Concurrency Theory vs Concurrent Languages

Silvia Crafa

Universita' di Padova

Bertinoro, OPCT 2014



Bisimulation inside

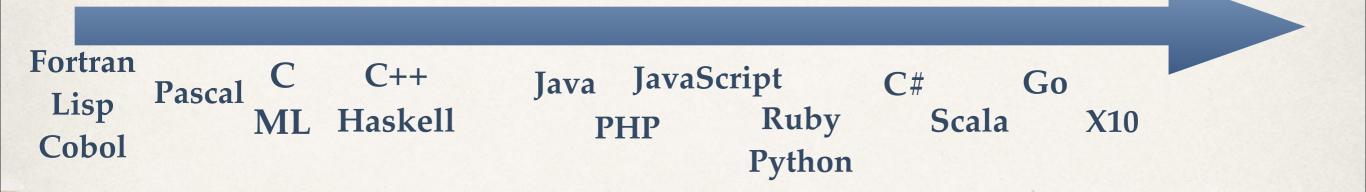
Concurrency Theory vs Concurrent Languages

Silvia Crafa

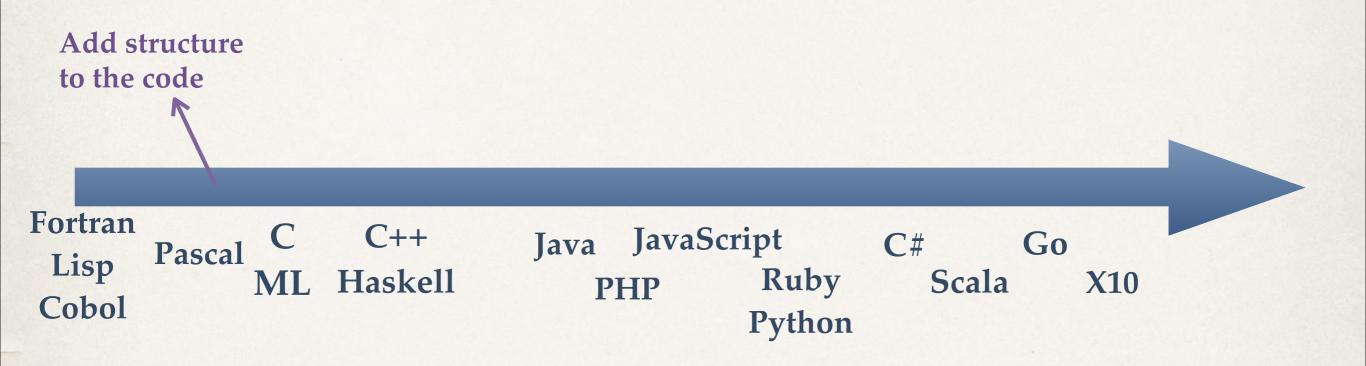
Universita' di Padova

Bertinoro, OPCT 2014

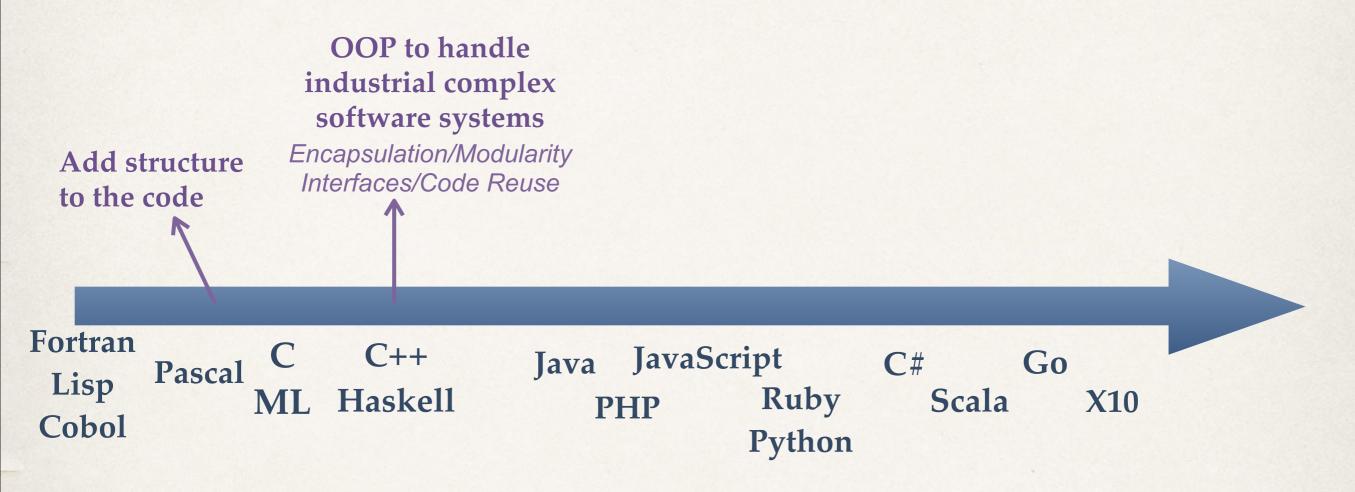




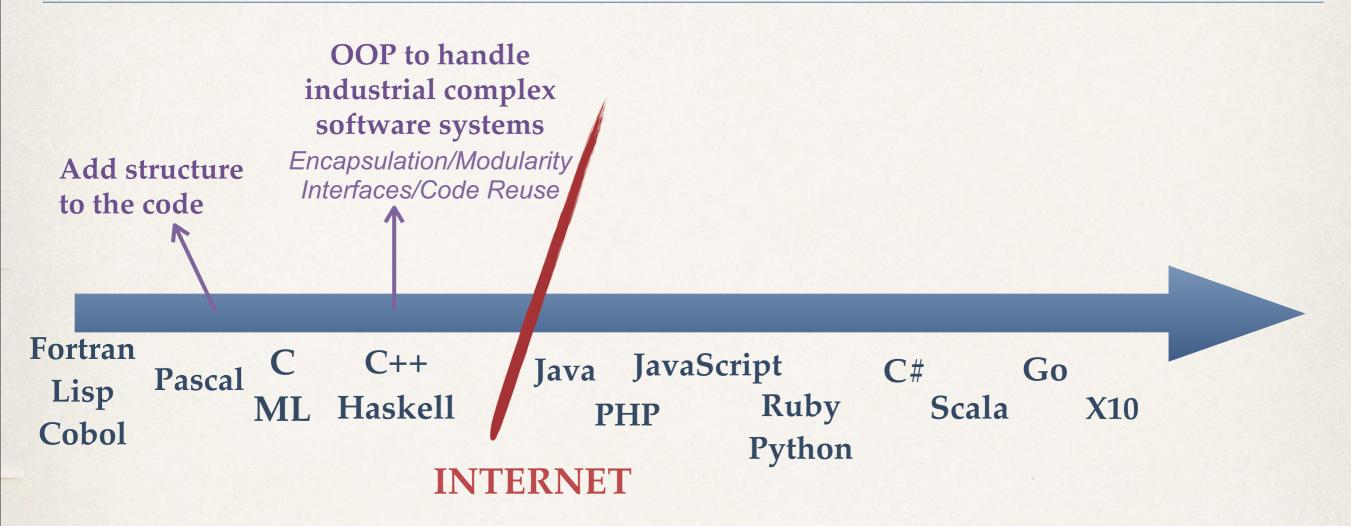
- When a language has been invented VS when became popular?
- Why has been invented VS why became popular?



