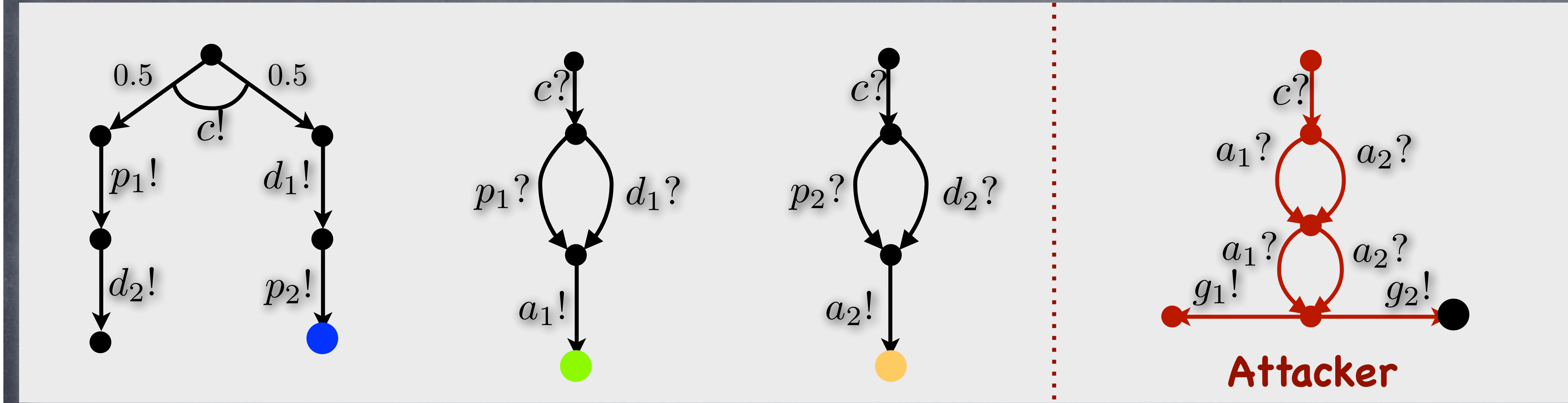
What about security?



$I(\underline{c\ p_1\ d_2}) = 1$

$I(c\ p_1\ d_2\ a_1) = 2$

$I(c\ p_1\ d_2\ \underline{a_1\ a_2}) =$ Atck

Atacker guesses 1

$I(\underline{c\ d_1\ p_2}) = 2$

$I(c\ d_1\ p_2\ a_2) = 1$

$I(c\ d_1\ p_2\ \underline{a_2\ a_1}) =$ Atck

Atacker guesses 2 ☹

$$[\,c\ p_1\ d_2\,]_1 = p_1 \neq d_1 = [\,c\ d_1\ p_2\,]_1$$

$$[\,c\ p_1\ d_2\,]_2 = p_2 \neq d_2 = [\,c\ d_1\ p_2\,]_2$$

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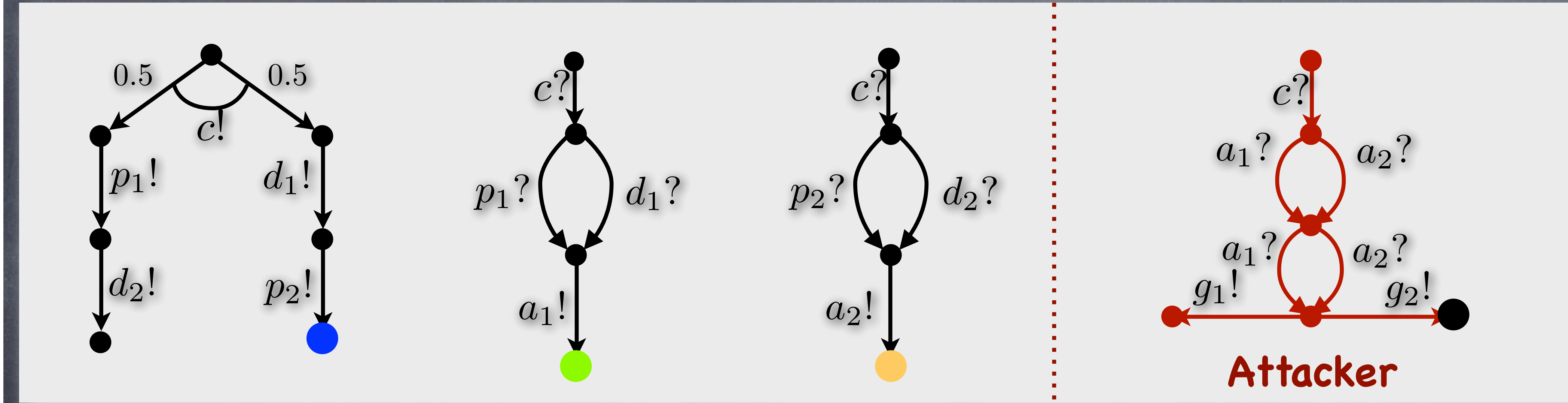
What about security?



