Abstract Cryptography

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FOSAD 2009, Bertinoro, Aug./Sept. 2009.

Abstract Cryptography

"I can only understand simple things."

JAMES MASSEY

Ueli Maurer

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Abstraction

Abstraction: eliminate irrelevant details from consideration

Examples: group, field, vector space, relation, graph, ...

Goals of abstraction:

- simpler definitions
- generality of results
- simpler proofs
- elegance
- didactic suitability

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- simpler definitions
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- elegance
- didactic suitability
- understanding

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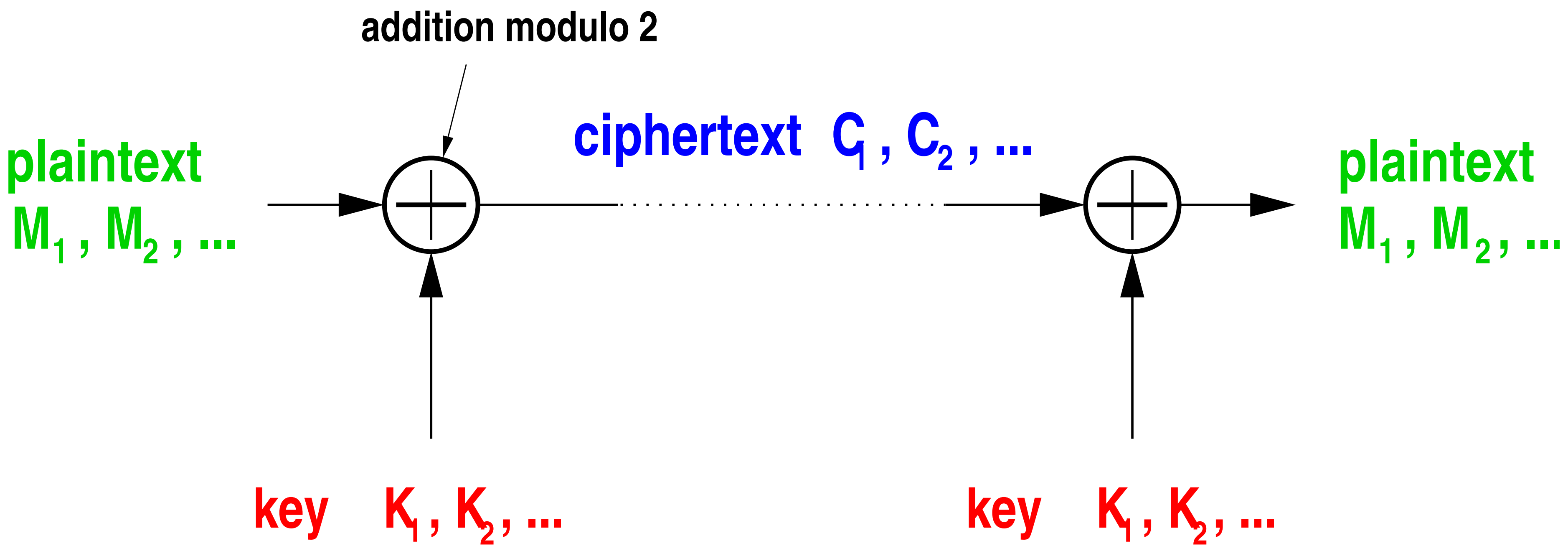
Goals of abstraction:

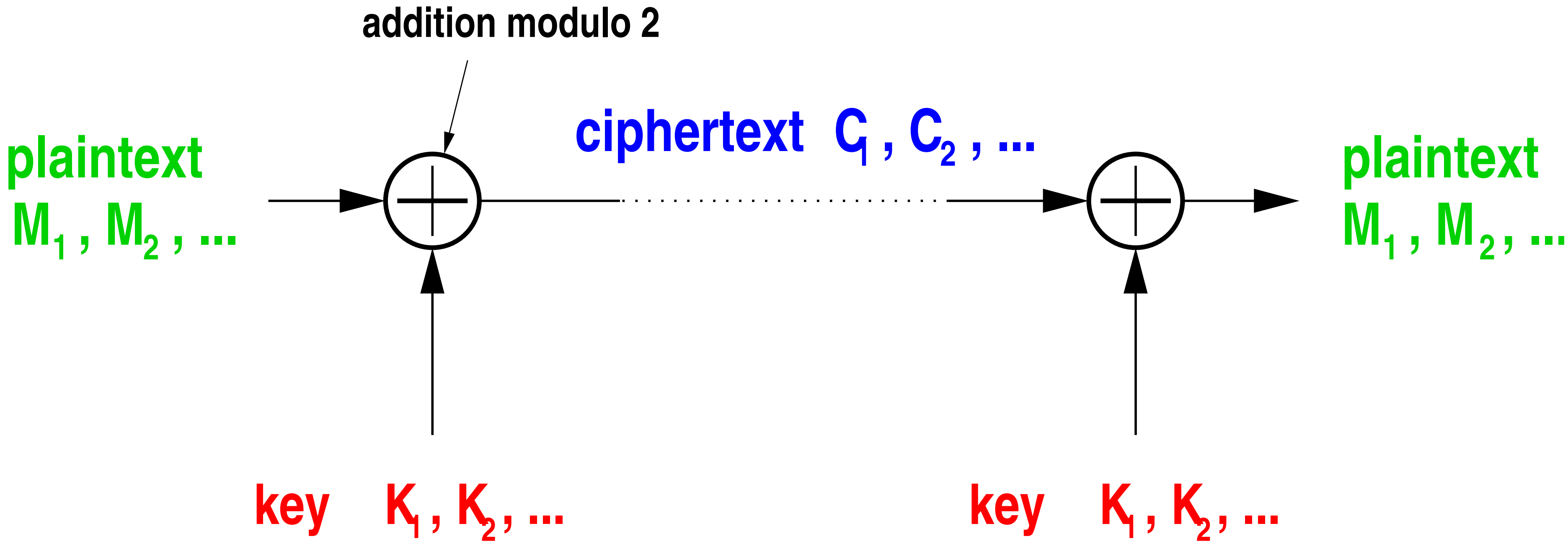
Goals of this talk:

- Introduce layers of abstraction in cryptography.
- Examples of abstract definitions and proofs.
- Announce a new security framework "abstract cryptography" (with Renato Renner).

Motivating example: One-time pad

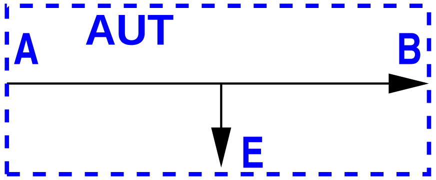


Motivating example: One-time pad

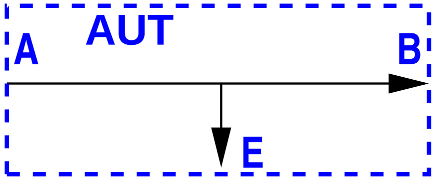


Perfect secrecy (Shannon): C and M statist. independent.

One-time pad in terms of systems

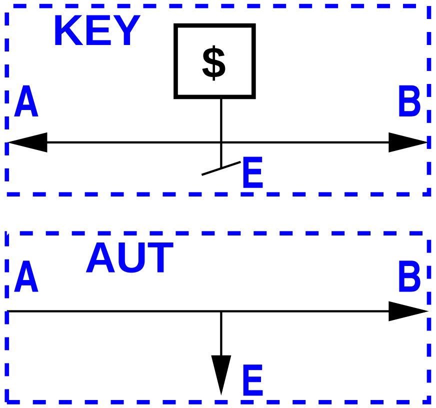


One-time pad in terms of systems



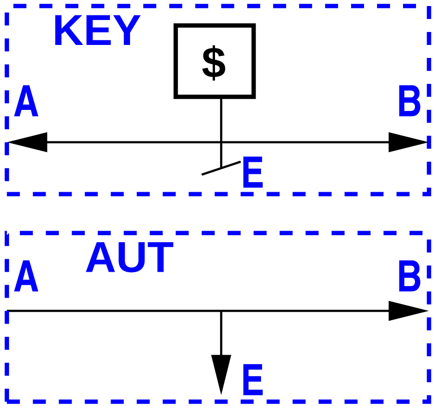
AUT

One-time pad in terms of systems



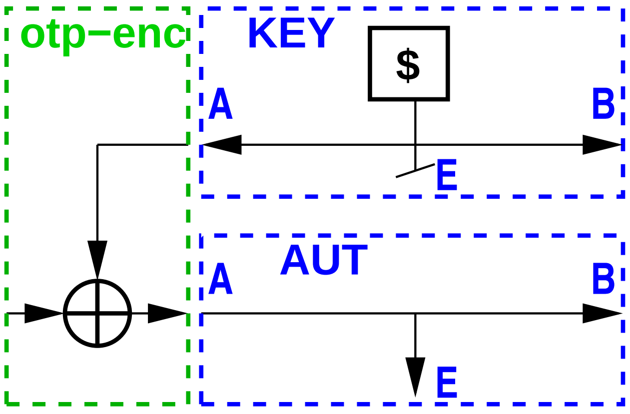
KEY||AUT

One-time pad in terms of systems



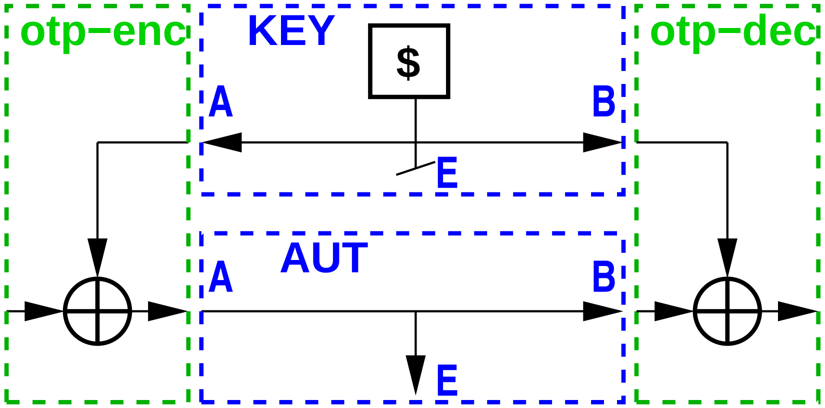
KEY || AUT

One-time pad in terms of systems



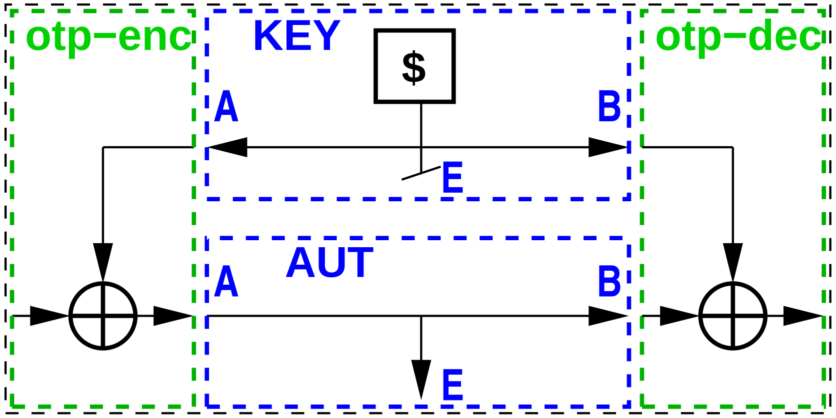
$\textbf{otp-enc}^{A}\,(\textbf{KEY}\,\|\,\textbf{AUT})$

One-time pad in terms of systems



otp-dec^B **otp-enc**^{A} (**KEY**||**AUT**)

