Verification of Security Protocols
Part II

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Advertisement

ProSecure project

Goal : analysis and design of security systems

→ five years project (2011-2015), founded by the European Research Council.

→ Regular job offers !

- PhD positions and Post-doc positions
- One research associate position (up to 5 years, with budget for PhD grant and other costs)
- Permanent positions (CNRS, INRIA, Universities)

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LORIA (Nancy)

Size : 500 researchers, among which about 150 permanent researchers and 150 PhD students.

Where is it ?

Well connected to :

- Paris, France (90 minutes)
- Luxembourg (90-120 minutes)
- Saarbrucken, Germany (120 minutes)

Yesterday course

How to use formal methods for analysing cryptographic protocols ?

- Messages are abstracted by terms
- Intruder can compute new terms using a deduction system
- Protocols can be described by rules of the form $u \rightarrow v$, where u, v are terms with variables.

What formal methods allow to do ?

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- For a bounded number of sessions, secrecy is co-NP-complete [RusinowitchTuruani CSFW01]
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What formal methods allow to do ?

- In general, secrecy preservation is undecidable.
- For a bounded number of sessions, secrecy is co-NP-complete [RusinowitchTuruani CSFW01]
  $\longrightarrow$ several tools for detecting attacks (Casper, Avispa platform...)
- For an unbounded number of sessions
  - for one-copy protocols, secrecy is DEXPTIME-complete [CortierComon RTA03] [SeildVerma LPAR04]
  - for message-length bounded protocols, secrecy is DEXPTIME-complete [Durgin et al FMSP99] [Chevalier et al CSL03]

  $\longrightarrow$ some tools for proving security (ProVerif, EVA Platform)

Limitations of this approach ?

Are you ready to use any protocol verified with this technique ?

- Only a finite scenario is checked.
  $\longrightarrow$ What happens if the protocol is used one more time ?
- The underlying mathematical properties of the primitives are abstracted away.
- The specification of the protocol is analysed, but not its implementation. $\longrightarrow$ Cédric Fournet course

Motivation

Back to our running example :

$$\begin{array}{rcl} A \rightarrow B & : & \{\mathrm{pin}\}_{k_a} \\ B \rightarrow A & : & \{\{\mathrm{pin}\}_{k_a}\}_{k_b} \\ A \rightarrow B & : & \{\mathrm{pin}\}_{k_b} \end{array}$$

We need the equation for the commutativity of encryption

$$\{\{z\}_x\}_y = \{\{z\}_y\}_x$$

Some other examples

Encryption-Decryption theory

$$\mathrm{dec}(\mathrm{enc}(x, y), y) = x \quad \pi_1(\langle x, y\rangle) = x \quad \pi_2(\langle x, y\rangle) = y$$

EXclusive Or

$$\begin{array}{rclcrcl} x \oplus (y \oplus z) & = & z & \quad & x \oplus y & = & y \oplus x \\ x \oplus x & = & 0 & \quad & x \oplus 0 & = & x \end{array}$$

Diffie-Hellmann

$$\exp(\exp(z, x), y) = \exp(\exp(z, y), x)$$

E-voting protocols

First phase :

$$V \rightarrow A : \quad \mathrm{sign}(\mathit{blind}(\mathrm{vote}, r), V)$$
$$A \rightarrow V : \quad \mathrm{sign}(\mathit{blind}(\mathrm{vote}, r), A)$$

Voting phase :

$$V \rightarrow C : \quad \mathrm{sign}(\mathrm{vote}, A)$$

...

Equational theory for blind signatures

[Kremer Ryan 05]

$$\begin{array}{rcl} \mathrm{checksign}(\mathrm{sign}(x, y), \mathrm{pk}(y)) & = & x \\ \mathrm{unblind}(\mathrm{blind}(x, y), y) & = & x \\ \mathrm{unblind}(\mathrm{sign}(\mathrm{blind}(x, y), z), y) & = & \mathrm{sign}(x, z) \end{array}$$

Deduction

$$\frac{}{T \vdash_E M}\ M \in T \qquad\qquad \frac{T \vdash_E M_1 \quad \cdots \quad T \vdash_E M_k}{T \vdash_E f(M_1, \ldots, M_k)}\ f \in \Sigma$$

$$\frac{T \vdash M}{T \vdash M'}\ M =_E M'$$

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Example : $E := \mathrm{dec}(\mathrm{enc}(x, y), y) = x$ and $T = \{\mathrm{enc}(secret, k),\ k\}$.

$$\frac{\dfrac{\dfrac{}{T \vdash \mathrm{enc}(secret, k)} \qquad \dfrac{}{T \vdash k}}{T \vdash \mathrm{dec}(\mathrm{enc}(secret, k), k)}\ f \in \Sigma}{T \vdash secret}\ \mathrm{dec}(\mathrm{enc}(x, y), y) = x$$

Rewriting system

For analyzing equational theories, we (try to) associate to E a finite convergent rewriting system $\mathcal{R}$ such that :

$$u =_E v \quad \text{iff} \quad u\downarrow = v\downarrow$$

Definition (Characterization of the deduction relation)

Let $t_1, \dots t_n$ and u be terms in normal form.