$$[\ c\ \ p_1\ \ d_2\ ]_1 = p_1 \neq d_1 = [\ c\ \ d_1\ \ p_2\ ]_1$$

$$[\ c\ \ p_1\ \ d_2\ ]_2 = p_2 \neq d_2 = [\ c\ \ d_1\ \ p_2\ ]_2$$

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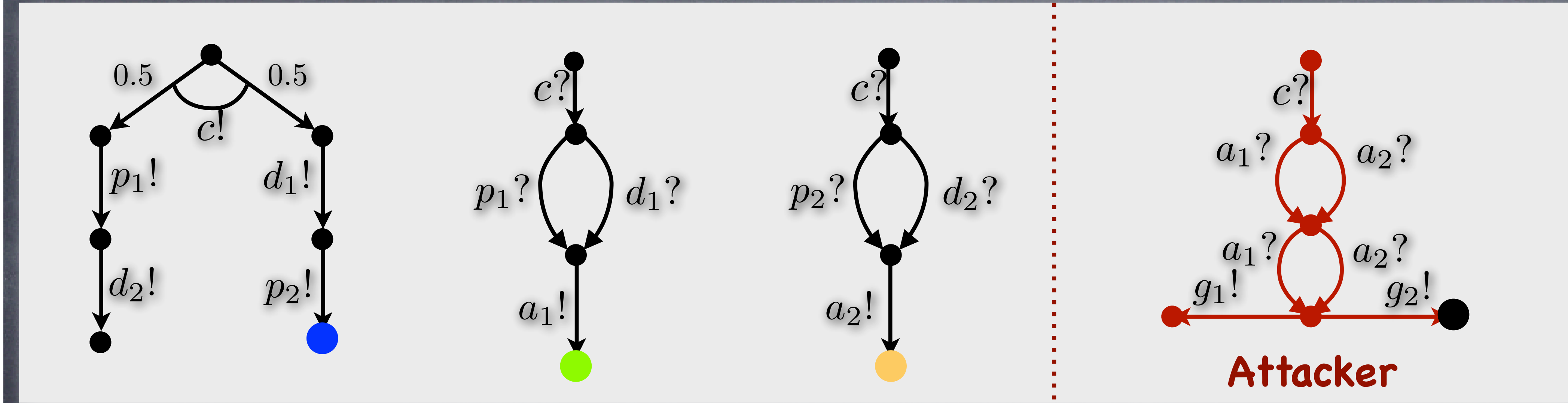
What about security?



$$[\ c\ \ p_1\ \ d_2\ ]_1 = p_1 \approx d_1 = [\ c\ \ d_1\ \ p_2\ ]_1$$

$$[\ c\ \ p_1\ \ d_2\ ]_2 = p_2 \approx d_2 = [\ c\ \ d_1\ \ p_2\ ]_2$$

secret actions should not be distinguished by I

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Distributed schedulers under secrecy

A distributed scheduler under secrecy is a distributed scheduler where I meets the following condition:

for all $\sigma, \sigma' \in \mathsf{Frag}$ and components A_i, A_j,

such that $[\sigma]_i \approx [\sigma']_i, [\sigma]_j \approx [\sigma']_j$, it holds that

$$\frac{\mathcal{I}(\sigma)(i)}{\mathcal{I}(\sigma)(i) + \mathcal{I}(\sigma)(j)} = \frac{\mathcal{I}(\sigma')(i)}{\mathcal{I}(\sigma')(i) + \mathcal{I}(\sigma')(j)}$$

provided $\mathcal{I}(\sigma)(i) + \mathcal{I}(\sigma)(j) \neq 0 \neq \mathcal{I}(\sigma')(i) + \mathcal{I}(\sigma')(j)$

$$(\forall a \in O_i : \mathsf{last}([\sigma]_i) \xrightarrow{a}_i) \text{ iff } (\forall a \in O_i : \mathsf{last}([\sigma']_i) \xrightarrow{a}_i)$$

$$(\forall a \in O_j : \mathsf{last}([\sigma]_j) \xrightarrow{a}_j) \text{ iff } (\forall a \in O_j : \mathsf{last}([\sigma']_j) \xrightarrow{a}_j)$$