One-time pad in terms of systems



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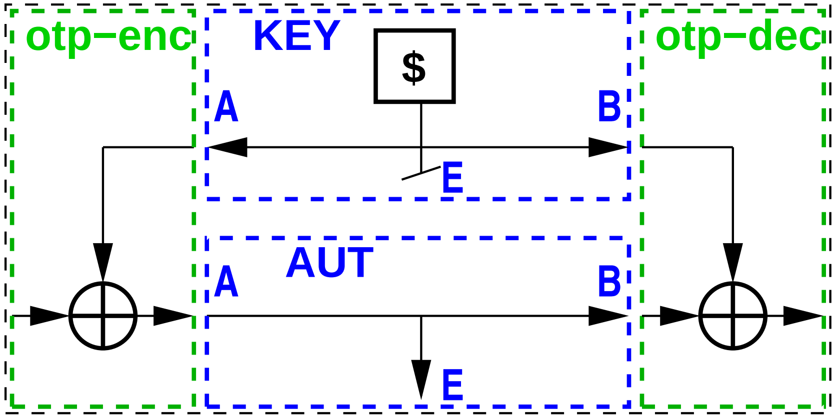
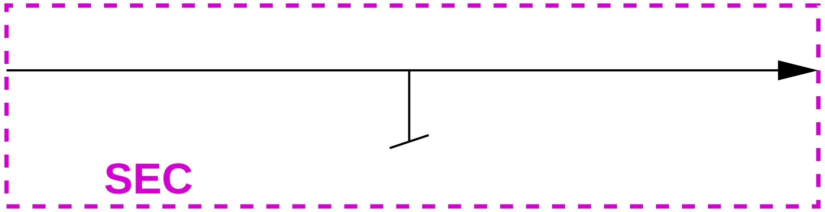
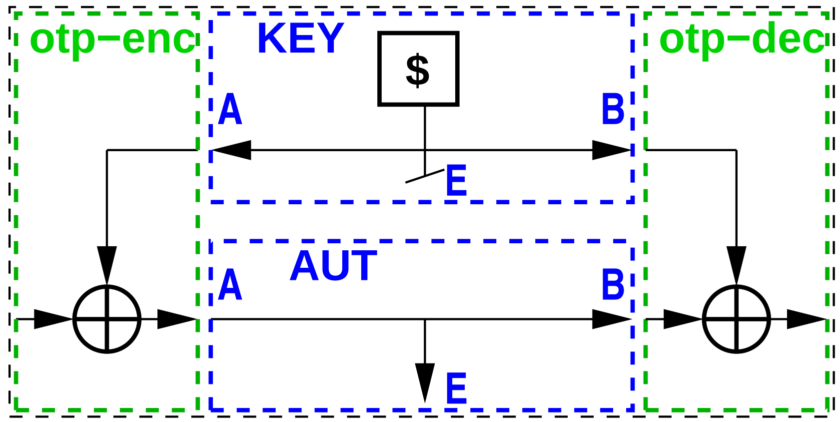
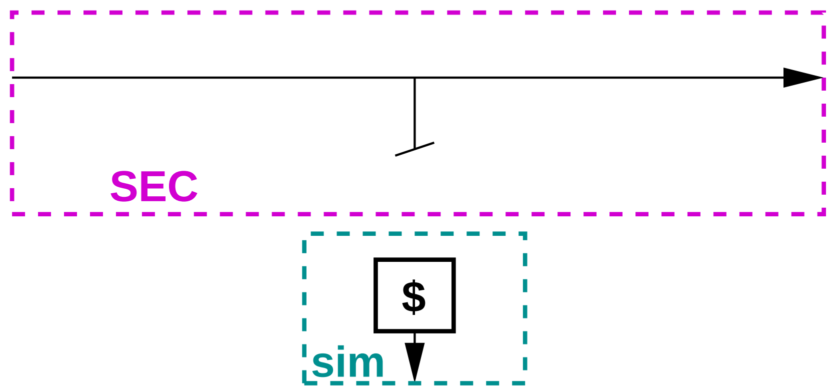




$\text{otp-dec}^{B}\ \text{otp-enc}^{A}\ (\text{KEY} \| \text{AUT})$ SEC

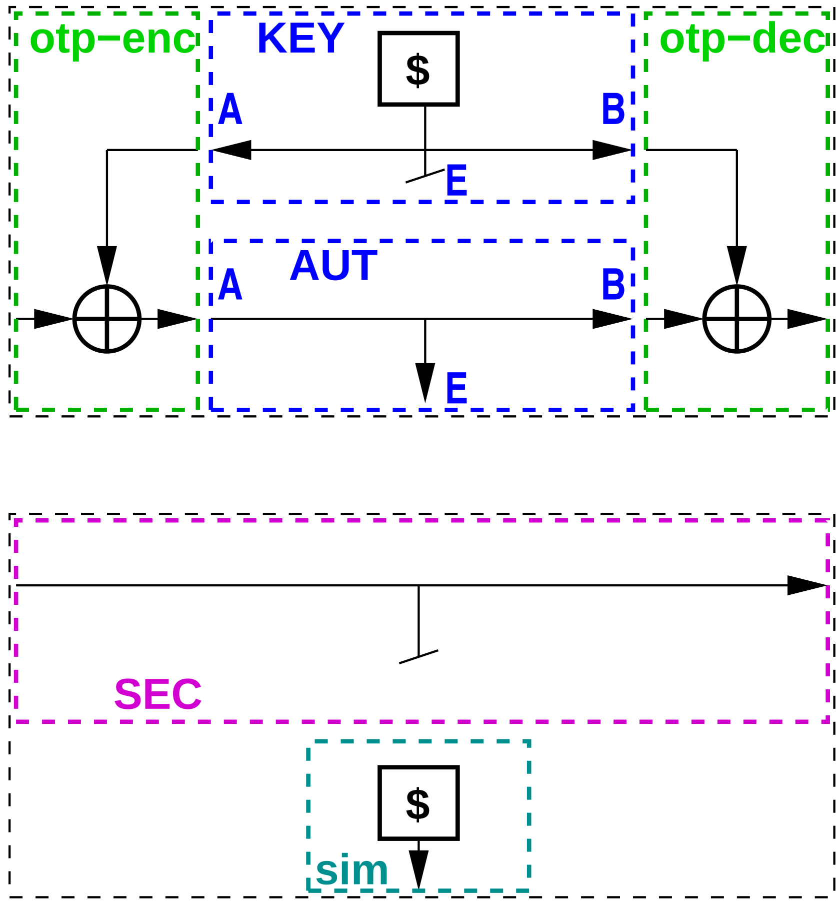
One-time pad in terms of systems





otp-dec^{B} **otp-enc**^{A} (**KEY**||**AUT**)   **sim**^{E} **SEC**

One-time pad in terms of systems



$$\text{otp-dec}^{B}\ \text{otp-enc}^{A}\ (\mathbf{KEY} \| \mathbf{AUT}) \equiv \text{sim}^{E}\ \mathbf{SEC}$$

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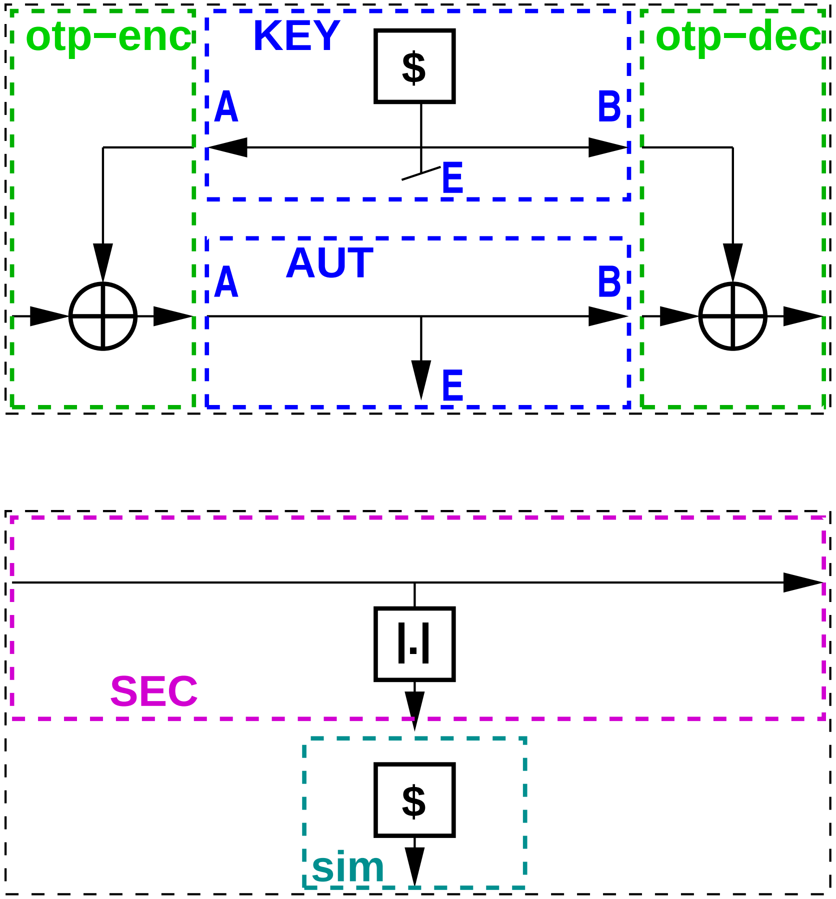
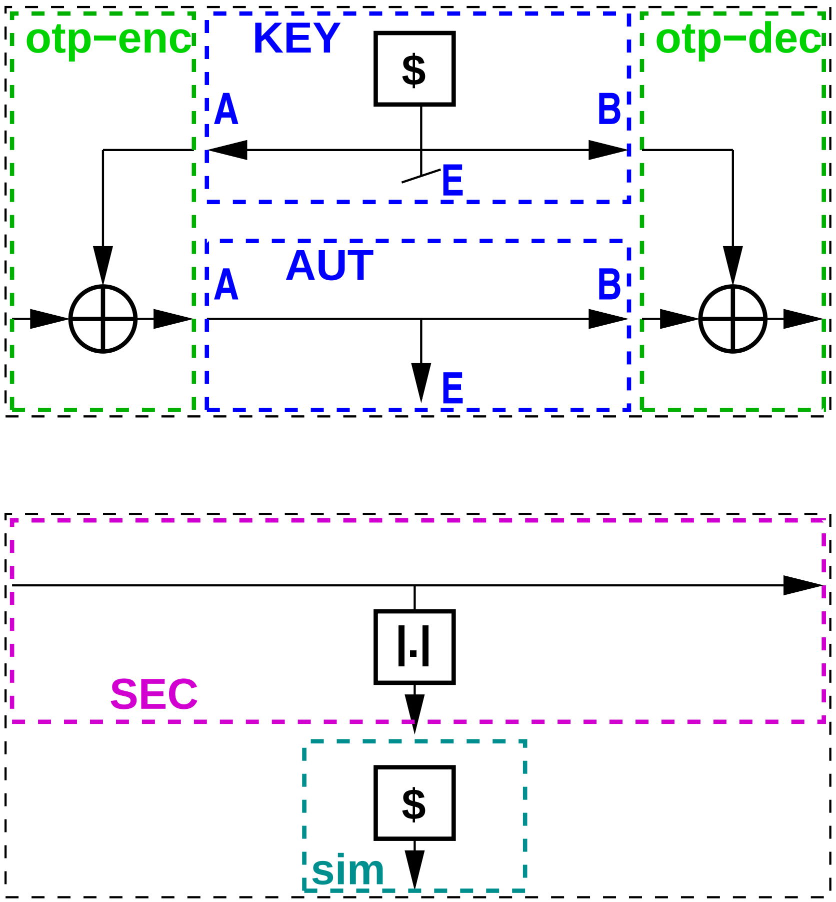