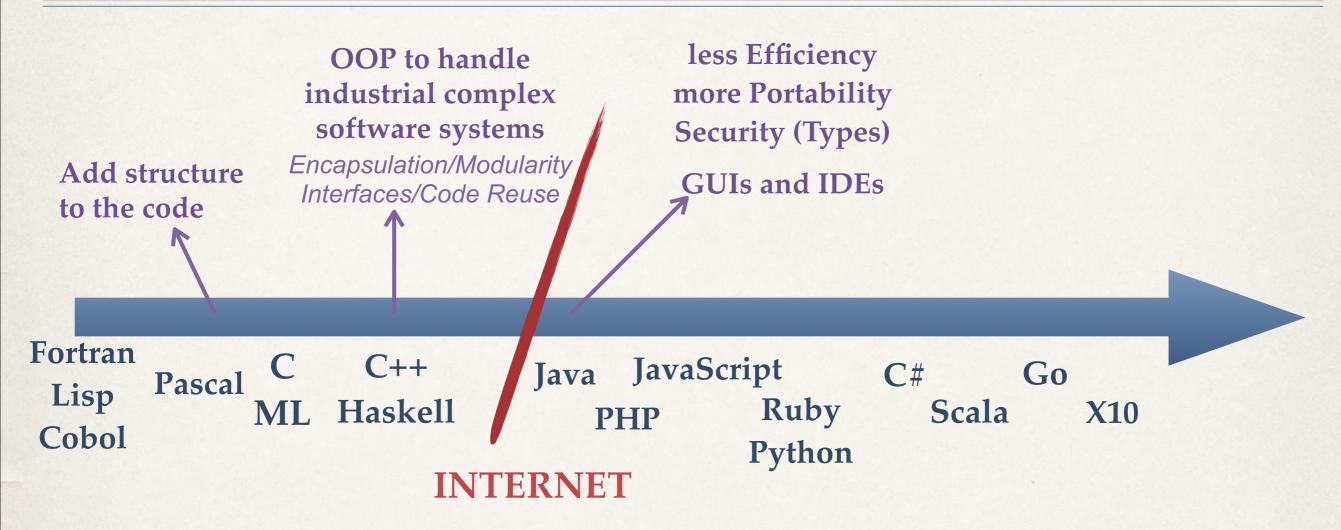
- When a language has been invented VS when became popular?
- Why has been invented VS why became popular?



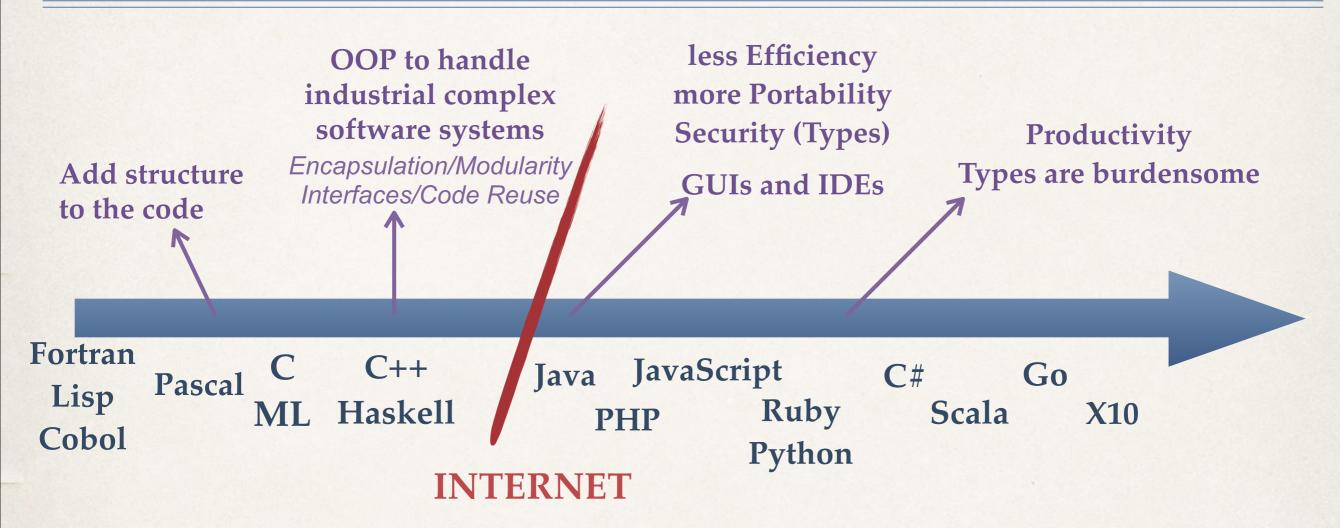
- When a language has been invented VS when became popular?
- Why has been invented VS why became popular?



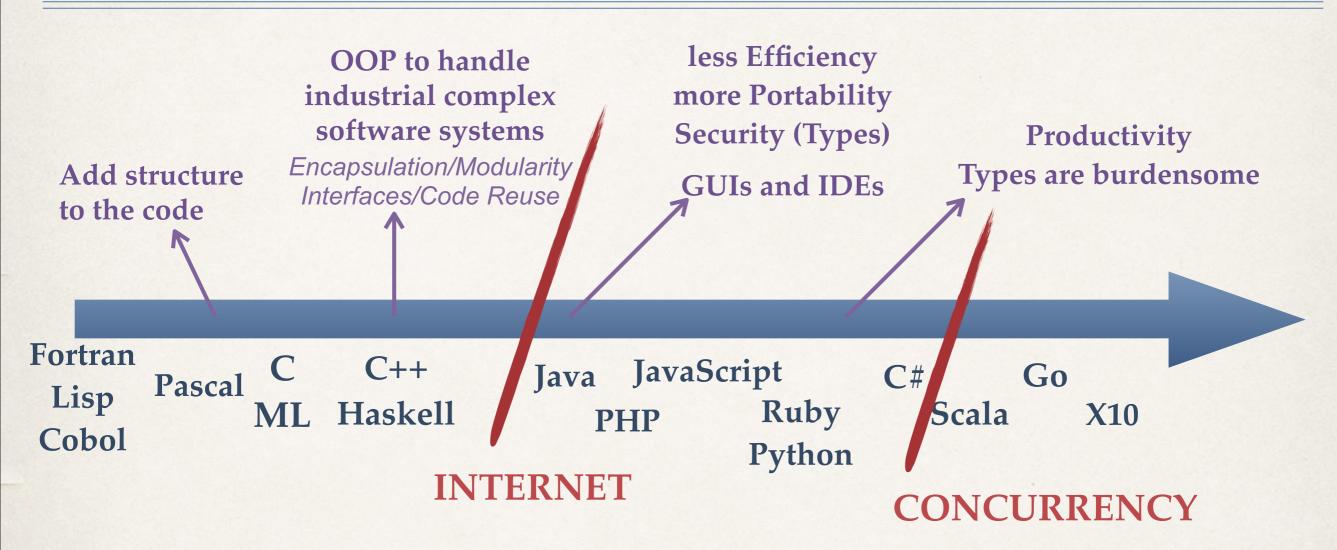
- When a language has been invented VS when became popular?
- Why has been invented VS why became popular?



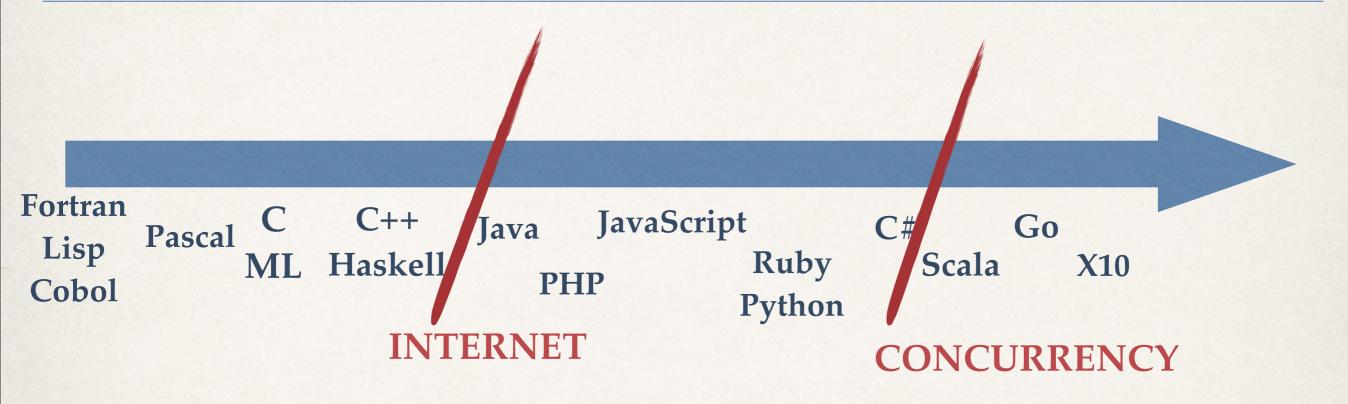
- * When a language has been invented VS when became popular?
- Why has been invented VS why became popular?