$$\{t_1, \dots t_n\} \vdash u \quad \text{iff} \quad \exists C \text{ s.t. } C[t_1, \dots, t_n] \to^* u$$

(Also called Cap Intruder problem [Narendran et al])

Some results with equational theories

	Security problem	
	Bounded number of sessions	**Unbounded number of sessions**
Commutative encryption	*co-NP-complete* [CKRT04]	**Ping-pong protocols :** *co-NP-complete* [Turuani04]
Exclusive Or	*Decidable* [CS03,CKRT03]	**One copy - No nonces :** *Decidable* [CLC03] **Two-way automata - No nonces :** *Decidable* [Verma03]
Abelian Groups	*Decidable* [Shmatikov04]	**Two-way automata - No nonces :** *Decidable* [Verma03]
Prefix encryption	*co-NP-complete* [CKRT03]	
Abelian Groups and Modular Exponentiation	**General case :** *Decidable* [Shmatikov04] **Restricted protocols :** *co-NP-complete* [CKRT03]	**AC properties of the Modular Exponentiation No nonces :** *Semi-Decision Procedure* [GLRV04]

And now are you ready to use any protocol verified with these techniques ?

Assuming :

- Analysis for an unbounded number of sessions
- With equational theories

Outline of the talk

Towards more cryptographic guarantees

1. Formal methods for protocols
   - Yesterday course
   - Adding equational theories
2. Cryptographic models
   - Encryption schemes
   - Security of encryption
   - Cryptographic models
   - Linking formal and cryptographic models
3. Passive case
   - Setting
   - Patterns
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4. Active case
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   - Trace mapping
   - A special case : computational secrecy
   - General computational indistinguishability

Specificity of cryptographic models

- Messages are bitstrings
- Real encryption algorithm
- Real signature algorithm
- General and powerful adversary

$\longrightarrow$ very little abstract model

Encryption : the old time

- Caesar encryption : $A \rightarrow E$, $B \rightarrow F$, $C \rightarrow G$, ...
- Cypher Disk (Léone Battista Alberti 1466)

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- Caesar encryption : $A \rightarrow E$, $B \rightarrow F$, $C \rightarrow G$, ...
- Cypher Disk (Léone Battista Alberti 1466)

$\rightarrow$ subject to statistical analysis (Analyzing letter frequencies)

Encryption : mechanized time

Automatic substitutions and permutations

Enigma

Encryption nowadays

$\rightarrow$ Based on algorithmically hard problems.

RSA Function $n = pq$, p et q primes.

e : public exponent

- $x \mapsto x^e \mod n$ easy (cubic)
- $y = x^e \mapsto x \mod n$ difficult
  $x = y^d$ où $d = e^{-1} \mod \phi(n)$

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Diffie-Hellman Problem

- Given $A = g^a$ and $B = g^b$,
- Compute $\mathrm{DH}(A, B) = g^{ab}$

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Diffie-Hellman Problem

- Given $A = g^a$ and $B = g^b$,
- Compute $\mathrm{DH}(A, B) = g^{ab}$

$\rightarrow$ Based on hardness of integer factorization.

Estimations for integer factorization

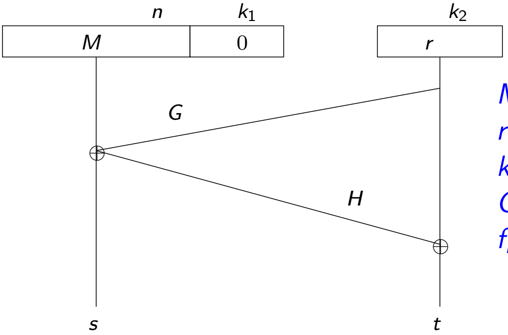
Module (bits)	Operations (in $\log_2$)
512	58
1024	80
2048	111
4096	149
8192	156

$\approx 2^{60}$ years

$\longrightarrow$ Lower bound for RSA and Diffie-Hellman.

How does an (asymmetric) encryption algorithm look like ?

Example : OAEP [Bellare Rogaway]



M : plaintext of length n
r : randomness of length
k_0
G, H : hash function
f_k : trapdoor function

$$E_K(x; r) = f_K(s||t)$$

What is a secure encryption scheme ?

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Intuitively :

- An adversary
- should not know the underlying plaintext.

Security of asymmetric encryption

Public data :

- $c = E_{k_e}(m, r)$ cyphertext
- k_e encryption key

There exists a unique message m satisfying the relation (with possible several relevant r)
$\rightarrow$ An exhaustive search on m and r yields m !

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$\Rightarrow$ Unconditional secrecy is impossible, one has to rely on algorithmic assumptions.

How to define an attacker/adversary

We wish to model an attacker :

- as clever as possible
  $\rightarrow$ he/she should be able to perform any operation
- with a limited time.
  - E.g. we do not wish to consider attacks that require 2^{60} years
  - Otherwise, the adversary could enumerate all keys (exponential time in $2^{size(keys)}$)

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Model : we consider any Turing machine

- that models any algorithm
- probabilistic : The adversary can generate keys and chose randomly his behavior
- polynomial in the size of the keys : which represents a reasonable execution time.