otp-dec^B otp-enc^A (KEY||AUT) $\equiv$ sim^E SEC

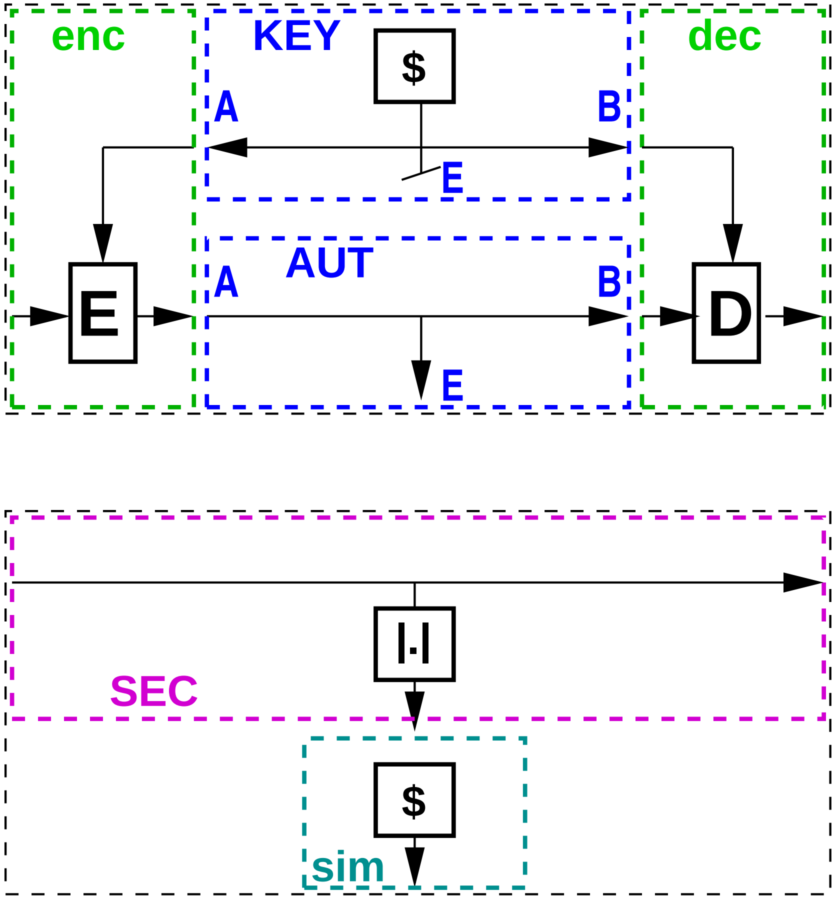
One-time pad in terms of systems



$\text{otp-dec}^B\ \text{otp-enc}^A\ (\mathbf{KEY}||\mathbf{AUT}) \equiv \text{sim}^E\ \mathbf{SEC}$

written as a reduction: $(\mathbf{KEY}||\mathbf{AUT}) \xrightarrow{\text{otp}} \mathbf{SEC}$

Symmetric encryption



$\text{dec}^{B}\,\text{enc}^{A}\,(\text{KEY}||\text{AUT}) \approx \text{sim}^{E}\,\text{SEC}$

written as a reduction: $(\text{KEY}||\text{AUT}) \xrightarrow{\text{sym}} \text{SEC}$

Constructive cryptography

Reduction concept:

$$\textbf{real system R} \xrightarrow{\textbf{protocol } \pi} \textbf{ideal system S}$$

Resource **S** is constructed from (reduced to) **R** by protocol π

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Example: Alice-Bob-Eve setting $\pi = (\pi_1, \pi_2)$

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Example: Alice-Bob-Eve setting $\pi = (\pi_1, \pi_2)$

$$\mathbf{R} \xrightarrow{\pi} \mathbf{S} \quad :\Leftrightarrow \quad \exists \sigma : \ \pi_1^{A} \pi_2^{B} \mathbf{R} \approx \sigma^{E} \mathbf{S}$$

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$$\mathbf{R} \xrightarrow{\pi} \mathbf{S} \quad :\Leftrightarrow \quad \exists \sigma : \; \pi_1^A \, \pi_2^B \, \mathbf{R} \approx \sigma^E \, \mathbf{S}$$

and

$$\pi_1^A \, \pi_2^B \, \bot^E \, \mathbf{R} \approx \bot^E \, \mathbf{S}$$

Constructive cryptography

Reduction concept:

$$\textbf{real system R} \xrightarrow{\text{protocol } \pi} \textbf{ideal system S}$$

Re … π

Composability of a reduction:

$$\mathbf{R} \xrightarrow{\alpha} \mathbf{S} \;\wedge\; \mathbf{S} \xrightarrow{\beta} \mathbf{T} \;\Rightarrow\; \mathbf{R} \xrightarrow{\alpha\circ\beta} \mathbf{T}$$

Example: Alice-Bob-Eve setting $\pi = (\pi_1, \pi_2)$

$$\mathbf{R} \xrightarrow{\pi} \mathbf{S} \;:\Leftrightarrow\; \exists \sigma : \; \pi_1^{\mathsf{A}} \pi_2^{\mathsf{B}} \mathbf{R} \approx \sigma^{\mathsf{E}} \mathbf{S}$$

and

$$\pi_1^{\mathsf{A}} \pi_2^{\mathsf{B}} \perp^{\mathsf{E}} \mathbf{R} \approx \perp^{\mathsf{E}} \mathbf{S}$$

Levels of abstraction in cryptography

#	possible name	concepts treated at this level
1.	Reductions	def. of (universal) composability
2.	Abstract resources	isomorphism
3.	Abstract systems	distinguisher, hybrid argument, secure reduction, compos. proof
4.	Discrete systems	games, equivalence, indistinguishability proofs
5.	System implem.	complexity, efficiency notion
6.	Physical models	timing, power, side-channels

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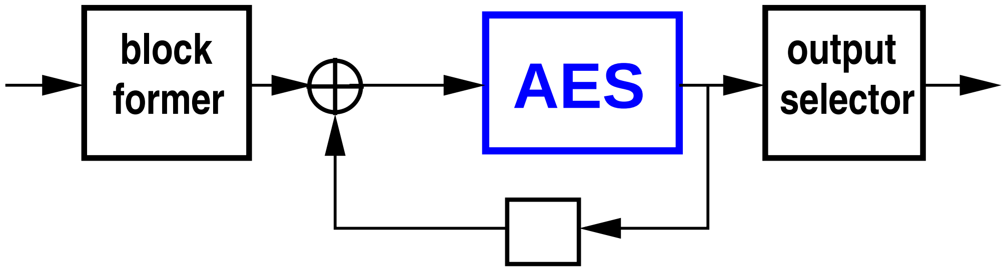
Example: CBC-MAC



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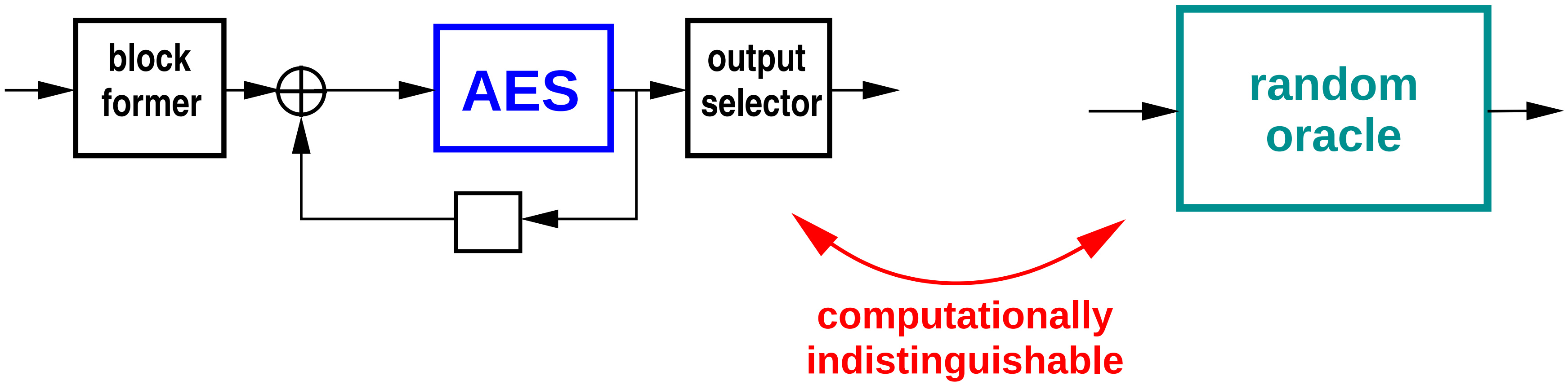

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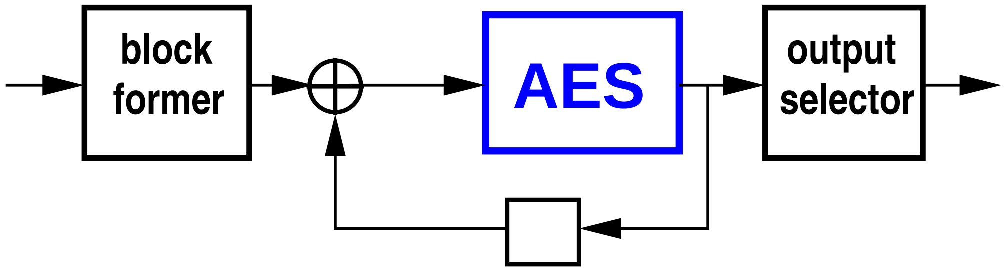

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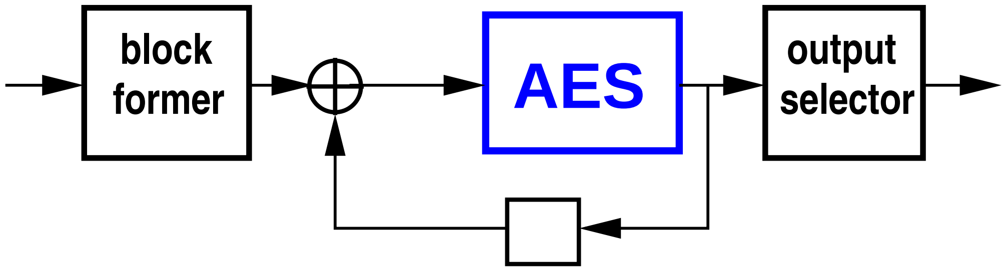

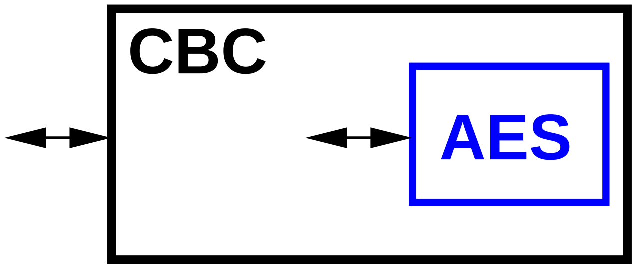

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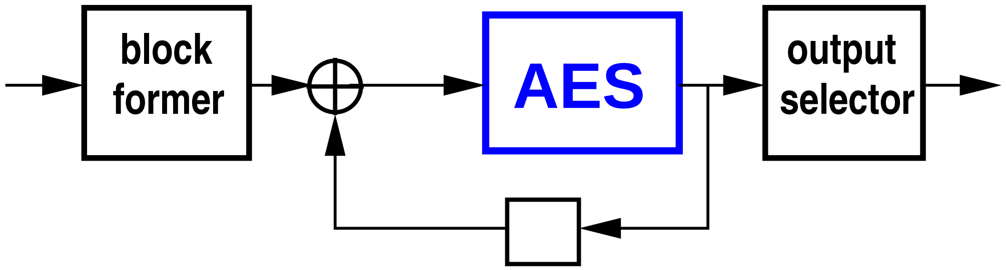
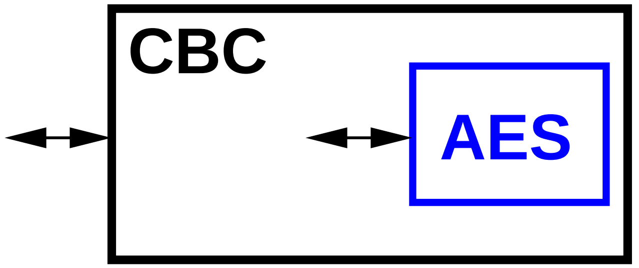
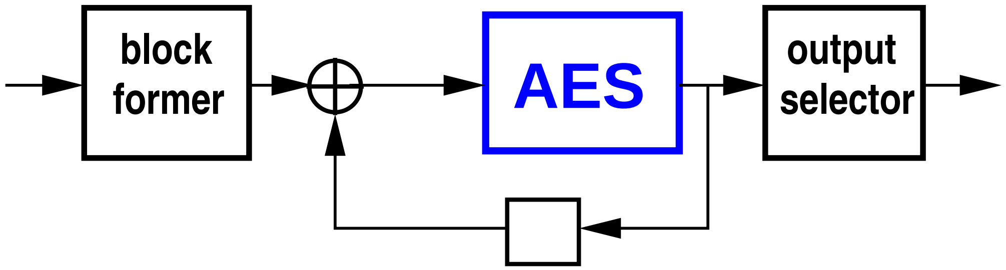
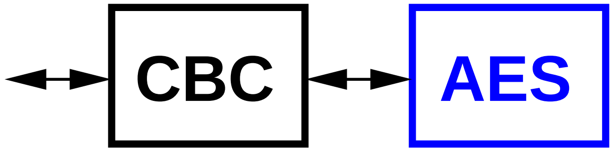