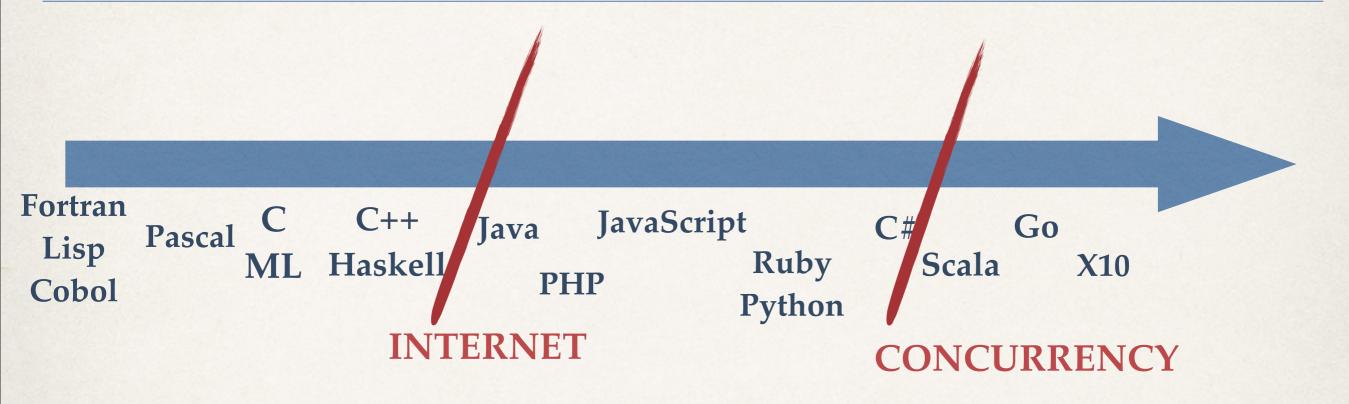
- When a language has been invented VS when became popular?
- Why has been invented VS why became popular?



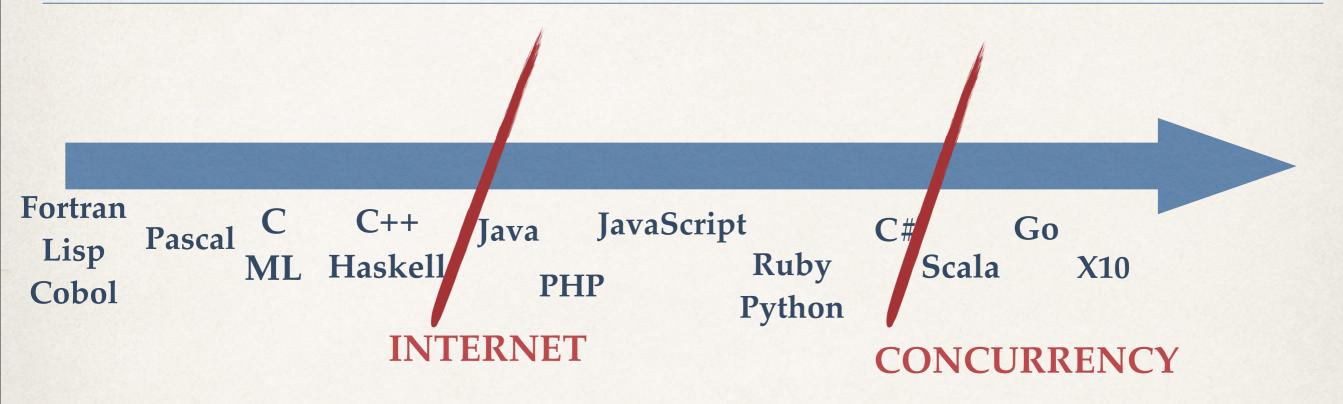
- When a language has been invented VS when became popular?
- Why has been invented VS why became popular?



Changes need a catalyser



- Changes need a catalyser
 - new hardware can only be parallel
 - new software must be concurrent



- Changes need a catalyser
 - new hardware can only be parallel
 - new software must be concurrent

Popular
Parallel Programming
Grand Challenge

- (correct) concurrent programming is difficult
- Adding concurrency to sequential code is even harder

Intrinsic reasons nondeterminism

Accidental reasons improper programming model

- (correct) concurrent programming is difficult
- Adding concurrency to sequential code is even harder

Intrinsic reasons nondeterminism

Accidental reasons improper programming model

Think concurrently (Concurrent Algorithm)



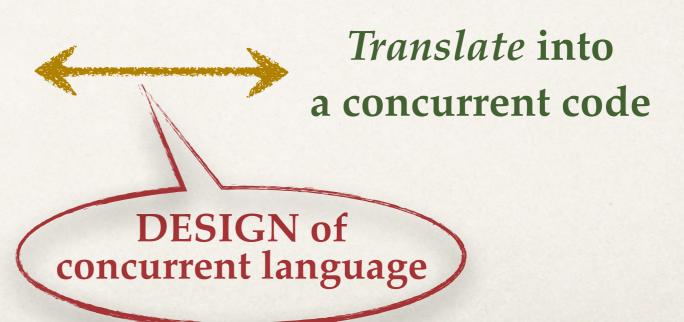
Translate into a concurrent code

- (correct) concurrent programming is difficult
- Adding concurrency to sequential code is even harder

Intrinsic reasons nondeterminism

Accidental reasons improper programming model

Think concurrently (Concurrent Algorithm)

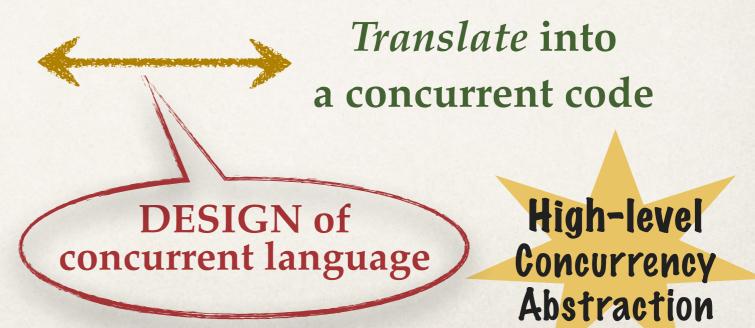


- (correct) concurrent programming is difficult
- Adding concurrency to sequential code is even harder

Intrinsic reasons nondeterminism

Accidental reasons improper programming model

Think concurrently (Concurrent Algorithm)



Easy to think
Easy to reason about



Expressiveness Performance

Easy to think Easy to reason about



Expressiveness Performance

* OOP

- encapsulation
- memory management
- multiple inheritance

Easy to think Easy to reason about



Expressiveness Performance

* OOP

- encapsulation
- memory management
- multiple inheritance

Easy to think Easy to reason about



Expressiveness Performance

* OOP

- encapsulation
- memory management
- multiple inheritance

- Types
 - documentation vs verbosity

Easy to think Easy to reason about



Expressiveness Performance

* OOP

- encapsulation
- memory management
- multiple inheritance

Types

documentation vs verbosity

Functional Programming

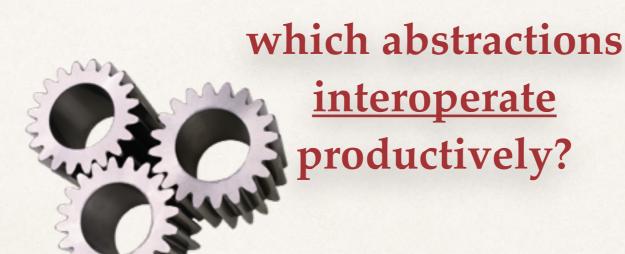
- composing and passing behaviours
- sometimes imperative style is easier to reason about

Easy to think
Easy to reason about



Expressiveness Performance

* OOP



Types

Functional Programming

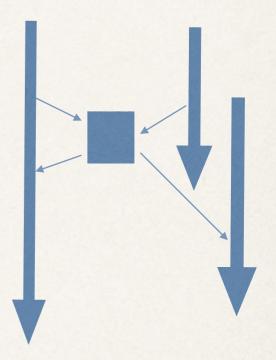
Concurrency Abstractions?

Many Concurreny Models...

Concurrency Abstractions?

Many Concurreny Models...

Shared Memory Model and "Java Threads"



finish{}

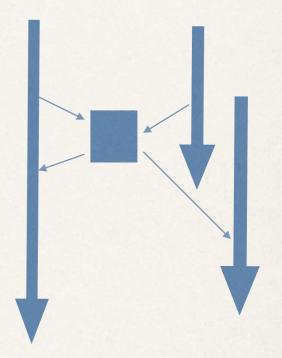
Concurrency Abstractions?