Security proof in a nutshell

Proof by reduction

1. Hypothesis : The algorithmic problem P is hard = there is no polynomial algorithm ($P = RSA, DL, DDH, CDH$...)

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   - If there exists a (polynomial) adversary A un adversaire (polynomial) breaking the encryption scheme,
   - Then one can build upon A for solving P in polynomial time.

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2. Reduction :
   - If there exists a (polynomial) adversary A un adversaire (polynomial) breaking the encryption scheme,
   - Then one can build upon A for solving P in polynomial time.
3. Conclusion : the encryption scheme is secure, there is no polynomial adversary.

What is a secure encryption scheme ?

- An adversary
- should not know the underlying plaintext.

$\longrightarrow$ several possible definitions of knowledge

One-Wayness (OW)

Basic security property : One-Wayness (OW)
without the inverse key, one cannot retrieve the underlying plaintext :

$$\Pr_{m,r}[c = E(m; r) \mid \mathcal{A}(c) = m]$$

is negligible.

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is negligible.

Negligibility : f is negligible if for any polynomial p, there exists η_0 s.t. for all $\eta \geq \eta_0$

$$f(\eta) \leq 1/p(\eta)$$

Not strong enough !

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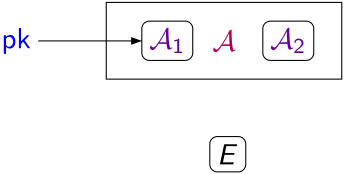
$\longrightarrow$ Introduction of a notion of indistinguishability. :
The adversary shall not guess even one bit of the underlying plaintext.

Indistinguabilité (IND)

Game Adversary : $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$

1. the adversary $\mathcal{A}_1$ is given the public key pk.



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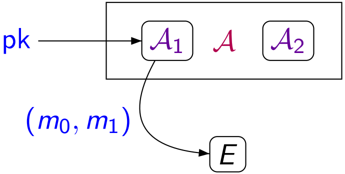

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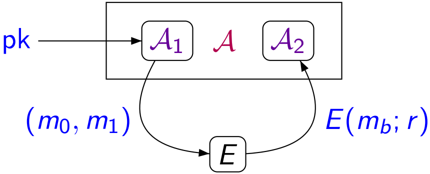

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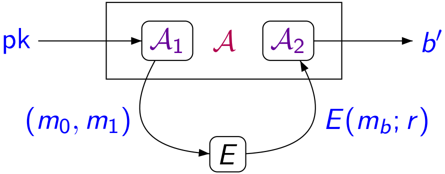

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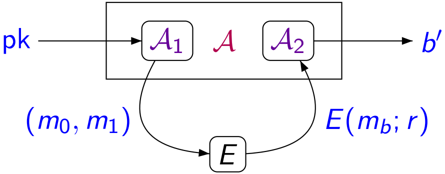

The probability $\Pr[b = b'] - \frac{1}{2}$ should be negligible.

Even stronger !

Non Malleability (NM)

Given a cyphertext $E(m; r)$, the adversary should not be able to create a cyphertext $E(m'; r')$ such that messages m and m' have a meaningful relation.

Definition of Non Malleability (NM)

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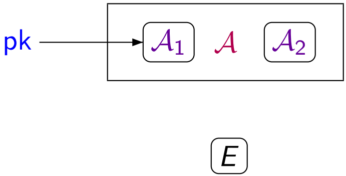

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Game Adversary : $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$

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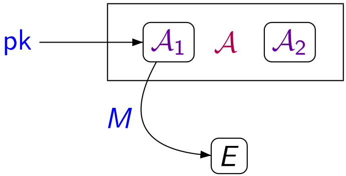

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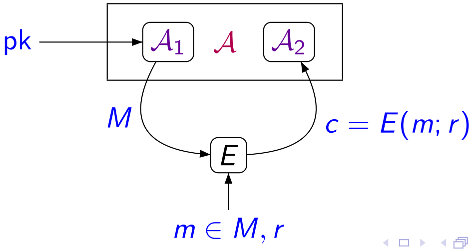

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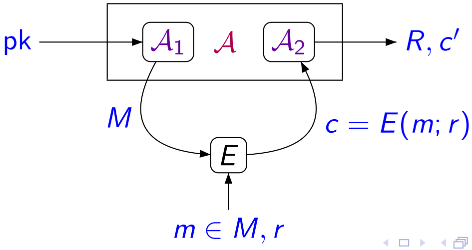
4. The adversary $\mathcal{A}_2$ outputs a binary relation R and a cyphertext c'.