Notation: **CBC(AES)**

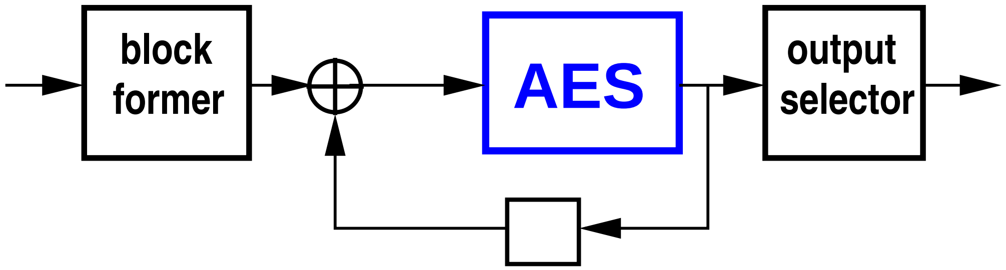
Example: CBC-MAC

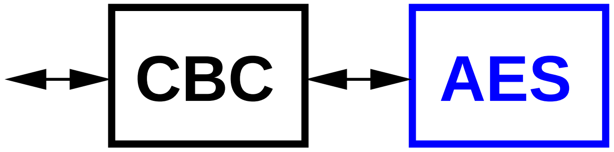




Notation: CBC ○ AES

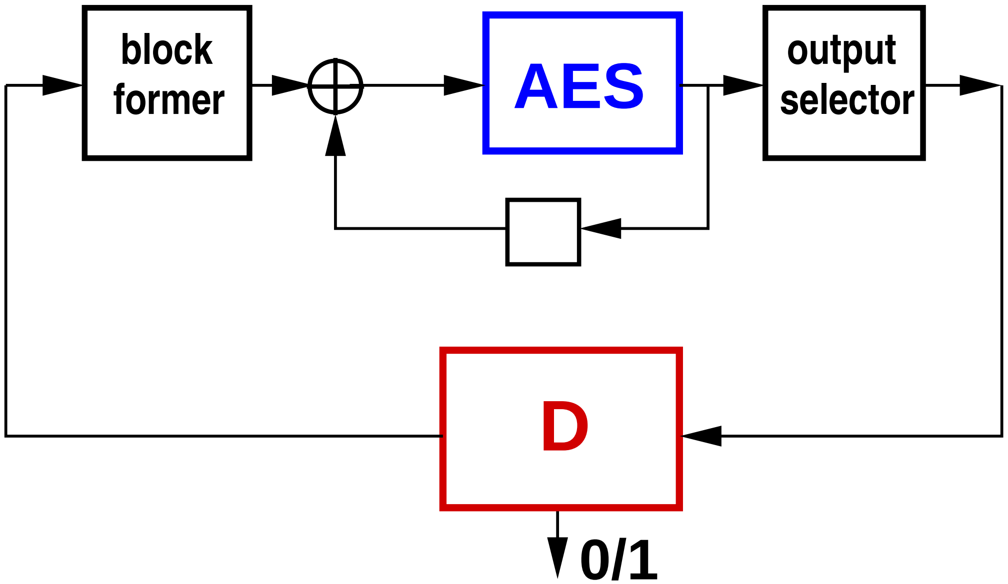
Example: CBC-MAC

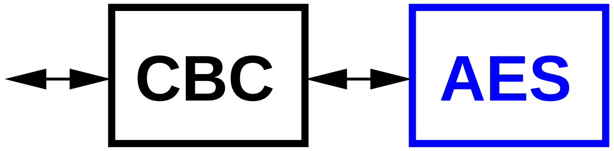




Notation: **CBC AES**

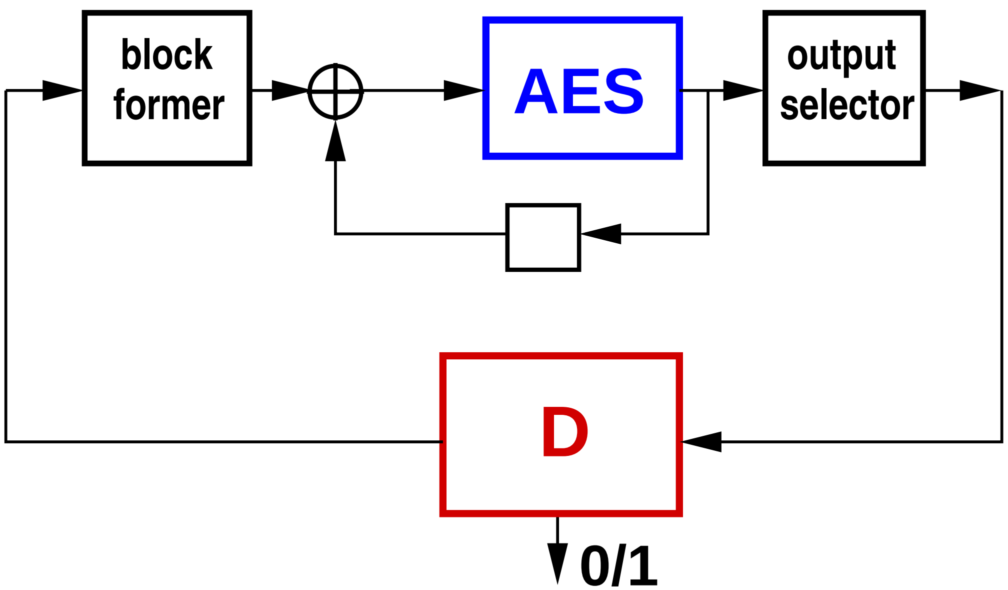
Example: CBC-MAC

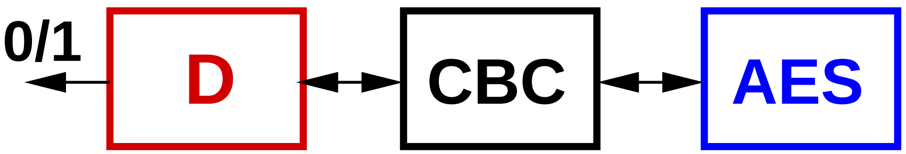




CBC AES

Example: CBC-MAC





D CBC AES

Security proof for CBC-MAC

[3]

CBC AES ≈ RO

Security proof for CBC-MAC

D CBC AES $\approx$ D RO

Security proof for CBC-MAC

D CBC AES $\approx$ D RO

To show: $\Delta^{D}(\text{CBC AES}, \text{RO}) \approx 0$

Security proof for CBC-MAC [3]

$$\mathbf{D}\ \mathbf{CBC}\ \mathbf{AES} \approx \mathbf{D}\ \mathbf{RO}$$

To show: $\Delta^{\mathbf{D}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

Note: $\Delta^{\mathbf{D}}(\mathbf{S}, \mathbf{T}) = |\mathbf{DS}, \mathbf{DT}|$ (stat. distance of binary r.v.)

Security proof for CBC-MAC

D CBC AES $\approx$ D RO

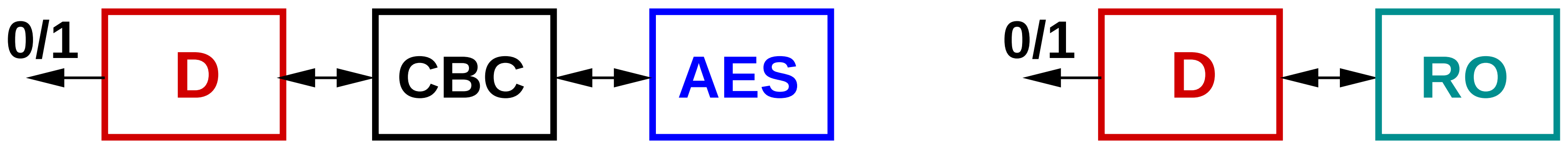
To show: $\Delta^{D}(\text{CBC AES}, \text{RO}) \approx 0$

Security proof for CBC-MAC

$$D\ CBC\ AES \approx D\ RO$$

To show: $\Delta^{\mathcal{E}}(CBC\,AES, RO) \approx 0$

Security proof for CBC-MAC



$$\mathbf{D}\ \mathbf{CBC}\ \mathbf{AES} \approx \mathbf{D}\ \mathbf{RO}$$

To show: $\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

$$\Delta^{\mathcal{E}}(\mathbf{S}, \mathbf{T}) := \max_{\mathbf{D} \in \mathcal{E}} \Delta^{\mathbf{D}}(\mathbf{S}, \mathbf{T})$$

Security proof for CBC-MAC

$$\mathbf{D}\ \mathbf{CBC}\ \mathbf{AES} \approx \mathbf{D}\ \mathbf{RO}$$

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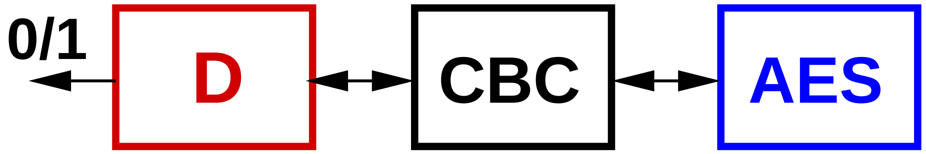

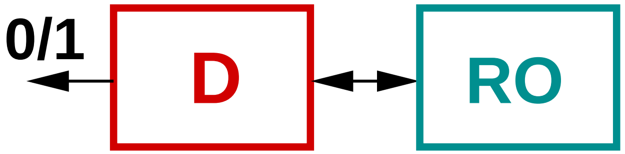

$$\mathbf{D}\ \mathbf{CBC}\ \mathbf{AES} \approx \mathbf{D}\ \mathbf{RO}$$

To show: $\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

Lemma: $\Delta^{\mathbf{D}}$ and $\Delta^{\mathcal{E}}$ are pseudo-metrics:

- $\Delta^{\mathcal{E}}(\mathbf{S}, \mathbf{S}) = 0$
- $\Delta^{\mathcal{E}}(\mathbf{R}, \mathbf{T}) \leq \Delta^{\mathcal{E}}(\mathbf{R}, \mathbf{S}) + \Delta^{\mathcal{E}}(\mathbf{S}, \mathbf{T})$

Security proof for CBC-MAC

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Security proof for CBC-MAC

$$D \; CBC \; AES \approx D \; RO$$

To show: $\Delta^{\mathcal{E}}(CBC\,AES, RO) \approx 0$

$$\Delta^{\mathcal{E}}(CBC\,AES, RO) \leq \Delta^{\mathcal{E}}(CBC\,AES, CBC\,RF) + \Delta^{\mathcal{E}}(CBC\,RF, RO)$$

Security proof for CBC-MAC

$$\mathbf{D}\,\mathbf{CBC}\,\mathbf{AES} \approx \mathbf{D}\,\mathbf{RO}$$

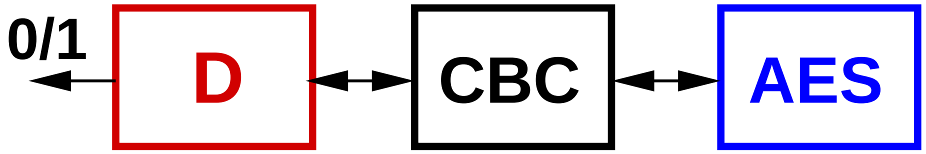
To show: $\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