Many Concurreny Models...

Shared Memory Model and "Java Threads"

```
synchronized(lock)
lock.wait()
lock.notify()
```

```
atomic {...}
when(cond){...}
```



Concurrency Abstractions?

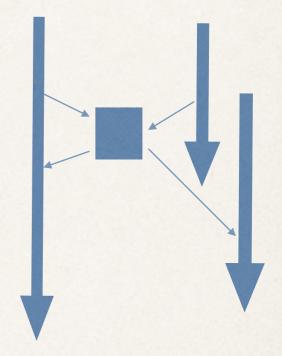
Many Concurreny Models...

Shared Memory Model and "Java Threads"

```
synchronized(lock)
lock.wait()
lock.notify()
```

```
atomic {...}
when(cond) {...}
```





* logical threads distinguished from executors (activities/tasks) (pool of thread workers)

Concurrency Abstractions?

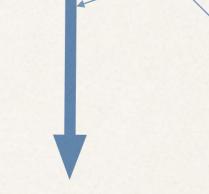
Many Concurreny Models...

Shared Memory Model and "Java Threads"

```
synchronized(lock) sill lock.wait() lock.notify()
```

```
atomic {...}
when(cond) {...}
```

```
#10
async{}
finish{}
```



```
new Thread().start()

JVM thread
```

Lightweight threads in the program Pool of Executors in the runtime

* logical threads distinguished from executors (activities/tasks) (pool of thread workers)

Many Concurrency Models

Shared Memory

- is very natural for "centralised algorithms" and components operating on shared data
- is error-prone when the sole purpose of SM is thread communication

Message Passing Model

- It is the message that carries the state!
- Channel based: Google's GO
- * Actor Model: Erlang, Scala. It fits well both OOP and FP
- * Sessions

* GPU Concurrency Model

- Massive data parallelism
- integration with high-level concurrent language (X10, Nova, Scala heterogeneous compiler)

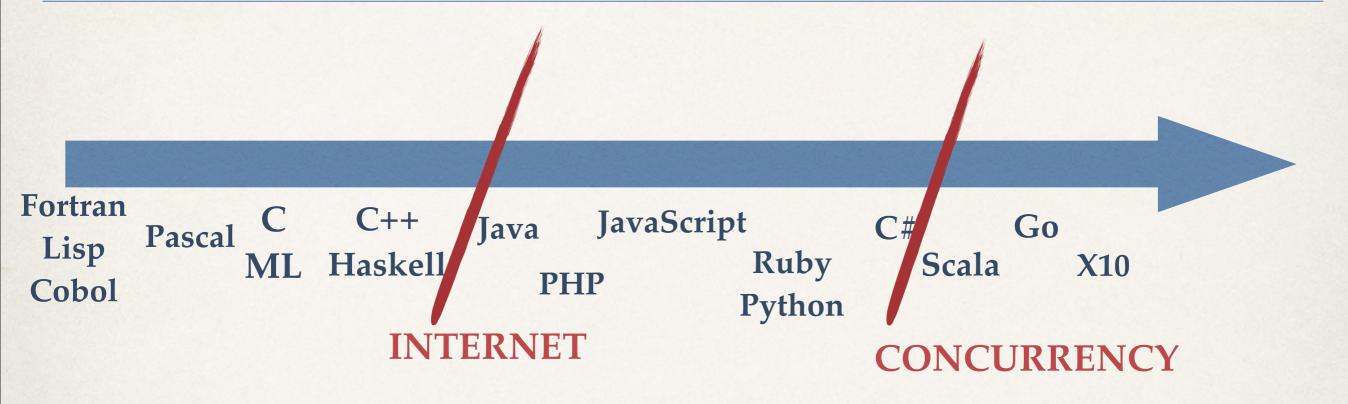
Many Concurrency Models

Shared Memory

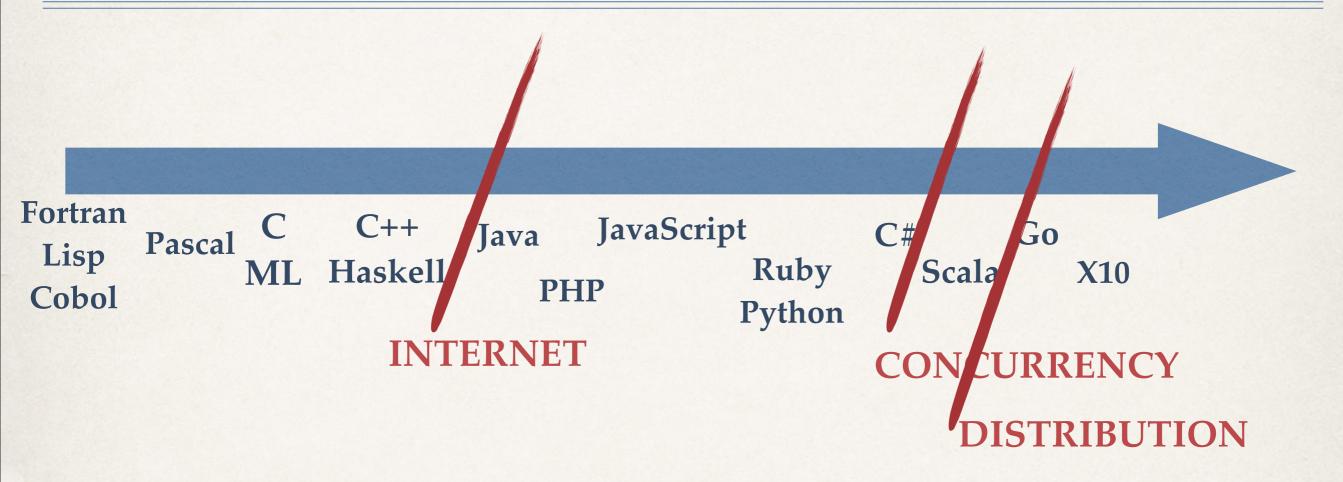
which abstractions interoperate productively?



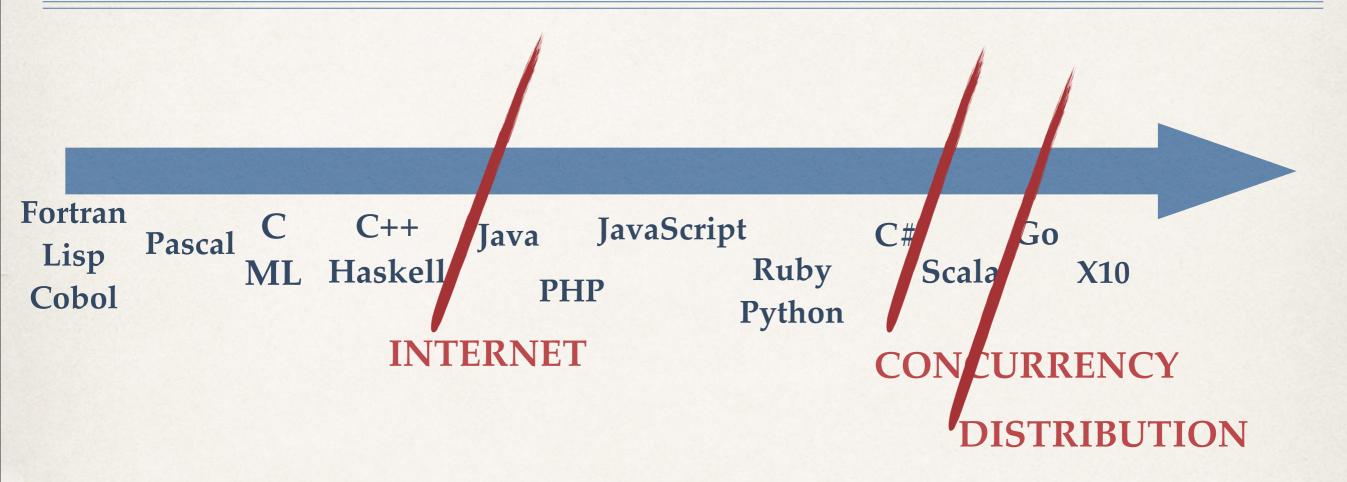
* GPU Concurrency Model



New catalyser:



- New catalyser:
 - multicore —> concurrent programming
 - cloud computing —> distributed programming



- New catalyser:
 - multicore —> concurrent programming
 - cloud computing —> distributed programming