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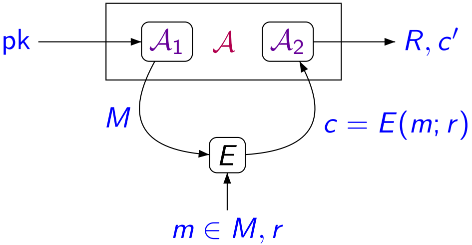

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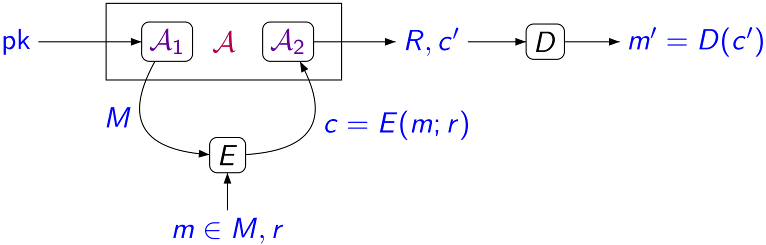

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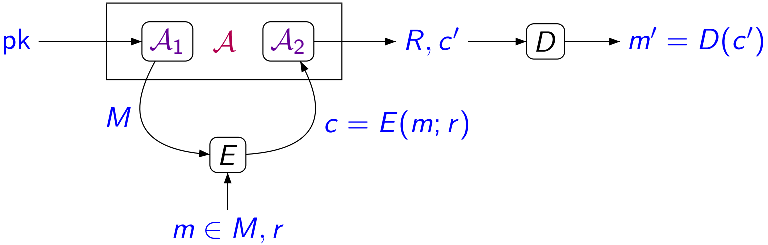

The probability $\Pr[R(m, m')] - \Pr[R(m, m^*)]$ should be negligible.

Relations

Non Malleability

$\Downarrow$

Indistinguishability

$\Downarrow$

One-Wayness

Exercise (medium) : show the implications.

Adding even more security

The adversary has access to oracles :

→ Encryption of all messages of his choice
→ Decryption of all messages of his choice

Three standard levels of security :

- Chosen-Plaintext Attacks (CPA)

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Three standard levels of security :

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- Non adaptive Chosen-Ciphertext Attacks (CCA1)
  → access to the (decryption) oracle before the challenge.
- Adaptive Chosen-Ciphertext Attacks (CCA2)
  → unlimited access to the (decryption) oracle (except for the challenge)

Relations

$$\begin{tikzcd}
\text{NM-CPA} \arrow[d] & \text{NM-CCA1} \arrow[l] \arrow[d] & \text{NM-CCA2} \arrow[l] \arrow[d, leftrightarrow] \\
\text{IND-CPA} \arrow[d] & \text{IND-CCA1} \arrow[l] & \text{IND-CCA2} \arrow[l] \\
\text{OW-CPA} & &
\end{tikzcd}$$

Outline of the talk

Towards more cryptographic guarantees

1. Formal methods for protocols
   - Yesterday course
   - Adding equational theories
2. Cryptographic models
   - Encryption schemes
   - Security of encryption
   - Cryptographic models
   - Linking formal and cryptographic models
3. Passive case
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Cryptographic models

Encryption is only one component of cryptographic models

- Cryptographic primitives : encryption, signatures, ...
- Protocol model
- Adversary
- Security notions

Setting for cryptographic protocols

Protocol :

- Message exchange program
- using cryptographic primitives

Adversary $\mathcal{A}$: any probabilistic polynomial Turing machine, *i.e.* any probabilistic polynomial program.

- polynomial : captures what is feasible
- probabilistic : the adversary may try to guess some information

Definition of secrecy preservation

$\rightarrow$ Several notions of secrecy :

One-Wayness : The probability for an adversary $\mathcal{A}$ to compute the secret s against a protocol $\mathcal{P}$ is negligible (smaller than any inverse of polynomial).

$$\forall p \text{ polynomial } \quad \exists \eta_0 \ \forall \eta \geq \eta_0 \quad \Pr^{\eta}_{m,r}[\mathcal{A}(\mathcal{P}_K) = s] \leq \frac{1}{p(\eta)}$$

η : security parameter = key length

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η : security parameter $=$ key length

$\rightarrow$ Not enough ! (why ?)

Computational secrecy

Computational secrecy of s is defined through the following game :

- Two values n_0 and n_1 are randomly generated instead of s ;
- The adversary interacts with the protocol where s is replaced by n_b, $b \in \{0, 1\}$;
- We give the pair (n_0, n_1) to the adversary ;
- The adversary gives b',

The data s is secret if $\Pr[b = b'] - \frac{1}{2}$ is a negligible function.

A typical cryptographic proof

1. Assume that some algorithmic problem P is difficult (E.g. RSA or integer factorization or Discrete Log or CDH, DDH, ...)
2. Suppose that a (polynomial probabilistic) adversary $\mathcal{A}$ breaks the protocol security with non negligible probability

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4. Conclude that the protocol is secure provided P is difficult.

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Formal and Cryptographic approaches

	Formal approach	Cryptographic approach
Messages	terms	bitstrings
Encryption	idealized	algorithm
Adversary	idealized	any polynomial algorithm
Secrecy property	reachability-based property	indistinguishability
Guarantees	unclear	strong
Protocol	may be complex	usually simpler

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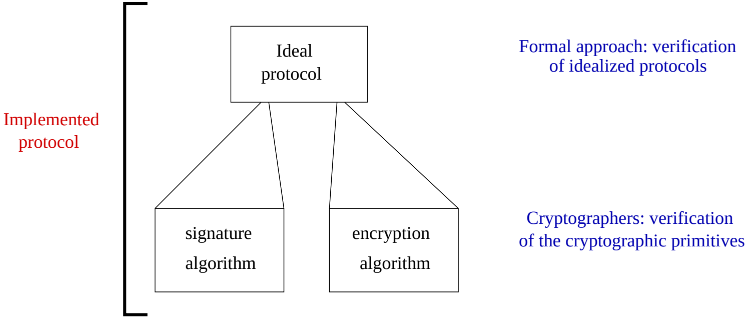
Protocol	may be complex	usually simpler
Proof	automatic	by hand, tedious and error-prone

Link between the two approaches ?