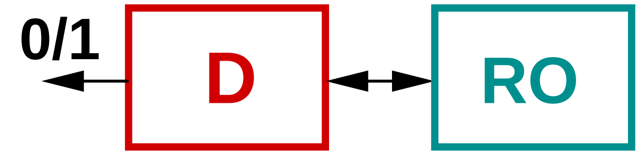
$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \leq \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) + \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{RF}, \mathbf{RO})$$

Absorption lemma: $\Delta^{\mathbf{D}}(\mathbf{C}\mathbf{S}, \mathbf{C}\mathbf{T}) = \Delta^{\mathbf{DC}}(\mathbf{S}, \mathbf{T})$

Proof: $\mathbf{DCS} = \mathbf{D}(\mathbf{CS}) = (\mathbf{DC})\mathbf{S}$

Security proof for CBC-MAC





$$\mathbf{D}\,\mathbf{CBC}\,\mathbf{AES} \approx \mathbf{D}\,\mathbf{RO}$$

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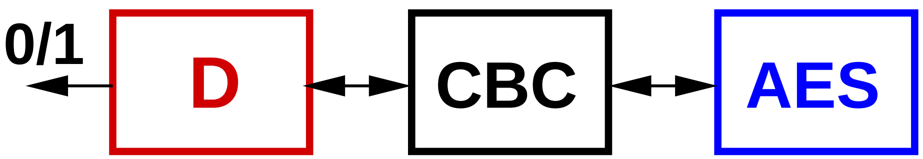
$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \leq \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) + \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{RF}, \mathbf{RO})$$

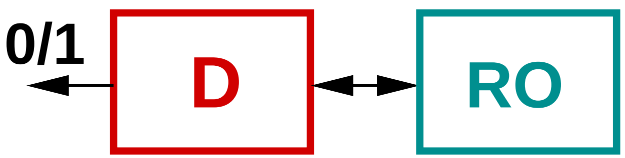
$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) = \Delta^{\mathcal{E}\mathbf{CBC}}(\mathbf{AES}, \mathbf{RF})$$

Absorption lemma: $\Delta^{\mathbf{D}}(\mathbf{C}\mathbf{S}, \mathbf{C}\mathbf{T}) = \Delta^{\mathbf{D}\mathbf{C}}(\mathbf{S}, \mathbf{T})$

Proof: $\mathbf{D}\mathbf{C}\mathbf{S} = \mathbf{D}(\mathbf{C}\mathbf{S}) = (\mathbf{D}\mathbf{C})\mathbf{S}$

Security proof for CBC-MAC [3]





$$\mathbf{D}\ \mathbf{CBC}\ \mathbf{AES} \approx \mathbf{D}\ \mathbf{RO}$$

To show: $\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \leq \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) + \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{RF}, \mathbf{RO})$$

$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) = \Delta^{\mathcal{E}\mathbf{CBC}}(\mathbf{AES}, \mathbf{RF})$$

Non-expansion lemma:

$$\mathcal{D}\mathbf{C} \subseteq \mathcal{D} \Rightarrow \Delta^{\mathcal{D}}(\mathbf{C}\mathbf{S}, \mathbf{C}\mathbf{T}) \leq \Delta^{\mathcal{D}}(\mathbf{S}, \mathbf{T})$$

Security proof for CBC-MAC [3]

$$\mathbf{D}\ \mathbf{CBC}\ \mathbf{AES} \approx \mathbf{D}\ \mathbf{RO}$$

To show: $\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \leq \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) + \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{RF}, \mathbf{RO})$$

$\mathcal{E}\,\mathbf{CBC} \subseteq \mathcal{E}$

$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) = \Delta^{\mathcal{E}\mathbf{CBC}}(\mathbf{AES}, \mathbf{RF})$$

Non-expansion lemma:

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Security proof for CBC-MAC

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To show: $\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \le \Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF} \ldots \mathbf{F}, \mathbf{RO})$$

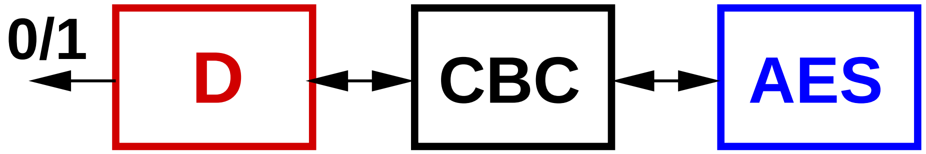
$$\mathcal{E}\,\mathbf{CBC} \subseteq \mathcal{E}$$

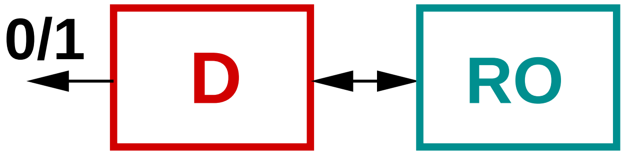
$$\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{CBC}\,\mathbf{RF}) = \Delta^{\mathcal{E}\mathbf{CBC}}(\mathbf{AES}, \mathbf{RF}) \le \Delta^{\mathcal{E}}(\mathbf{AES}, \mathbf{RF})$$

Non-expansion lemma:

$$\mathcal{D}\mathbf{C} \subseteq \mathcal{D} \;\Rightarrow\; \Delta^{\mathcal{D}}(\mathbf{C}\mathbf{S}, \mathbf{C}\mathbf{T}) \le \Delta^{\mathcal{D}}(\mathbf{S}, \mathbf{T})$$

Security proof for CBC-MAC





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To show: $\Delta^{\mathcal{E}}(\mathbf{CBC}\,\mathbf{AES}, \mathbf{RO}) \approx 0$

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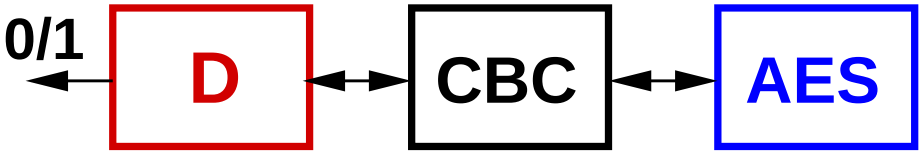

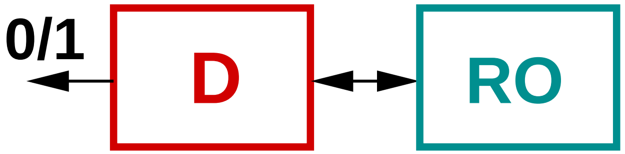

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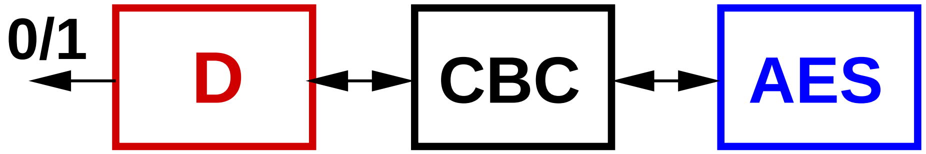

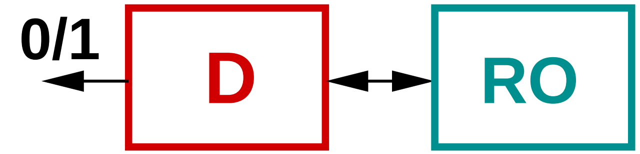

Note: Many security proofs can be phrased at this level of abstraction and become quite simple or even trivial.

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We need notions for

- the complexity of system implementation
- what is efficient (for the good guys)
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Note: The usual poly-time notions (i.e., $n^{O(1)}$) are of course composable, but so are many other notions, e.g. $n^{O(\log \log n)}$ or $n^{O(\sqrt{\log \log \log n})}$.

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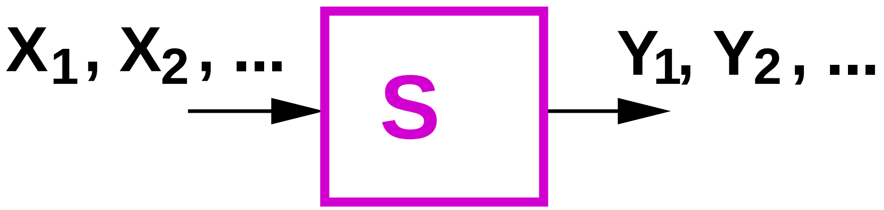

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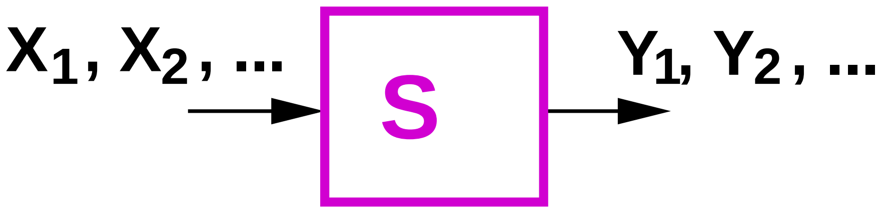

Description of **S**: figure, pseudo-code, text, ...

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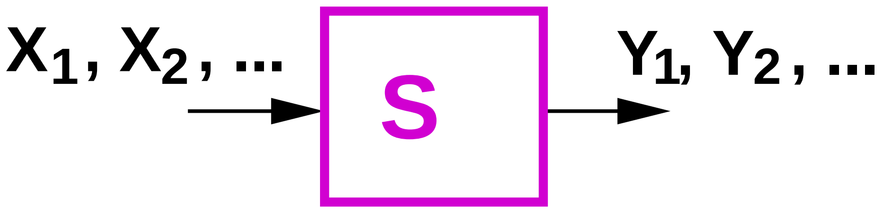

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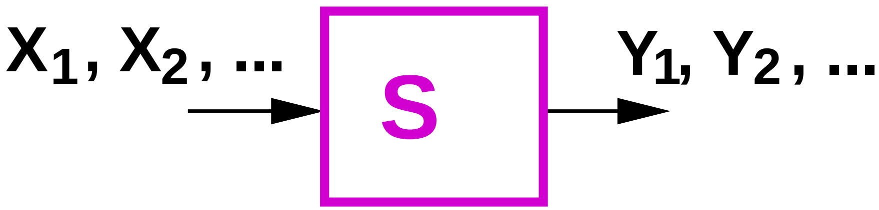

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(where $X^i = (X_1, \ldots, X_i)$)

This abstraction is called a random system [Mau02].