Reactive Programming

react to events

react lo load

react to failures

- react to events
 - event driven
 - asynchronous
- react lo load

instead of issuing a command that asks for a change, react to an event that indicates that something has changed

- * futures
- push data to consumers when available rather than polling

react to failures

- react to events
 - event driven
 - asynchronous
- react lo load
 - scalability
 - up/down +/- CPU nodes
 - in/out +/- server
- react to failures

instead of issuing a command that asks for a change, react to an event that indicates that something has changed

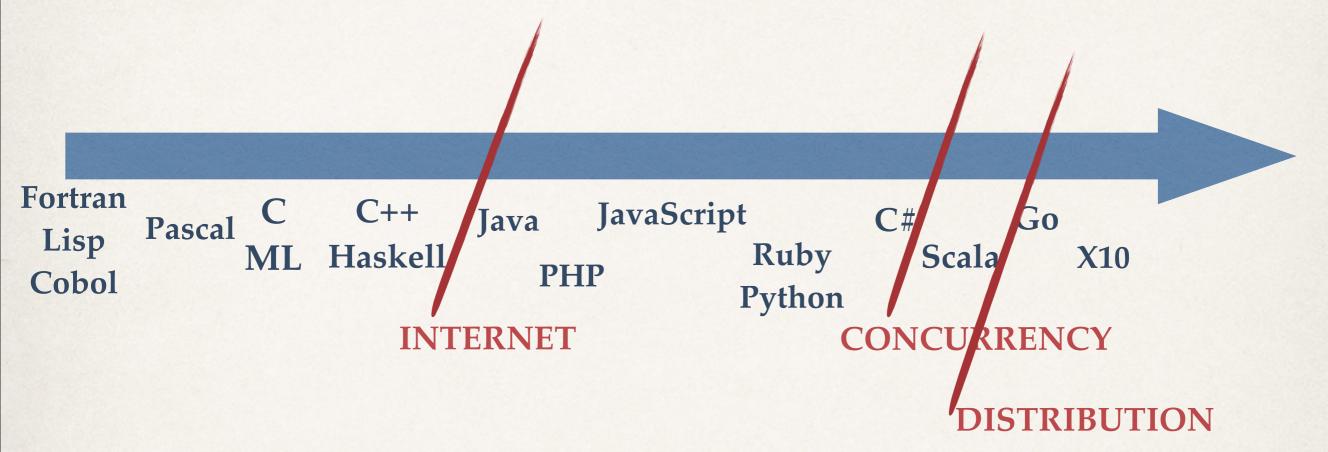
- futures
- * push data to consumers when available rather than polling

- react to events
 - event driven
 - asynchronous
- react lo load
 - scalability
 - up/down +/- CPU nodes
 - in/out +/- server
- react to failures
 - * resiliency

instead of issuing a command that asks for a change, react to an event that indicates that something has changed

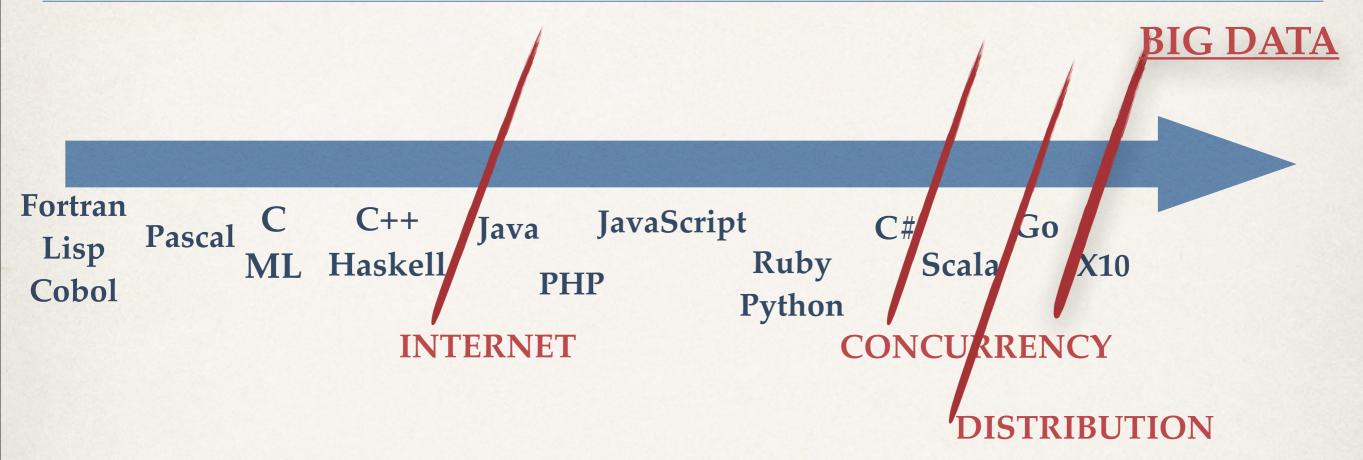
- futures
- * push data to consumers when available rather than polling

The Quest for good Abstractions



- New catalyser:
 - multicore —> concurrent programming
 - cloud computing —> distributed programming

The Quest for good Abstractions



- New catalyser:
 - multicore —> concurrent programming
 - cloud computing —> distributed programming
 - big data application —> High Performance Computing

High Performance Computing

- scale-out on massively parallel hardware
 - high-performance computing on supercomputers
 - analytic computations on big data
- a single program
 - runs on a collection of places on a cluster of computers
 - can create global data-structures spanning multiple places
 - can spawn tasks at remote places, detecting termination of arbitrary trees of spawned tasks

High Performance Computing

- scale-out on massively parallel hardware
 - high-performance computing on supercomputers
 - analytic computations on big data
- a single program
 - * runs on a collection of places on a cluster of computers
 - can create global data-structures spanning multiple places
 - can spawn tasks at remote places, detecting termination of arbitrary trees of spawned tasks
- Big Data Application Framework
 - Map Reduce Model
 - Bulk Synchronous Parallel Model

High Performance Computing

- scale-out on massively parallel hardware
 - high-performance computing on supercomputers
 - analytic computations on big data
- a single program
 - * runs on a collection of places on a cluster of computers
 - can create global data-structures spanning multiple places
 - can spawn tasks at remote places, detecting termination of arbitrary trees of spawned tasks
- Big Data Application Framework
 - Map Reduce Model
 - Bulk Synchronous Parallel Model

"Concurrent Patterns"
with their
distinctive abstractions

What about Theory?

The X10 experience



- open-source language for HPC programming
- key design features:
 - * scaling: code running on 100 10.000 multicore nodes (up to 50millions core)
 - productivity: high level abstractions (Java-like, Scala-like) + typing (constrained dependent types as contracts).
 - performance on heterogeneous hardware: it compiles to Java, to C++, to CUDA. Resilient extension
 - * concurrent abstractions: place-centric, asynchronous computing

```
// double in parallel all the array elements
val a:Array[Int]= ...
    for(i in 0..(a.size-1))
        async { a(i)*=2 }
println ("The End")
```

Spawns an asynchronous lightweight activity running in parallel

waits for the termination of all the spawned activities

Spawns an asynchronous lightweight activity running in parallel

```
// double in parallel all the array elements
val a:Array[Int]= ...
var b=0
finish for(i in 0..(a.size-1))
   async { a(i)*=2
        atomic { b=b+a(i) }
   }
println ("The End")
```

STM when (cond) s clocks

```
class HelloWholeWorld {
  public static def main(args:Rail[String]) {
    finish for (p in Place.places())
       async at(p)
       Console.OUT.printnl("Hello from place "+p)
       Console.OUT.printnl("Hello from everywhere")
}
```