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[Pelozo & D'Argenio 2012]

Results (finite state models)

	Dist. Sched.	Str. Dist. Sched.	Distr. Sched. with Secrecy
Det = Random sup P(F goal)	Yes	No	No
sup P(F goal)?	Undecidable	Undecidable	Undecidable
sup P(F goal) = 1	Undecidable	Undecidable	Undecidable

sup P(F goal)? is NP-Hard for

(locally | globally) memoryless (deterministic | randomized)

distributed schedulers

Results (finite state systems)

- Partial order reduction:
  - Peled's original conditions preserve strongly distributed schedulers
  - Apply classical algorithms for prob. MC on reduction
- Counterexample guided refinement:
  - Check sup P(F goal) ≤ p with classical Prob. MC
  - If the result is true => the model sat. property
  - If not and counterexample is a DS => error
  - If not and counterexample is not a DS

    => refine model and recalculate

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Results (finite behaviour systems)

- Bounded reachability reduces to a polynomial optimization problem
  - For dist. schedulers variables take value in {0,1}
  - For SDS/DSS => quadratic restrictions
- Anonymity can be encoded on SMTs
  - through a system of polynomial inequalities
- A good thing:
  - Usually, security protocols are of finite behavior
- A bad thing:
  - Inherently exponential

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Conclusion

Ignored by classical probabilistic model checking

- Distributed schedulers properly captures the idea of partial observation among components.
- Particularly suited for security.
- These observations has been acknowledge by other authors from different point of views:
  - Compositionality [de Alfaro, Henzinger, Jhala, 2001], [Cheung, Lynch, Segala, Vaandrager, 2006]
  - Security analysis [Chatzikokolakis, Palamidessi, 2007], [Andrés, Palamidessi, Rossum, Sokolova, 2011], [Chadha Sistla, Viswanathan, 2010]
  - Testing [Georgievska, Andova, 2010], [Hierons, Núñez 2012]
- Down side: undecidability and complexity

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Conclusion

Ignored by classical probabilistic model checking

- Distributed schedulers properly captures the idea of partial observation among components.
- Par... for security.

e.g.:tractable but interesting subclasses / abstraction techniques / appropriate data structures / etc.

...en acknowledge by other ...nt of views:

- ... [de Alfaro, Henzi... ...ch, Segala, Vaandrager, ...06]
- Security analys... [Chatz... Palamidessi, Rossum, Sokolova, 2011], ...
- Testing [Georgievska, Andova ...10], [Hierons ...nez 2012]

They did not stop the automated theorem proving or SAT solving communities (among others)

- Down side: undecidability and complexity

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Analysis of Distributed Probabilistic Systems: Limitations and Possibilities

Pedro R. D'Argenio

Universidad Nacional de Córdoba
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Joint work with Sergio Giro, Luis M. Ferrer Fioriti, Georgel Calin, Pepijn Crouzen, Ernst Moritz Hahn, Lijun Zhang, Silvia Pelozo

http://dsg.cs.famaf.unc.edu.ar/

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References:

S. Giro, P.R. D'Argenio: On the Expressive Power of Schedulers in Distributed Probabilistic Systems. QAPL 2009 (ENTCS 253(3):45-71).

S. Giro, P.R. D'Argenio: Quantitative Model Checking Revisited: Neither Decidable Nor Approximable. FORMATS 2007: 179-194.

S. Giro, P.R. D'Argenio, L.M. Ferrer Fioriti: Partial Order Reduction for Probabilistic Systems: A Revision for Distributed Schedulers. CONCUR 2009: 338-353.

S. Giro: On the Automatic Verification of Distributed Probabilistic Automata with Partial Information. PhD thesis, FaMAF, UNC, 2010.

L.M. Ferrer Fioriti: Reducción de orden parcial en model checking probabilista simbólico. Lic. thesis, FaMAF, UNC, 2010.

G. Calin, P. Crouzen, P.R. D'Argenio, E.M. Hahn, L. Zhang: Time-Bounded Reachability in Distributed Input/Output Interactive Probabilistic Chains. SPIN 2010: 193-211.

S. Pelozo, P.R. D'Argenio: Security analysis in probabilistic distributed protocols via bounded reachability. TGC 2012: 182-197.

S. Giro, M.N. Rabe: Verification of Partial-Information Probabilistic Systems Using Counterexample-Guided Refinements. ATVA 2012: 333-348.

S. Giro, P.R. D'Argenio, and L.M. Ferrer Fioriti: Distributed probabilistic input/output automata: Expressiveness, (un)decidability and algorithms. TCS 538:84-102, 2014.

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