Composition of the two approaches

Automatic cryptographically sound proofs



Passive Case

A first result : seminal result from M. Abadi and Ph. Rogaway

J. of Cryptology, 2002

How to symbolically abstract computational indistinguishability of distributions ?

Setting

- Messages are represented by terms

$$
\begin{array}{lllll}
T & ::= & & & \text{terms} \\
 & | & x & & \text{variable} \\
 & | & a & & \text{name/nonce} \\
 & | & f(T_1, \ldots, T_k) & & \text{application of symbol } f \in \mathcal{F}
\end{array}
$$

In the initial result of Abadi and Rogaway, $\mathcal{F} = \{\mathsf{enc}, \langle\ ,\rangle\}$

- Each functional symbol has a concrete implementation
$\Rightarrow$ a sequence of messages

$$n,\ \mathsf{enc}(n, k),\ \mathsf{enc}(\langle n\ , n\rangle, k)$$

generates a distribution : uniform distribution for nonces and application of the functions (symmetric encryption and pairing).

Distinguishing distributions

The two distributions $[\![\psi]\!]$ and $[\![\psi']\!]$ are indistinguishable, $[\![\psi]\!] \approx [\![\psi']\!]$, if

$$\mathbb{P}\left[\widehat{\psi} \leftarrow [\![\psi]\!]; \mathcal{A}(\eta, \widehat{\psi}) = 1\right] - \mathbb{P}\left[\widehat{\psi} \leftarrow [\![\psi']\!]; A(\eta, \widehat{\psi}) = 1\right]$$

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$$\phi_1 = n_0, n_1, \mathrm{enc}(n_0, k) \qquad \phi_2 = n_0, n_1, \mathrm{enc}(n_1, k)$$

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$$\phi_3 = n_0, n_1, \mathsf{enc}(n_0, k), k \qquad \phi_4 = n_0, n_1, \mathsf{enc}(n_1, k), k$$

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$$\phi_3 = n_0, n_1, \mathrm{enc}(n_0, k), k \not\approx \phi_4 = n_0, n_1, \mathrm{enc}(n_1, k), k$$

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Patterns : Definition of what is visible to an intruder

Given a sequence $S = M_1, M_2, \ldots, M_k$, we define

$$\mathsf{Pat}(S) = \{\mathsf{Pat}^S(M_1), \mathsf{Pat}^S(M_2), \ldots, \mathsf{Pat}^S(M_k)\} \text{ with}$$

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\mathrm{Pat}^S(a) &= \begin{cases} a & \text{if } S \vdash a \\ \square & \text{otherwise} \end{cases} \\
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$$\mathrm{Pat}^S(\{M\}_k) = \begin{cases} \{\mathrm{Pat}^S(M)\}_k & \text{if } S \vdash k \\ \square & \text{otherwise} \end{cases}$$

Reminder : deduction system

Standard "Dolev Yao" deduction system, seen Part I of this course.

$$\frac{T \vdash u \quad T \vdash v}{T \vdash \langle u\,,v\rangle} \qquad \frac{T \vdash u \quad T \vdash v}{T \vdash \mathrm{enc}(u,v)}$$

$$\frac{}{T \vdash u}\; u \in T \qquad \frac{T \vdash \langle u\,,v\rangle}{T \vdash u} \qquad \frac{T \vdash \langle u\,,v\rangle}{T \vdash v}$$

$$\frac{T \vdash \mathrm{enc}(u,v) \quad T \vdash v}{T \vdash u}$$

Examples

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\end{array}$$

Definition

Two patterns are equivalent, denoted by $\equiv$ if they are equal up-to bijective renaming.

Soundness of indistinguishability

Theorem (Abadi-Rogaway)

Equivalence of patterns implies computational indistinguishability

$$\mathrm{Pat}(S_1) \equiv \mathrm{Pat}(S_2) \quad \Rightarrow \quad [\![S_1]\!] \approx [\![S_2]\!]$$

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  - message length-concealing
    $\mathrm{Pat}(\mathrm{enc}(n,k)) = \square = \mathrm{Pat}(\mathrm{enc}(\langle n\ , n, n, n\rangle, k))$

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  - which key-concealing

$$\mathrm{Pat}(\mathrm{enc}(n, k), \mathrm{enc}(n', k)) = \Box = \mathrm{Pat}(\mathrm{Pat}(\mathrm{enc}(n, k), \mathrm{enc}(n', k')))$$

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    $\mathrm{Pat}(\mathrm{enc}(n, k), \mathrm{enc}(n', k)) = \Box = \mathrm{Pat}(\mathrm{Pat}(\mathrm{enc}(n, k), \mathrm{enc}(n', k')))$
- S_1, S_2 contain no key cycles
  Examples : $\mathrm{enc}(k, k)$ or $\mathrm{enc}(k_1, k_2), \mathrm{enc}(k_2, k_1)$

Proof of soundness of indistinguishability

Lemma (Main lemma)

$$[\![S]\!] \approx [\![\mathrm{Pat}(S)]\!]$$

We can then easily deduce the main theorem.