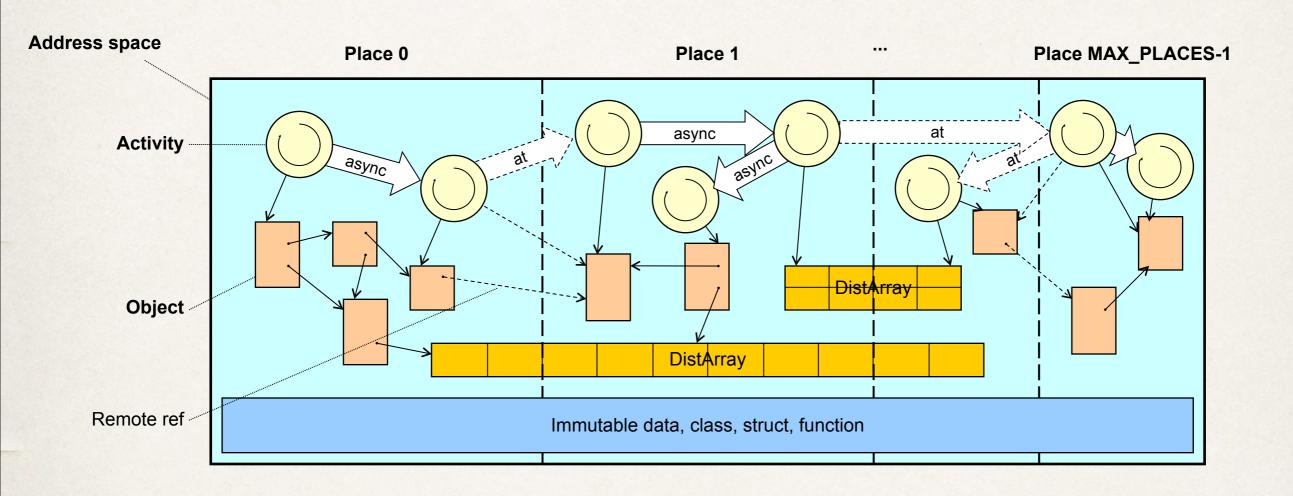
```
class HelloWholeWorld {
  public static def main(args:Rail[String]) {
    finish for (p in Place.places())
        async at(p)
        Console.OUT.printnl("Hello from place "+p)
        Console.OUT.printnl("Hello from everywhere")
}
```

```
%X10_NPLACES=4
Hello from place 1
Hello from place 2
Hello from place 0
Hello from place 3
Hello from everywhere
```

```
class HelloWholeWorld {
  public static def main(args:Rail[String]) {
    finish for (p in Place.places())
        async at(p) @CUDA
        Console.OUT.printnl("Hello from place "+p)
        Console.OUT.printnl("Hello from everywhere")
}
```

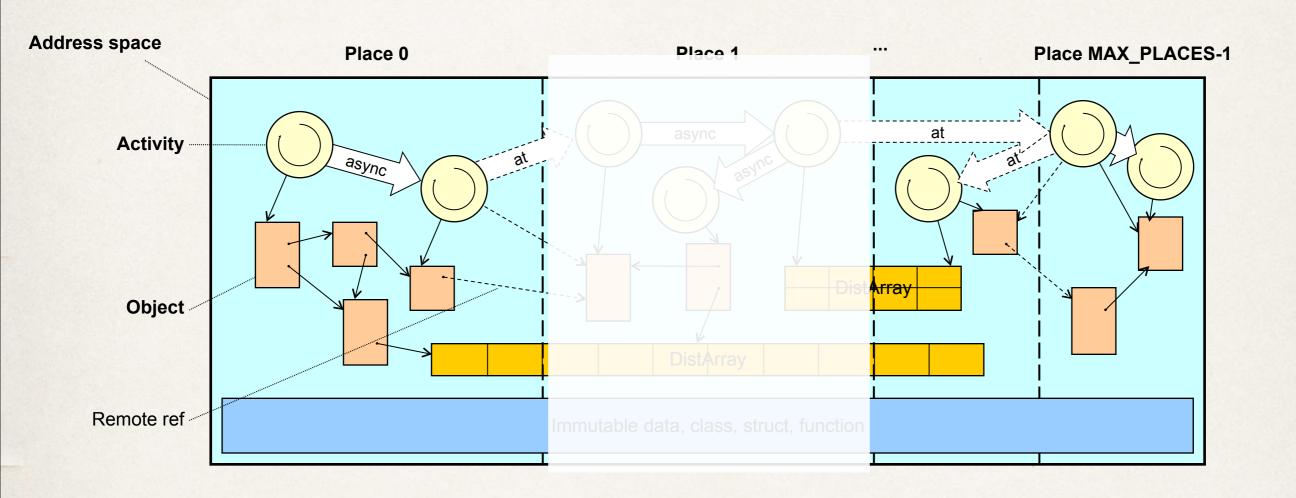
```
%X10_NPLACES=4
Hello from place 1
Hello from place 2
Hello from place 0
Hello from place 3
Hello from everywhere
```

Async Partitioned Global Address Space

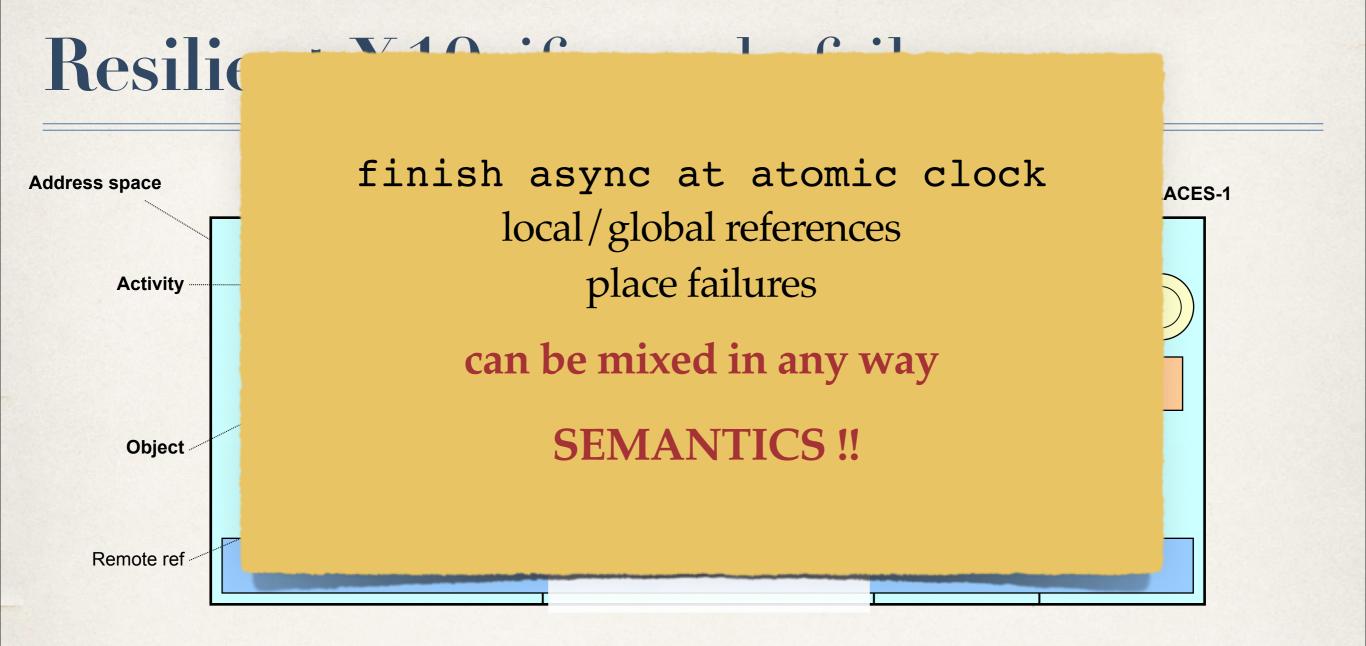


- * A global address space is divided into multiple *places* (=computing nodes)
 - Each place can contain activities and objects
- An object belongs to a specific place, but can be remotely referenced
- DistArray is a data structure whose elements are scattered over multiple places

Resilient X10: if a node fails....



- * it is relatively easy to localize the impact of place death
 - Objects in other places are still alive, but remote references become inaccessible
 - Execution continues using the remaining nodes
 - Happens Before Relation between remaining statements is preserved (HB Invariance) – no new race conditions, or sequentialization induced by failure.



- * it is relatively easy to localize the impact of place death
 - Objects in other places are still alive, but remote references become inaccessible
 - Execution continues using the remaining nodes
 - Happens Before Relation between remaining statements is preserved (HB Invariance) – no new race conditions, or sequentialization induced by failure.