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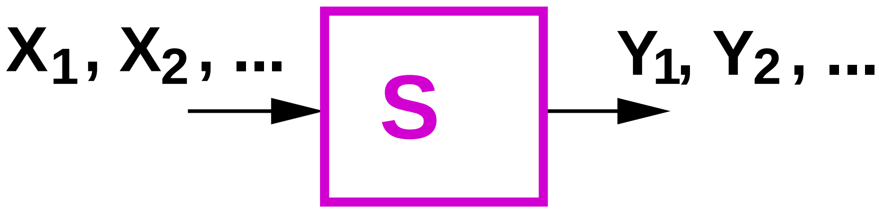

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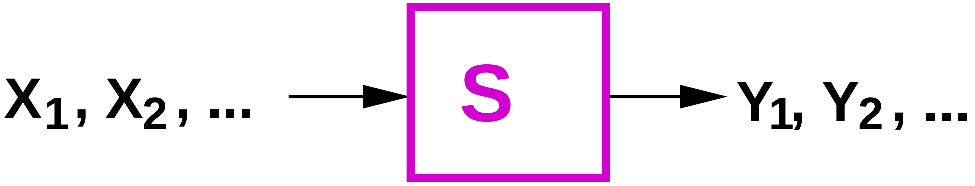
Equivalence of systems: $\mathbf{S} \equiv \mathbf{T}$ if same behavior

Games



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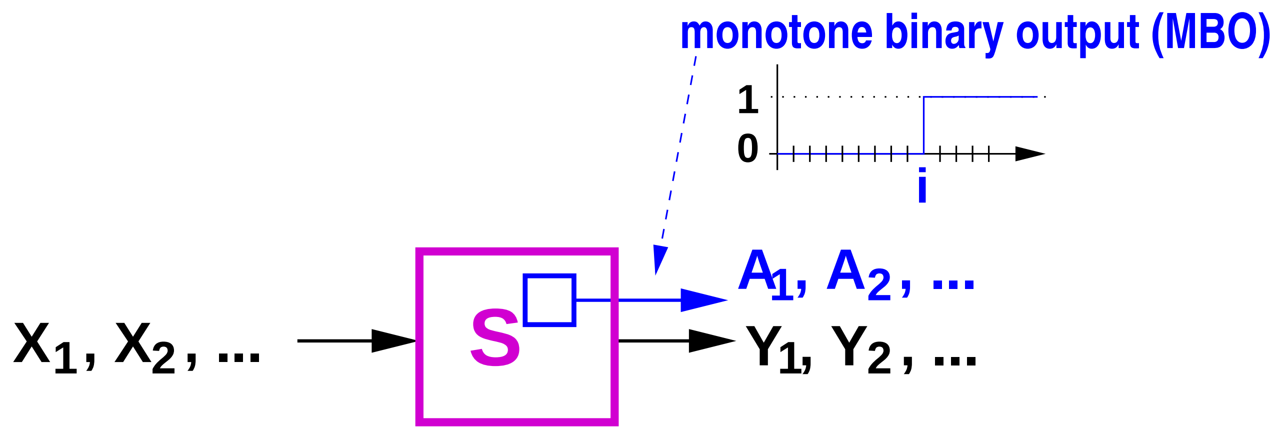

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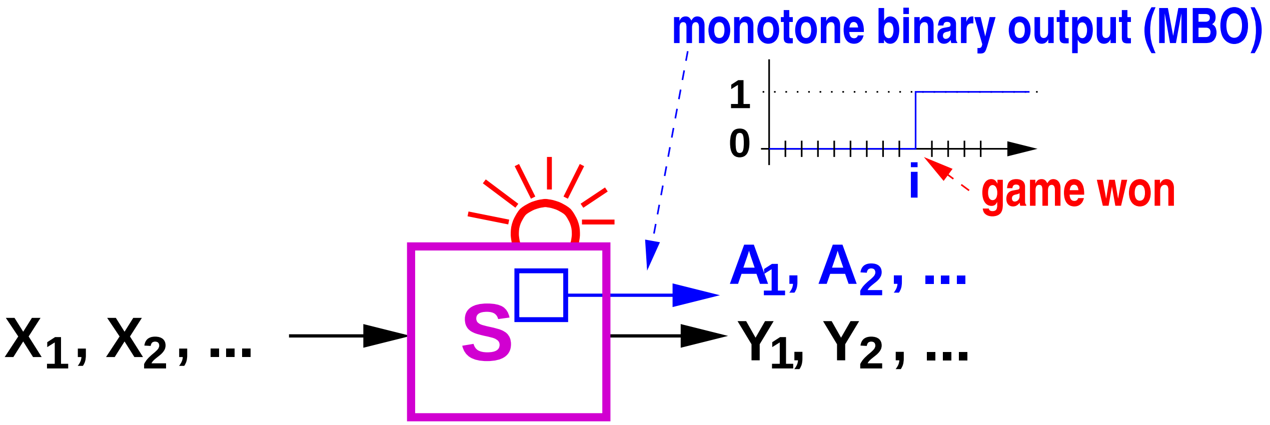

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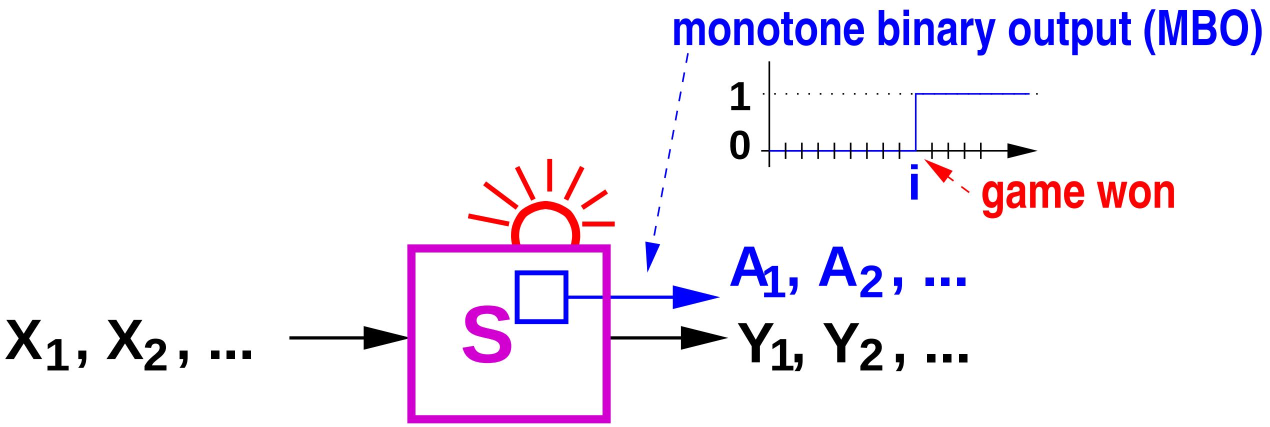

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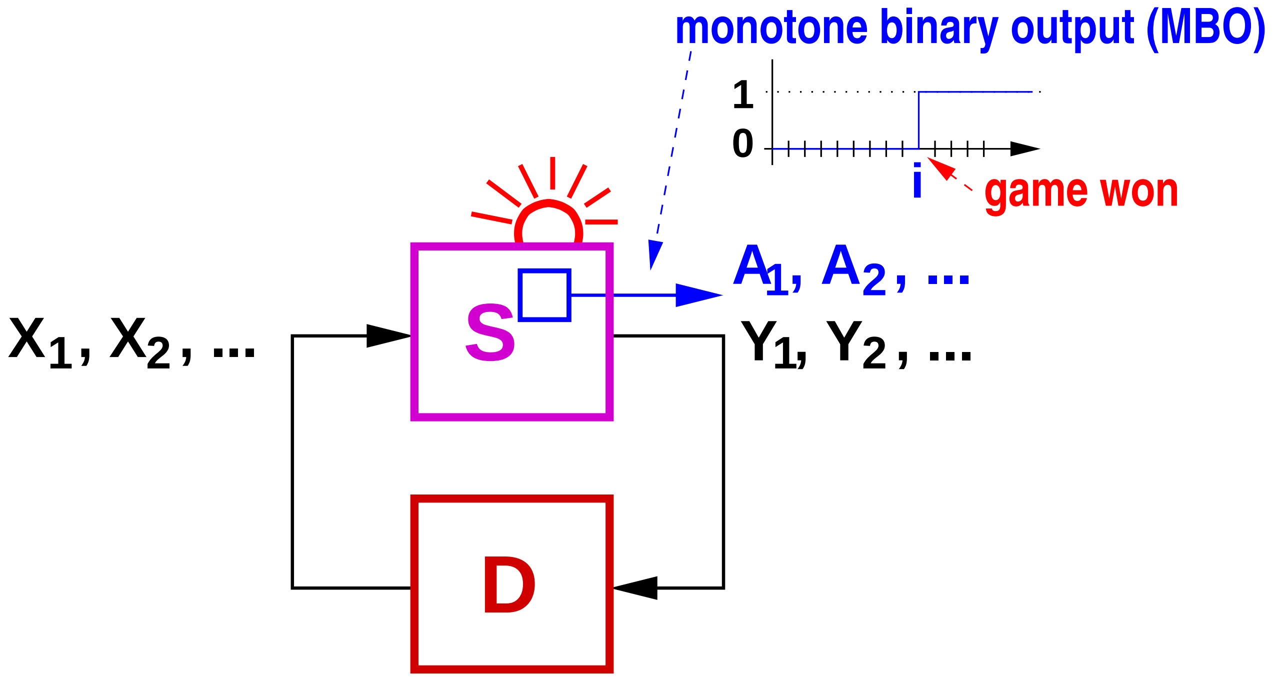

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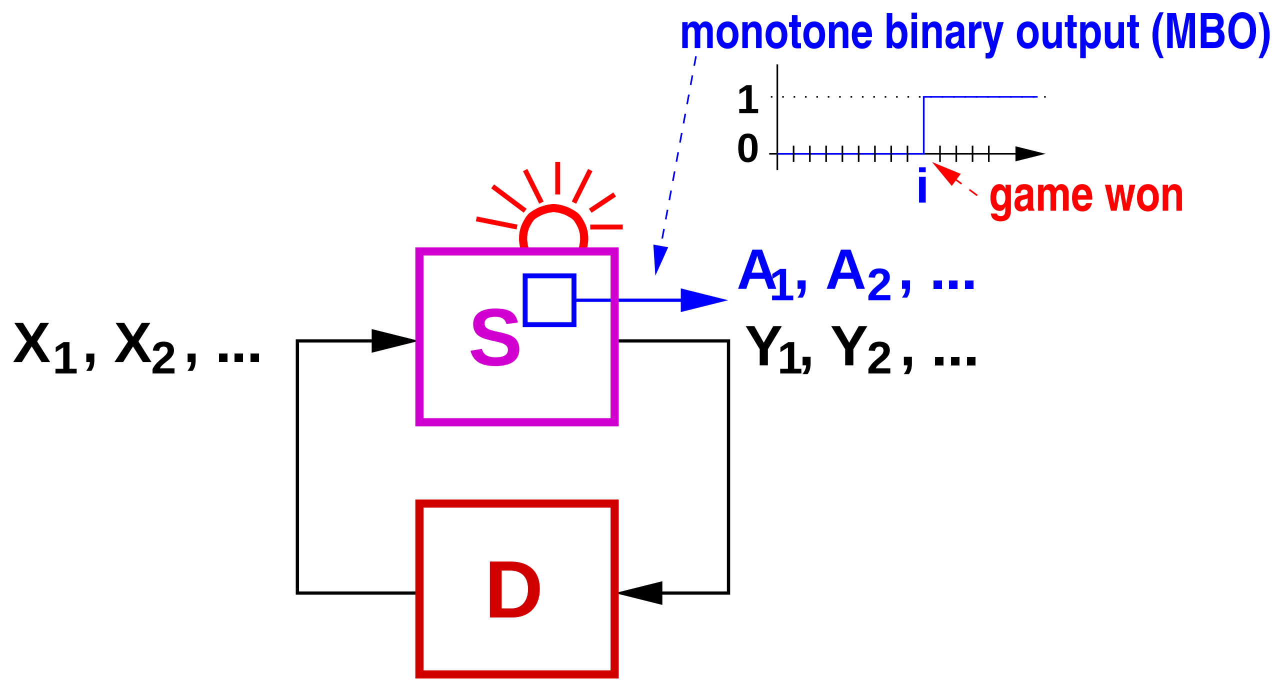