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We can then easily deduce the main theorem.

Indeed, assume $\mathrm{Pat}(S_1) \equiv \mathrm{Pat}(S_2)$.

1. By the lemma, we have $[\![S_1]\!] \approx [\![\mathrm{Pat}(S_1)]\!]$ and $[\![S_2]\!] \approx [\![\mathrm{Pat}(S_2)]\!]$.
2. Then $\mathrm{Pat}(S_1) \equiv \mathrm{Pat}(S_2)$ implies $[\![\mathrm{Pat}(S_1)]\!] \approx [\![\mathrm{Pat}(S_2)]\!]$.

Proof of the main lemma $[\![S]\!] \approx [\![\mathrm{Pat}(S)]\!]$

Main steps :

Renaming Let $K_1, \ldots, k_n$ be the hidden (non deducible) keys of S and $J_1, \ldots, J_l$ be the visible (deducible) keys of S.

Since S contain no key cycles,
we may assume that K_j does not encrypt k_i whenever $i < j$.

Proof of the main lemma $[\![S]\!] \approx [\![\mathrm{Pat}(S)]\!]$

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Intermediate patterns We define a sequence $\mathrm{Pat}_o(S), \ldots, \ldots \mathrm{Pat}_n(S)$ such that $\mathrm{Pat}_o(S) = \mathrm{Pat}(S)$ and $\mathrm{Pat}_n(S) = S$

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Hybrid argument If $[\![S]\!] \not\approx [\![\mathrm{Pat}(S)]\!]$ then there exists i such that
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Security of encryption $[\![\mathrm{Pat}_i(S)]\!] \not\approx [\![\mathrm{Pat}_{i+1}(S)]\!]$ contradicts the security of encryption.

Intermediate patterns

Let $K_1, \ldots, k_n$ be the hidden (non deducible) keys of S and $J_1, \ldots, J_l$ be the visible (deducible) keys of S such that K_j does not encrypt k_i whenever $i < j$.

Definition (Intermediate patterns)

$$\mathrm{Pat}_i(S) = \mathrm{Pat}_{S \cup \{K_1, \ldots, K_i\}}(S)$$

$\mathrm{Pat}_i(S)$: what is visible to an intruder, with the extra knowledge $K_1, \ldots, K_i$.

Example of intermediate patterns

Visible keys : J_1, J_2
Hidden keys : K_1, K_2

$$S = \mathrm{Pat}_2(S) = \quad \mathrm{enc}(\langle \mathrm{enc}(J_1, K_2)\ , J_1\rangle, K_1), \quad \mathrm{enc}(J_1, J_2), \quad J_2$$

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$$\mathrm{Pat}_1(S) = \quad \mathrm{enc}(\langle \Box\ , J_1\rangle, K_1), \quad \mathrm{enc}(J_1, J_2), \quad J_2$$

Example of intermediate patterns

Visible keys : J_1, J_2
Hidden keys : K_1, K_2

$$\begin{array}{rcccc}
S = \mathsf{Pat}_2(S) = & \mathsf{enc}(\langle \mathsf{enc}(J_1, K_2)\ , J_1\rangle, K_1), & \mathsf{enc}(J_1, J_2), & J_2 \\
\mathsf{Pat}_1(S) = & \mathsf{enc}(\langle \square\ , J_1\rangle, K_1), & \mathsf{enc}(J_1, J_2), & J_2 \\
\mathsf{Pat}(S) = \mathsf{Pat}_0(S) = & \square, & \mathsf{enc}(J_1, J_2), & J_2
\end{array}$$

Hybrid argument

$$\mathrm{Pat}(S) = \mathrm{Pat}_0(S) \qquad \mathrm{Pat}_1(S) \quad \cdots \quad \mathrm{Pat}_{n-1}(S) \qquad \mathrm{Pat}_n(S) = S$$

Assume by contradiction that $[\![\mathrm{Pat}(S)]\!] \not\approx [\![S]\!]$.

Then, since the number n of intermediate steps is fixed, there must exist i such that

$$[\![\mathrm{Pat}_i(S)]\!] \not\approx [\![\mathrm{Pat}_{i+1}(S)]\!]$$

Exercises

Abstracting indistinguishability in various contexts

1. How to adapt the definition of patterns for encryption schemes that are not which key-concealing ?
2. How to adapt the definition of patterns for encryption schemes that are not message length-concealing ?
3. How to adapt the definition of patterns for asymmetric encryption schemes ?