TX10

Semantics of (Resilient) X10 [ECOOP 2014] S.Crafa, D.Cunningham, V.Saraswat, A.Shinnar, O.Tardieu

$$v ::= o \mid o\$p \mid \mathsf{E} \mid \mathsf{DPE}$$

$$Expressions \qquad e ::= v \mid x \mid e.f \mid \{f:e,\dots,f:e\} \mid \mathsf{globalref}\,e \mid \mathsf{valof}\,e$$

$$Statements \qquad s ::= \mathsf{skip}; \mid \mathsf{throw}\,v \mid \mathsf{val}\,x = e \mid s \mid e.f = e; \mid \{s \mid t\} \mid \mathsf{at}(p)\mathsf{val}\,x = e \mid \mathsf{in}\,s \mid \mathsf{async}\,s \mid \mathsf{finish}\,s \mid \mathsf{try}\,s\,\mathsf{catch}\,t \mid \mathsf{at}(p)\,s \mid \mathsf{async}\,s \mid \mathsf{finish}\,s \mid \mathsf{try}\,s\,\mathsf{catch}\,t$$

$$\mathsf{at}(p)\,\mathsf{s} \mid \mathsf{async}\,\mathsf{s} \mid \mathsf{finish}\,\mu\,s$$

$$\mathsf{Configurations} \quad k \quad ::= \langle s,g \rangle \mid g \qquad \qquad \mathsf{error}\,\mathsf{propagation}\,\mathsf{and}\,\mathsf{handling}$$

Global heap $g := \emptyset \mid g \cdot [p \mapsto h]$ Local heap $h := \emptyset \mid h \cdot [o \mapsto (\tilde{f}_i : \tilde{v}_i)]$

- Small-step transition system, mechanised in Coq
- non in ChemicalAM style (better fits the centralised view of the distributed program)

$$\langle s, g \rangle \longrightarrow_p \langle s', g' \rangle \mid g' \qquad \langle s, g \rangle \xrightarrow{\mathsf{E} \times}_p \langle s', g' \rangle \mid g' \qquad \langle s, g \rangle \xrightarrow{\mathsf{E} \otimes}_p \langle s', g' \rangle \mid g'$$

- Small-step transition system, mechanised in Coq
- non in ChemicalAM style (better fits the centralised view of the distributed program)

$$\langle s, g \rangle \longrightarrow_p \langle s', g' \rangle \mid g'$$
 $\langle s, g \rangle \xrightarrow{\mathsf{E} \times}_p \langle s', g' \rangle \mid g'$ $\langle s, g \rangle \xrightarrow{\mathsf{E} \otimes}_p \langle s', g' \rangle \mid g'$

Async failures arise in parallel threads and are caught by the inner finish waiting for their termination finish {async throw E async s2}

- Small-step transition system, mechanised in Coq
- non in ChemicalAM style (better fits the centralised view of the distributed program)

$$\langle s, g \rangle \longrightarrow_p \langle s', g' \rangle \mid g'$$
 $\langle s, g \rangle \xrightarrow{\mathsf{E} \times}_p \langle s', g' \rangle \mid g'$ $\langle s, g \rangle \xrightarrow{\mathsf{E} \otimes}_p \langle s', g' \rangle \mid g'$

Async failures arise in parallel threads and are caught by the inner finish waiting for their termination finish {async throw E async s2}

Synch failures lead to the failure of any sync continuation leaving async (remote) running code free to terminate {async at(p)s1 throw E s2}

- Small-step transition system, mechanised in Coq
- non in ChemicalAM style (better fits the centralised view of the distributed program)

$$\langle s,g \rangle \longrightarrow_p \langle s',g' \rangle \mid g' \qquad \langle s,g \rangle \stackrel{\mathsf{E} \times}{\longrightarrow}_p \langle s',g' \rangle \mid g' \qquad \langle s,g \rangle \stackrel{\mathsf{E} \otimes}{\longrightarrow}_p \langle s',g' \rangle \mid g'$$

Proved

Async failures arise in parallel threads

and are caught by the inner finish waiting for their termination

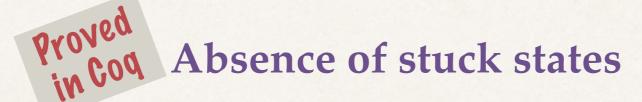


Synch failures lead to the failure of any sync continuation leaving async (remote) running code free to terminate

{async at(p)s1 throw E s2}

- Small-step transition system, mechanised in Coq
- non in ChemicalAM style (better fits the centralised view of the distributed program)

$$\langle s, g \rangle \longrightarrow_p \langle s', g' \rangle \mid g' \qquad \langle s, g \rangle \xrightarrow{\mathsf{E} \times}_p \langle s', g' \rangle \mid g' \qquad \langle s, g \rangle \xrightarrow{\mathsf{E} \otimes}_p \langle s', g' \rangle \mid g'$$



(the proof can be run, yielding an interpreter for TX10)

smoothly scales to node failure, with

- global heap is a partial map: dom(g) collects non failed places
- executing a statement at failed place results in a DPE
- place shift at failed place results in a DPE
- remote exceptions flow back at the remaining finish masked as DPE

contextual rules modified accordingly

(Place Failure)

$$p \in dom(g)$$

$$\langle s, g \rangle \longrightarrow_p \langle s, g \setminus \{(p, g(p))\}\rangle$$

$$p
otin dom(g)$$
 $\langle \operatorname{skip}, g
angle \xrightarrow{\mathsf{DPE} \otimes}_{p} g$ $\langle \operatorname{async} s, g
angle \xrightarrow{\mathsf{DPE} \otimes}_{p} g$ $\langle \operatorname{at}(p) s, g
angle \xrightarrow{\mathsf{DPE} \otimes}_{q} g$

Happens Before Invariance

 failure of place q does not alter the happens before relationship between statement instances at places other than q

```
\overline{\operatorname{at}}(0) \{ \overline{\operatorname{at}}(p) \text{ finish } \overline{\operatorname{at}}(q) \overline{\operatorname{async}} s_1 \quad s_2 \}
```

s2 runs at 0 after s1

$$\overline{\operatorname{at}}(0)\operatorname{finish}\left\{\overline{\operatorname{at}}(p)\{\overline{\operatorname{at}}(q)\ \overline{\operatorname{async}}\,s_1\}\right.$$
 $s_2\}$

s2 runs at 0 in parallel with s1

Happens Before Invariance

 failure of place q does not alter the happens before relationship between statement instances at places other than q

```
\overline{\operatorname{at}}(0) { \overline{\operatorname{at}}(p) finish \overline{\operatorname{at}}(q) \overline{\operatorname{async}}\,s_1 s_2} s_2 runs at 0 after s_1 p fails while s_1 is running at q
```

$$\overline{\mathtt{at}}(0) \, \mathtt{finish} \, \{ \, \overline{\mathtt{at}}(p) \, \{ \overline{\mathtt{at}}(q) \, \, \overline{\mathtt{async}} \, s_1 \} \quad s_2 \} \quad s2 \, runs \, at \, 0 \, in \, \underline{parallel} \, with \, s1$$

Happens Before Invariance

 failure of place q does not alter the happens before relationship between statement instances at places other than q

```
\overline{\operatorname{at}}(0) {\overline{\operatorname{at}}(p) finish \overline{\operatorname{at}}(q) \overline{\operatorname{async}}\,s_1 s_2} s_2 runs at 0 after s1 p fails while s1 is running at q same behaviour!
```

$$\overline{\operatorname{at}}(0)$$
 finish $\{\overline{\operatorname{at}}(p)\{\overline{\operatorname{at}}(q)\ \overline{\operatorname{async}}\ s_1\}\ s_2\}$ s2 runs at 0 in parallel with s1

Happens Before Invariance

 failure of place q does not alter the happens before relationship between statement instances at places other than q

DPE slows at place 0 discarding s1

$$\overline{\operatorname{at}}(0) \, \{\, \overline{\operatorname{at}}(p) \, \operatorname{finish} \, \overline{\operatorname{at}}(q) \, \overline{\operatorname{async}} \, s_1 \quad s_2 \}$$

$$\operatorname{throws} \, \mathbf{v}$$

$$\overline{\operatorname{at}}(0) \, \operatorname{finish} \, \{\, \overline{\operatorname{at}}(p) \, \{ \overline{\operatorname{at}}(q) \, \overline{\operatorname{async}} \, s_1 \} \quad s_2 \}$$

 $v \times$ flows at place 0 while s2 is running

 $\langle s,g\rangle\cong\langle t,g\rangle$ equivalent configurations when

- transition steps are weakly bi-simulated
- * under any modification of the shared heap by current activities (object field update, object creation, place failure)