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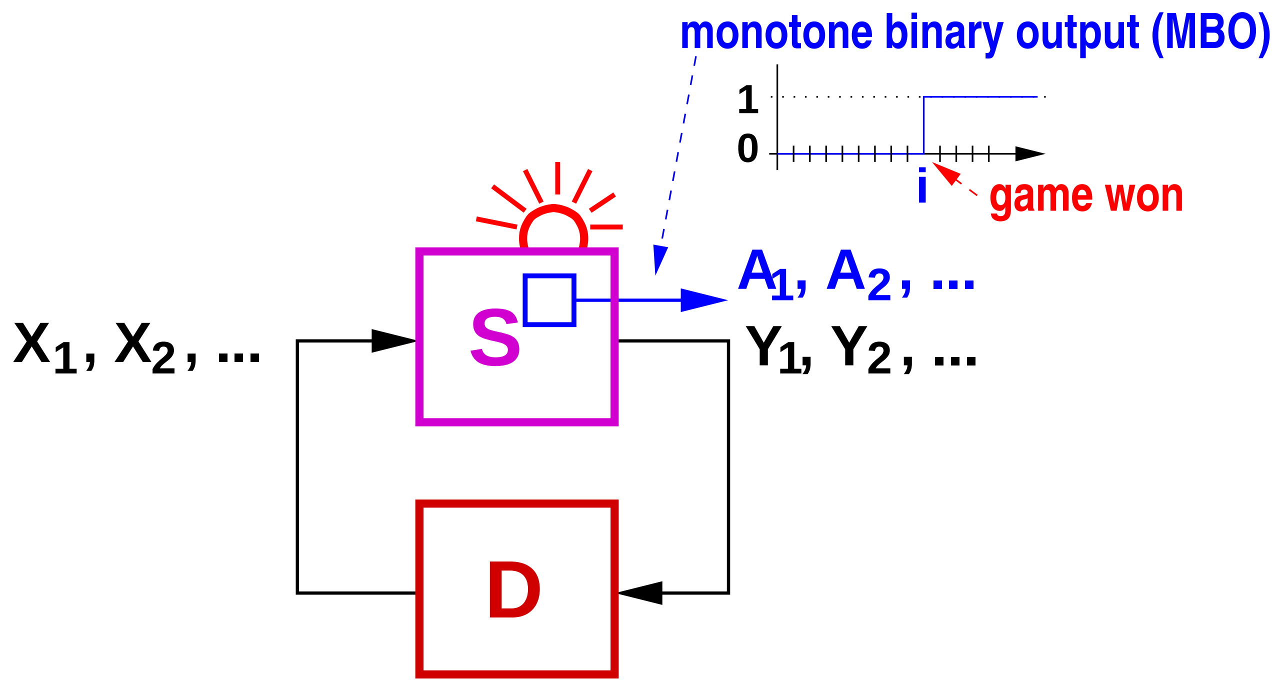

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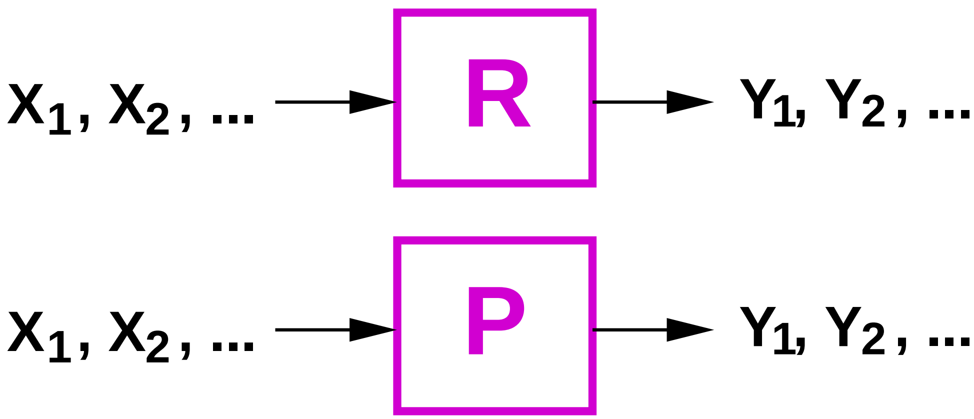

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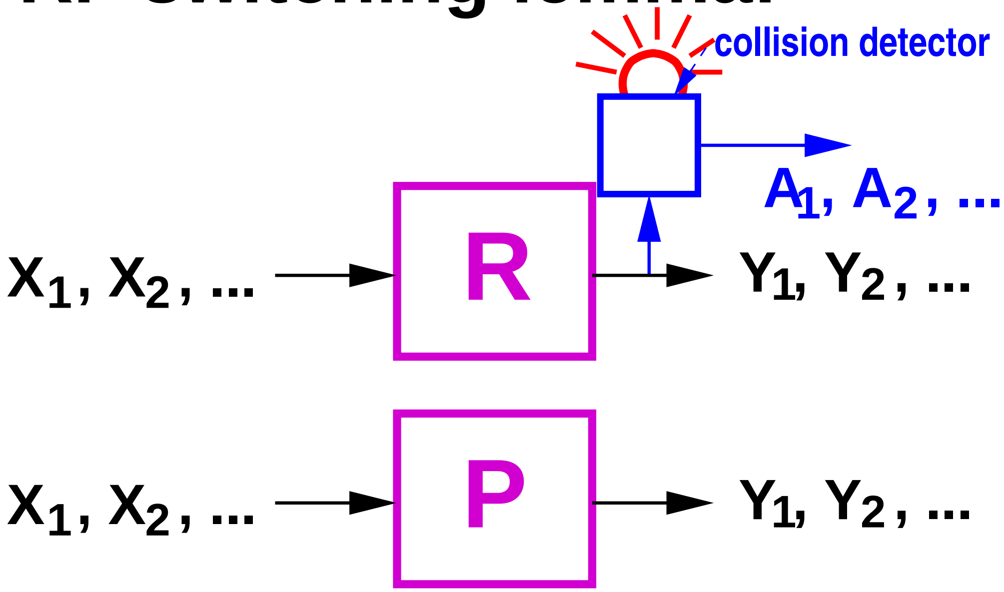

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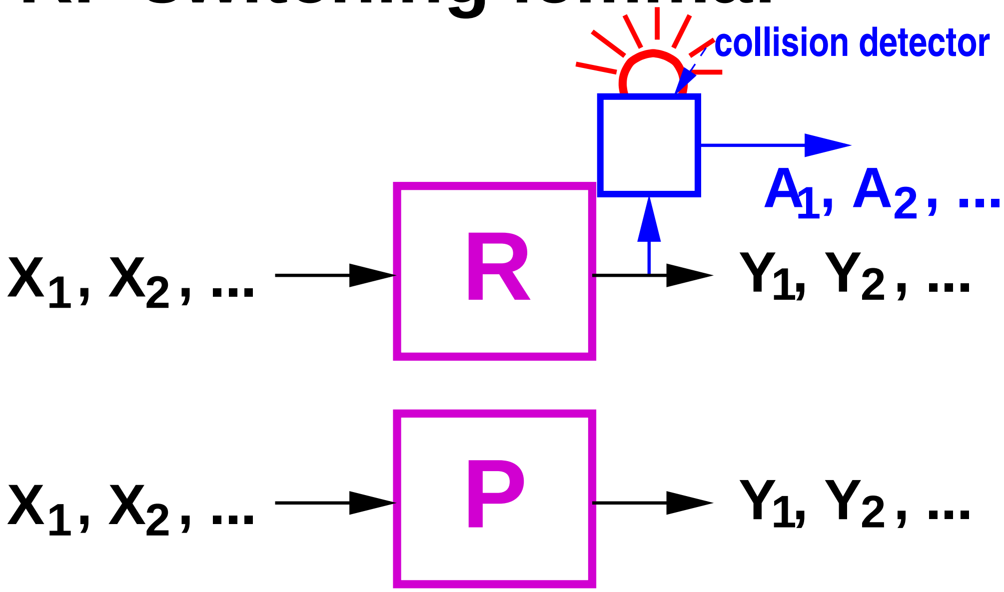

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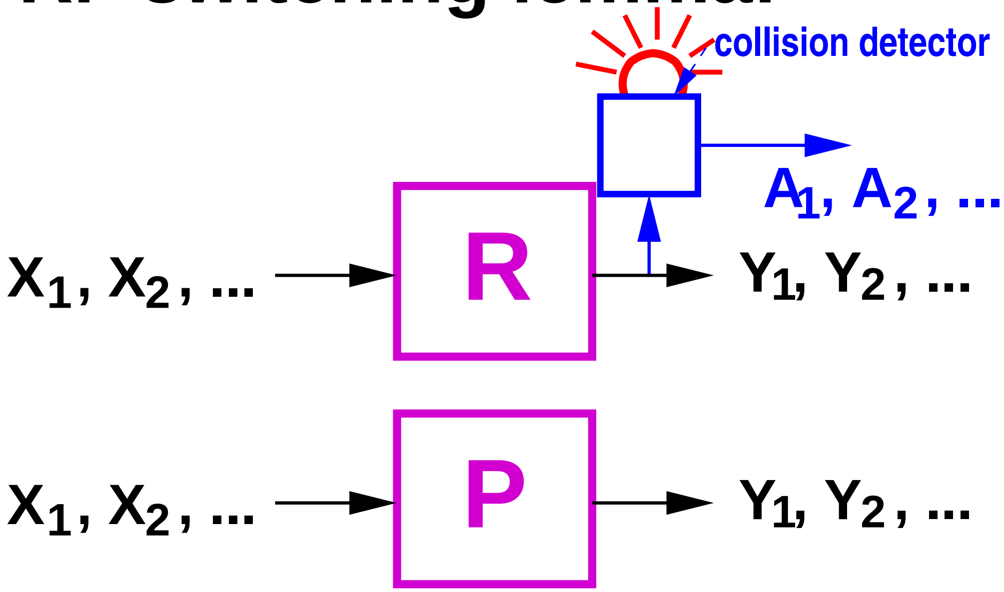

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C

X^i

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Similarly simple proof of CBC-MAC security:

$$(\mathbf{CBCRF})|\mathcal{A} \equiv \mathbf{RO} \;\Rightarrow\; \Delta(\mathbf{CBCRF},\mathbf{RO}) \;\leq\; \tfrac{1}{2}\ell^2 2^{-n}$$

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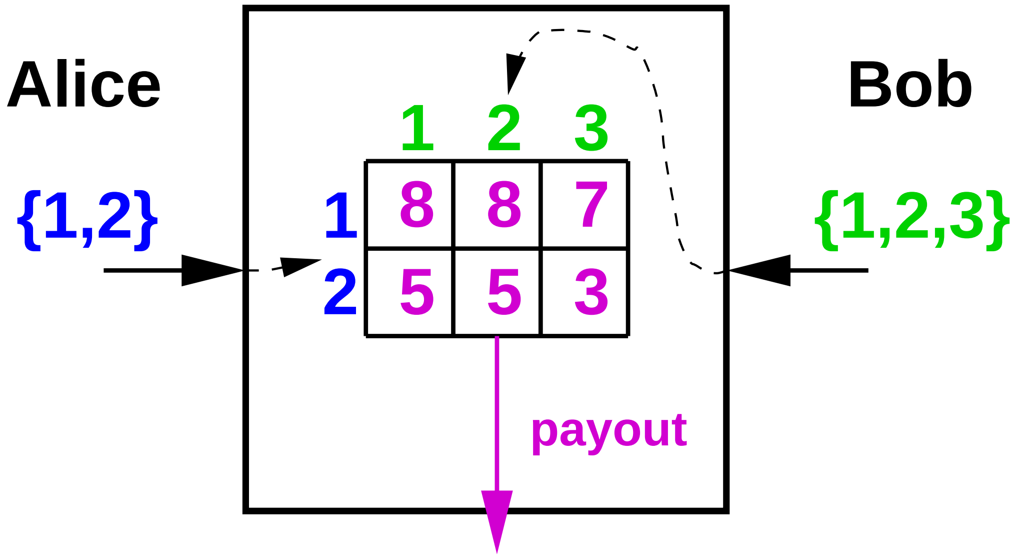
- capture scenarios that could previously not be modeled.