Active Case

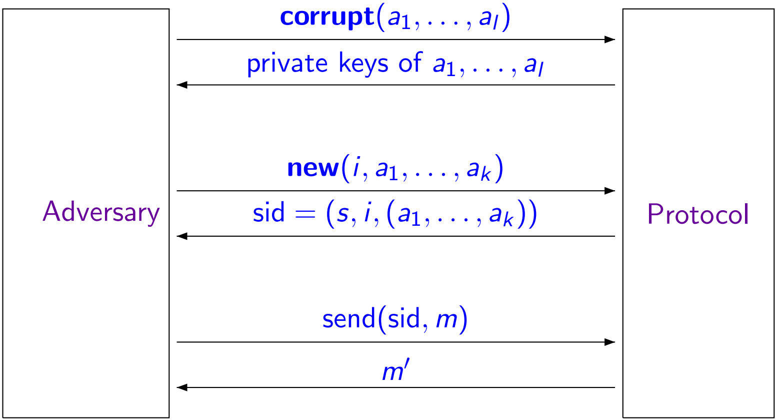
Can we extend the work to the active case ?

that is,

Are standard Dolev-Yao models sound w.r.t. to computational ones ?

A common setting

Same setting in formal and cryptographic models



Formal Intruder Deduction Rules

$$\frac{S\vdash m_1 \quad S\vdash m_2}{S\vdash \langle m_1, m_2\rangle} \qquad \frac{S\vdash \langle m_1, m_2\rangle}{S\vdash m_i}\, i \in \{1,2\}$$

$$\frac{S\vdash \mathrm{ek}(b) \quad S\vdash m}{S\vdash \{m\}_{\mathrm{ek}(b)}^{\mathrm{adv}(i)}}\, i \in \mathbb{N} \qquad \frac{S\vdash \{m\}_{\mathrm{ek}(b)}^{l} \quad S\vdash \mathrm{dk}(b)}{S\vdash m}$$

$$\frac{S\vdash \mathrm{sk}(b) \quad S\vdash m}{S\vdash [m]_{\mathrm{sk}(b)}^{\mathrm{adv}(i)}}\, i \in \mathbb{N} \qquad \frac{S\vdash [m]_{\mathrm{sk}(b)}^{l}}{S\vdash m}$$

Result : Soundness of trace properties

Theorem (extension of [Micciancio Warinschi TCC'04])

Every concrete trace is the image of a valid formal trace, except with negligible probability.

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Corollary :

Let Π *be protocol,* P^s *an arbitrary predicate on formal traces and* P^c *its corresponding predicate on concrete traces.*

Then $\Pi \models^s P^s$ *implies* $\Pi \models^c P^c$.

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Let Π be protocol, P^s an arbitrary predicate on formal traces and P^c its corresponding predicate on concrete traces.

Then $\Pi \models^s P^s$ implies $\Pi \models^c P^c$.

Applications : authentication, secrecy, ...

Hypotheses on the Implementation

- encryption : IND-CCA2
  $\longrightarrow$ the adversary cannot distinguish between $\{n_0\}_k$ and $\{n_1\}_k$ even if he has access to encryption and decryption oracles.
- signature : randomized and existentially unforgeable under chosen-message attack *i.e.* one can not produce a valid pair (m, σ)
- parsing :
  - each bit-string has a label which indicates his type (identity, nonce, key, signature, ...)
  - one can retrieve the (public) encryption key from an encrypted message.
  - one can retrieve the signed message from the signature

skip the proof

Proof idea

Proof technique : Reducing the protocol security to the robustness of the primitives (which itself reduces to hardness of algorithmic problem like integer factorization).

$$\mathcal{A} \text{ breaks } \mathcal{P} \Rightarrow \mathcal{A}' \text{ breaks } \{\ \} \text{ or sign}$$

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Example : If a computational (concrete) adversary $\mathcal{A}$ is able to compute $\{n_a\}_{K_a}$ out of $\{< A, n_a >\}_{K_a}$,
Then we can build an adversary $\mathcal{A}'$ that breaks the encryption $\{\ \}_{K_a}$.

Proof idea

Key result : every concrete trace is the image of a valid formal trace, except with negligible probability.

$$
\begin{array}{ccccccc}
 & \mathit{init}(1,a,b) & \rightarrow & \{a,n_a\}_{K_b} & & \{n_a\}_{K_b}\text{non valid !} \\
 & \uparrow & & \downarrow & & \uparrow \\
\mathcal{A}: & \mathit{init}(1,a,b) & & m_1 & \rightarrow & \mathit{send}(m_2)
\end{array}
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Using the adversary $\mathcal{A}$, we build an adversary $\mathcal{A}'$ that breaks encryption.

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\mathcal{A}' : \quad (\langle a, n_a^0\rangle, \langle a, n_a^1\rangle) \rightarrow \begin{array}{c}\text{encryption}\\ \text{oracle}\end{array} \rightarrow \{a, n_a^\alpha\}_{K_b}
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$$
\rightarrow \mathcal{A} \rightarrow \{n_a^\alpha\}_{K_b} \rightarrow \text{decryption oracle} \rightarrow n_a^\alpha \rightarrow \alpha
$$

Trace properties vs observational equivalence

Fact 1 : Computational security properties are often stated as indistinguishability games rather than trace properties.
Example : secrecy, ideal functionalities, ...

Trace properties vs observational equivalence

Fact 1 : Computational security properties are often stated as indistinguishability games rather than trace properties.
Example : secrecy, ideal functionalities, ...

Fact 2 : Some security properties cannot be expressed as trace properties.
Example : Privacy properties of e-voting protocols

$$P(A, a) \| P(B, b) \sim_o P(A, b) \| P(B, a)$$

Correspondence of computational secrecy

Theorem

Symbolic secrecy implies computational secrecy.