$\langle s,g\rangle\cong\langle t,g\rangle$ equivalent configurations when

- transition steps are weakly bi-simulated
- under any modification of the shared heap by current activities (object field update, object creation, place failure)

$$\langle s,g \rangle \mathcal{R} \langle t,g \rangle$$
 whenever

- 1. \vdash isSync s iff \vdash isSync t
- 2. $\forall p, \forall \Phi \ environment \ move$

if
$$\langle s, \Phi(g) \rangle \xrightarrow{\lambda}_{p} \langle s', g' \rangle$$
 then $\exists t'. \langle t, \Phi(g) \rangle \xrightarrow{\lambda}_{p} \langle t', g' \rangle$
with $\langle s', g' \rangle \mathcal{R} \langle t', g' \rangle$ and viceversa

 $\langle s,g\rangle\cong\langle t,g\rangle$ equivalent configurations when

- transition steps are weakly bi-simulated
- * under any modification of the shared heap by current activities (object field update, object creation, place failure)

 $\langle s, g \rangle \mathcal{R} \langle t, g \rangle whenever$

- 1. \vdash isSync s iff \vdash isSync t
- 2. $\forall p, \forall \Phi \ environment \ move$

if
$$\langle s, \Phi(g) \rangle \xrightarrow{\lambda}_{p} \langle s', g' \rangle$$
 then $\exists t'. \langle t, \Phi(g) \rangle \xrightarrow{\lambda}_{p} \langle t', g' \rangle$ with $\langle s', g' \rangle \mathcal{R} \langle t', g' \rangle$ and viceversa

models the update of *g*:

$$dom(\Phi(g)) = dom(g) \ and$$

$$\forall p \in dom(g) \ dom(g(p)) \subseteq dom(\Phi(g)(p))$$

$\langle s,g\rangle\cong\langle t,g\rangle$ equivalent configurations when

- transition steps are weakly bi-simulated
- * under any modification of the shared heap by current activities (object field update, object creation, place failure)

$$\langle s,g \rangle \mathcal{R} \langle t,g \rangle$$
 whenever

- 1. \vdash isSync s iff \vdash isSync t
- 2. $\forall p, \forall \Phi \ environment \ move$

if
$$\langle s, \Phi(g) \rangle \xrightarrow{\lambda}_{p} \langle s', g' \rangle$$
 then $\exists t'. \langle t, \Phi(g) \rangle \xrightarrow{\lambda}_{p} \langle t', g' \rangle$ with $\langle s', g' \rangle \mathcal{R} \langle t', g' \rangle$ and viceversa

 $dom(\Phi(g)) = dom(g) \ and$ $\forall p \in dom(g) \ dom(g(p)) \subseteq dom(\Phi(g)(p))$

models the update of g:

Bisimulation whose Bisimilarity is a congruence

 $\mathtt{finish}\;\mathtt{at}(p)\,s\;\ncong\;\mathtt{at}(p)\,\mathtt{finish}\,s$

$$\{\{s\ t\}\ u\}\cong \{s\ \{t\ u\}\}$$

$$\vdash \mathsf{isAsync}\ s \ \mathsf{try}\ \{s\ t\}\ \mathsf{catch}\ u\cong \{\mathsf{try}\ s\ \mathsf{catch}\ u\ \mathsf{try}\ t\ \mathsf{catch}\ u\}$$

$$\mathsf{at}(p)\{s\ t\} \not\cong \{\mathsf{at}(p)s\ \mathsf{at}(p)t\}$$

$$\mathsf{at}(p)\mathsf{at}(q)s \not\cong \mathsf{at}(q)s$$

$$\mathsf{async}\ \mathsf{at}(p)s \not\cong \mathsf{at}(p)\ \mathsf{async}\ s$$
 if s throws a sy and home is fathen l.h.s. throwasked DPEx r.h.s. re-throwshince synch ex not masked by finish $\{s\ t\}\cong \mathsf{finish}\ \{s\ \mathsf{async}\ t\}\cong \mathsf{finish}\ \{s\ t\}$

if s throws a sync exc. and home is failed, then l.h.s. throws a masked DPEx while r.h.s. re-throws vx since synch exc are not masked by DPE

Conclusions

- * Concurrecy is critical for Programming Languages
 - heterogeneous concurrency models (Distribution)

- What is the right level of abstraction?
 - * What are good abstractions? Expressive, flexible, easy to reason about, easy to implement in a scalable/resilient way

- Formal method to experiment!
 - test new primitive, new mix of primitives
 - tool to reason about programs

(Par Left)

$$\langle s, g \rangle \xrightarrow{\lambda}_p \langle s', g' \rangle \mid g'$$

$$\lambda = \epsilon, v \times \langle \{s \ t\}, g \rangle \xrightarrow{\lambda}_{p} \langle \{s' \ t\}, g' \rangle \mid \langle t, g' \rangle$$

$$\lambda = v \otimes \langle \{s \ t\}, g \rangle \xrightarrow{\lambda}_p \langle s', g' \rangle \mid g'$$

(Par Right)

$$\frac{\vdash \mathsf{isAsync}\,t \quad \langle s,g \rangle \overset{\lambda}{\longrightarrow}_p \langle s',g' \rangle \mid g'}{\langle \{t \ s\},g \rangle \overset{\lambda}{\longrightarrow}_p \langle \{t \ s'\},g' \rangle \mid \langle t,g' \rangle}$$

(Place Shift)

$$(v',g') = \mathsf{copy}(v,q,g)$$

$$\langle \operatorname{at}(q) \operatorname{val} x = v \operatorname{in} s, g \rangle \longrightarrow_p \langle \overline{\operatorname{at}}(q) \{ s[v'/x] \operatorname{skip} \}, g' \rangle$$

(At)

$$\frac{\langle s, g \rangle \xrightarrow{\lambda}_{q} \langle s', g' \rangle \mid g'}{\langle \overline{\mathtt{at}}(q) \, s, g \rangle \xrightarrow{\lambda}_{p} \langle \overline{\mathtt{at}}(q) \, s', g' \rangle \mid g'}$$

(Spawn)

$$\langle \operatorname{async} s, g \rangle \longrightarrow_p \langle \overline{\operatorname{async}} s, g \rangle$$

(Async)

$$\begin{array}{c|c} \langle s,g \rangle \stackrel{\lambda}{\longrightarrow}_p \langle s',g' \rangle \mid g' \\ \\ \lambda = \epsilon & \langle \overline{\mathtt{async}} \, s,g \rangle \stackrel{\lambda}{\longrightarrow}_p \langle \overline{\mathtt{async}} \, s',g' \rangle \mid g' \\ \\ \lambda = v \times, v \otimes & \langle \overline{\mathtt{async}} \, s,g \rangle \stackrel{v \times}{\longrightarrow}_p \langle \overline{\mathtt{async}} \, s',g' \rangle \mid g' \\ \end{array}$$

(Finish)

$$\frac{\langle s,g\rangle \stackrel{\lambda}{\longrightarrow_p} \langle s',g'\rangle}{\langle \mathtt{finish}_{\mu} \, s,g\rangle \longrightarrow_p \langle \mathtt{finish}_{\mu \cup \lambda} \, s',g'\rangle}$$

(End Finish)

$$\langle s,g\rangle \xrightarrow{\lambda}_p g' \quad \lambda' = \mathsf{E} \otimes if \lambda \cup \mu \neq \emptyset \ else \ \epsilon$$

$$\langle \mathtt{finish}_{\mu} s,g\rangle \xrightarrow{\lambda'}_p g'$$

(Exception)

$$\langle \mathtt{throw}\, v, g \rangle \overset{v \otimes}{\longrightarrow}_p g$$

(Skip)

$$\langle \mathtt{skip}, g \rangle \longrightarrow_p g$$

(Try)

$$\langle s, g \rangle \xrightarrow{\lambda}_p \langle s', g' \rangle \mid g'$$

$$\lambda = \epsilon, v \times \qquad \langle \operatorname{try} s \operatorname{catch} t, g \rangle \xrightarrow{\lambda}_{p} \langle \operatorname{try} s' \operatorname{catch} t, g' \rangle \mid g'$$

$$\lambda = v \otimes \qquad \langle \operatorname{try} s \operatorname{catch} t, g \rangle \xrightarrow{\rho} \langle \{s' \mid t\}, g' \rangle \mid \langle t, g' \rangle$$

Plus rules for expression evaluation