- For protocols with only public key encryption, signatures and nonces
- Provided the public key encryption and the signature algorithms verify strong existing cryptographic properties (IND-CCA2, existentially unforgeable),

The previous result does not work in general

Example

$A \rightarrow B : h(s)$

s is inaccessible but not indistinguishable to an attacker :

$h(n_b), n_0, n_1 \rightarrow b$

The previous result does not work in general

Example

$A \rightarrow B : h(s)$

s is inaccessible but not indistinguishable to an attacker :
$h(n_b), n_0, n_1 \rightarrow b$

Results :

1. Design of a new formal secrecy property
2. Proof of its soundness and its faithfulness w.r.t. indistinguishability in our new setting :
   - pairing
   - asymmetric encryption
   - hashes (random oracle model)
3. NP-completeness of the secrecy property

Patterns : extension to hashes

Given $S = \{M_1, M_2, \ldots, M_k\}$ and some addintional knowledge T, we define

$\mathrm{Pat}_T(S) = \{\mathrm{Pat}^{S\cup\{T\}}(M_1), \mathrm{Pat}^{S\cup\{T\}}(M_2), \ldots, \mathrm{Pat}^{S\cup\{T\}}(M_k)\}$ with

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\mathrm{Pat}^S(h(M)) & = & \left\{\begin{array}{ll} h(\mathrm{Pat}^S(M)) & \text{if } S \vdash M \\ \square & \text{otherwise} \end{array}\right.
\end{array}$$

Examples

- $\phi_1 = \{h(\langle n_b, n' \rangle)\}$. Then $\mathsf{Pat}_{n_b}(\phi_1) = \{\Box\}$
  $\longrightarrow$ n_b is intuitively hidden by n'.

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  $\rightarrow$ n_b is intuitively hidden by n'.

- $\phi_2 = \{h(\langle n_b, \{n'\}^r_{ek(a)} \rangle), n'\}$. $\mathrm{Pat}_{n_b}(\phi_2) = \{\Box, n'\}$.
  $\rightarrow$ The encryption of n' does hide n_b.

Pattern-based secrecy definition

Π protocol
$X^j_{A_i}$ nonce variable occurring in some role A_i.
$\mathcal{M}$ set of sent messages
s session number

Definition

$X^j_{A_i}$ is secret in Π, written $\Pi \models^f \mathrm{Invisible}(i,j)$, if :

$$n^{a_i,j,s} \text{ does not occur in } \mathrm{Pat}_{n^{a_i,j,s}}(\mathcal{M}) \qquad \forall \mathcal{M} \in \mathrm{Exec}(\Pi)\ \forall s$$

Soundness and decidability of the secrecy property

Theorem

$$\Pi \models^f \mathsf{Invisible}^f(i,j) \quad \textit{iff} \quad \Pi \models^c \mathsf{Indist}(i,j)$$

Remark : Our formal secrecy definition is both sufficient and necessary for indistinguishability in the computational world.

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Theorem

Deciding $\Pi \models^f \mathsf{Invisible}^f(i,j)$ *is NP-complete for a finite number of sessions.*

General computational indistinguishability

Observational equivalence is a sound abstraction of computational indistinguishability.

$$P \sim_o Q \Rightarrow [\![P]\!] \approx [\![Q]\!]$$

- For simple processes
  (A fragment of applied pi-calculus that captures most security protocols)
- For symmetric encryption implemented using IND-CC2 schemes

General computational indistinguishability

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- For symmetric encryption implemented using IND-CC2 schemes

Limitation : No dishonest keys !

(currently solved by Guillaume Scerri)

Related Work

Abadi-Rogaway, followed by several extensions : passive case.

Backes-Pfitzmann

- very general results : symmetric and asymmetric encryption, pairing, signatures, MACs.
- less abstract model than classical Dolev-Yao models,

Laud : specialized decision procedure for symmetric encryption

Datta-Derek-Mitchell-Shmatikov-Turuani : symbolic deduction system for proofs in the concrete model (asymmetric encryption, no automatic procedure)

Blanchet : direct automation of the (game-based) cryptographic proofs in the concrete model → tool CryptoVerif

Conclusion

Formal methods form a powerful approach for analyzing security protocols

- Makes use of classical techniques in formal methods : term algebra, equational theories, clauses and resolution techniques, tree automata, etc.
  $\Rightarrow$ Many decision procedures
- Several automatic tools
  - For successfully detecting attacks on protocols (e.g. Casper, Avispa)
  - For proving security for an arbitrary number of sessions (e.g. ProVerif)
- Provides cryptographic guarantees under classical assumptions on the implementation of the primitives

Some current directions of research

- Enriching the symbolic model
  - Considering more equational theories (e.g. theories for e-voting protocols)
  - Adding more complex structures for data (list, XML, ...)
  - Considering recursive protocols (e.g. group protocol) where the number of message exchanges in a session is not fixed
  - Proving more complex security properties like equivalence-based properties (e.g. for anonymity or e-voting protocols)
- With cryptographic guarantees
  - Combining formal and cryptographic models for more complex primitives and security properties.
  - How far can we go ?
  - Is it possible to consider weaker cryptographic primitives ?