Resources and isomorphisms



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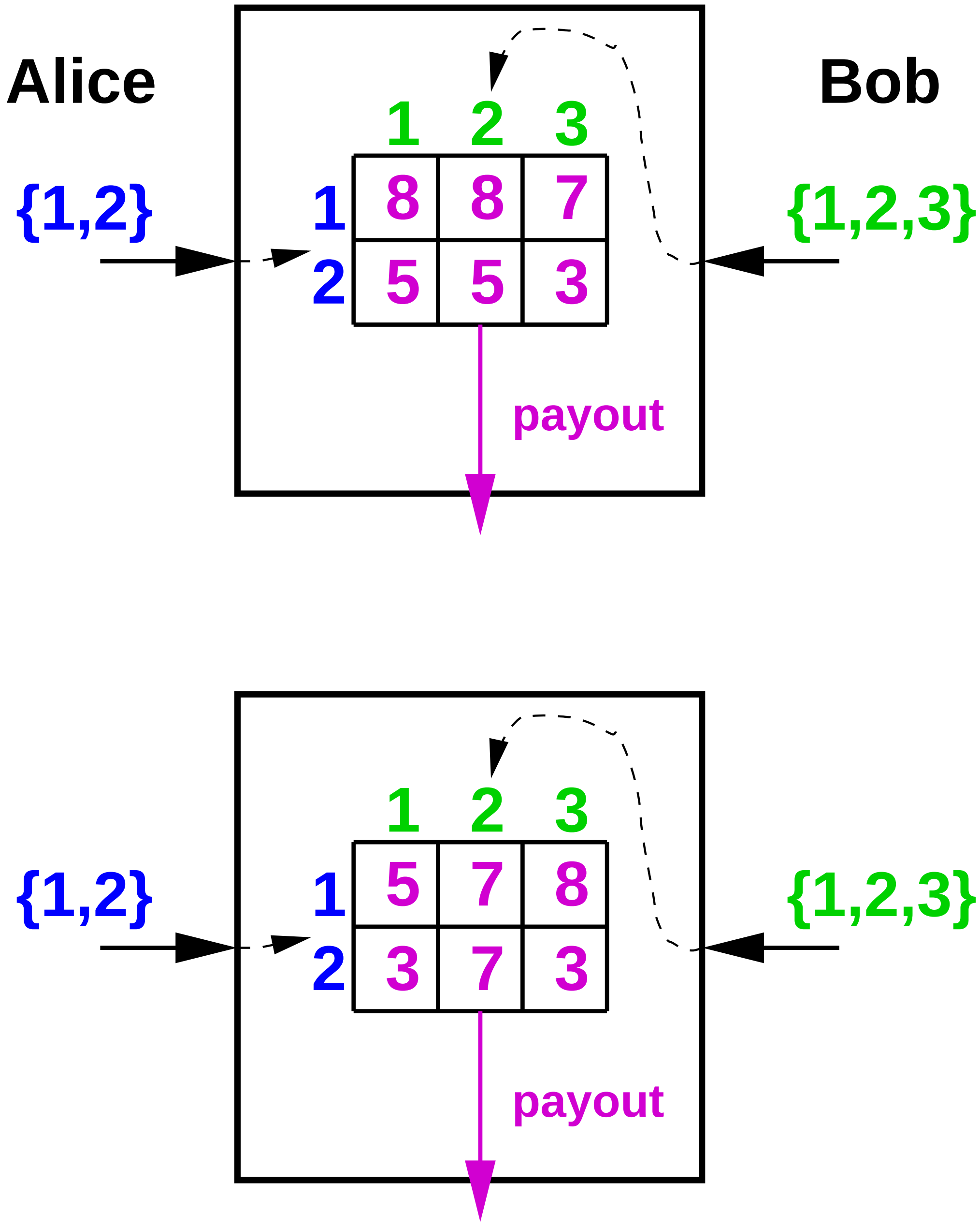

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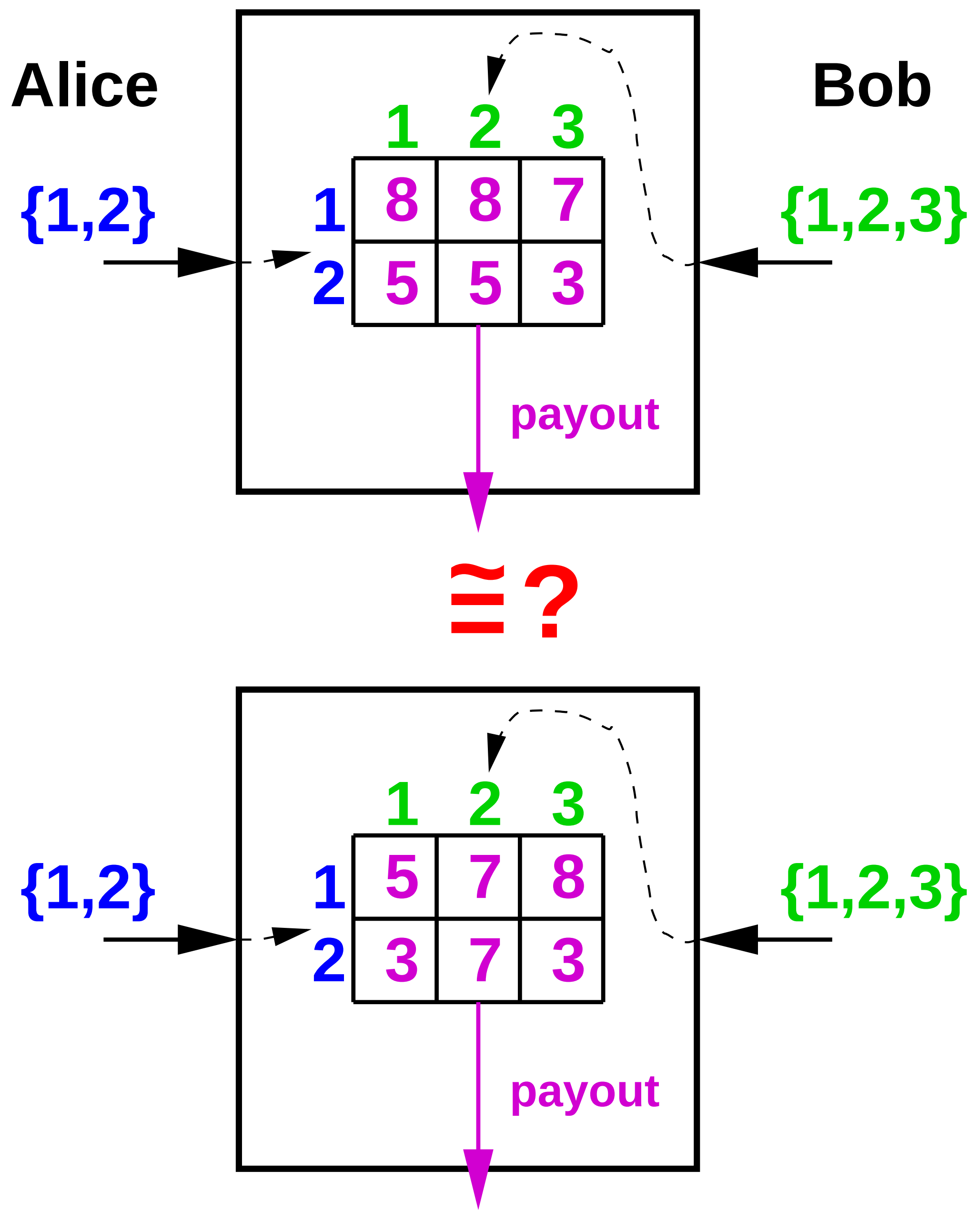

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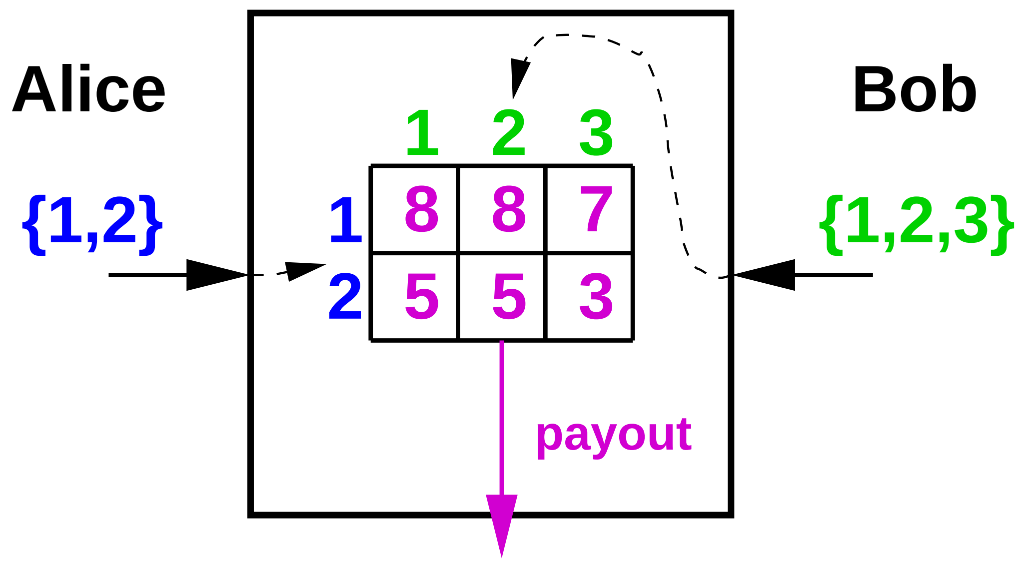

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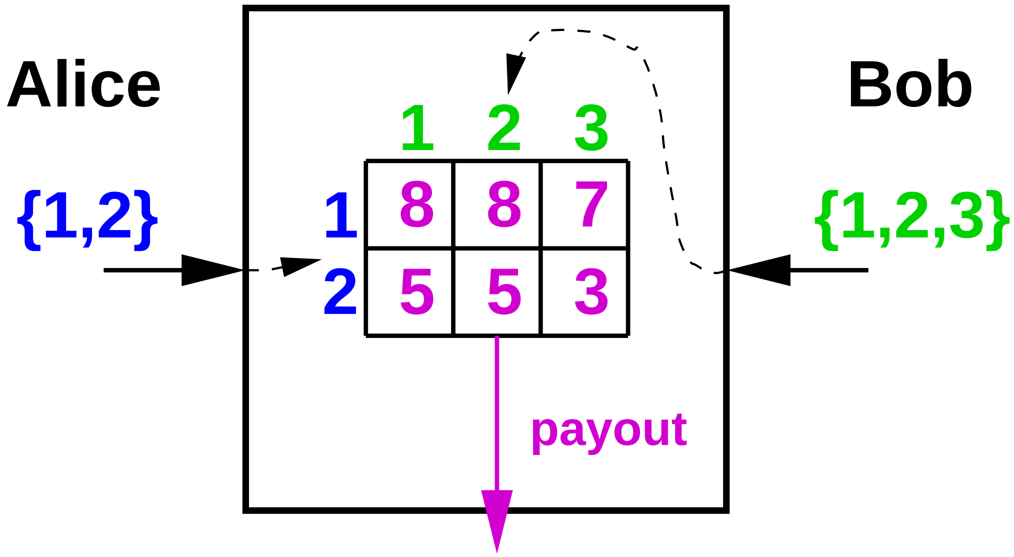

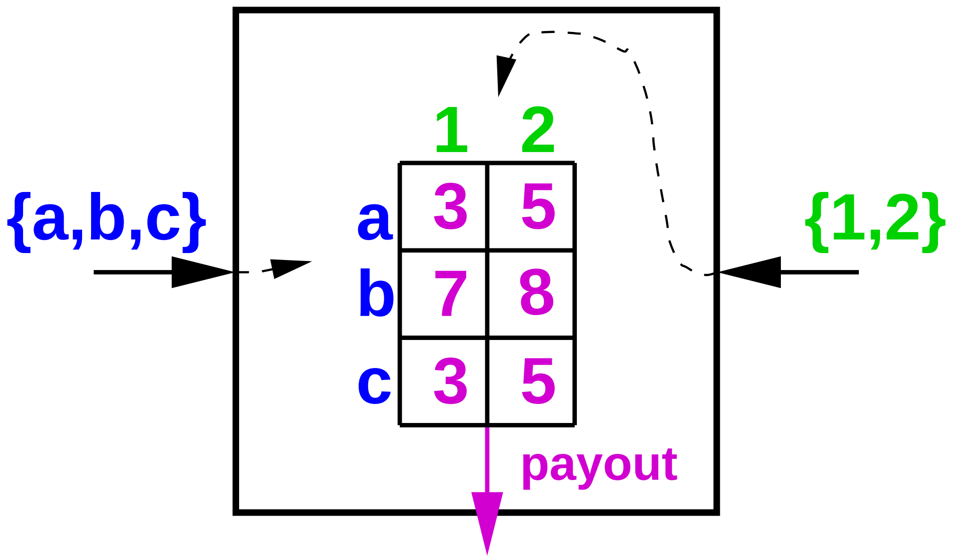

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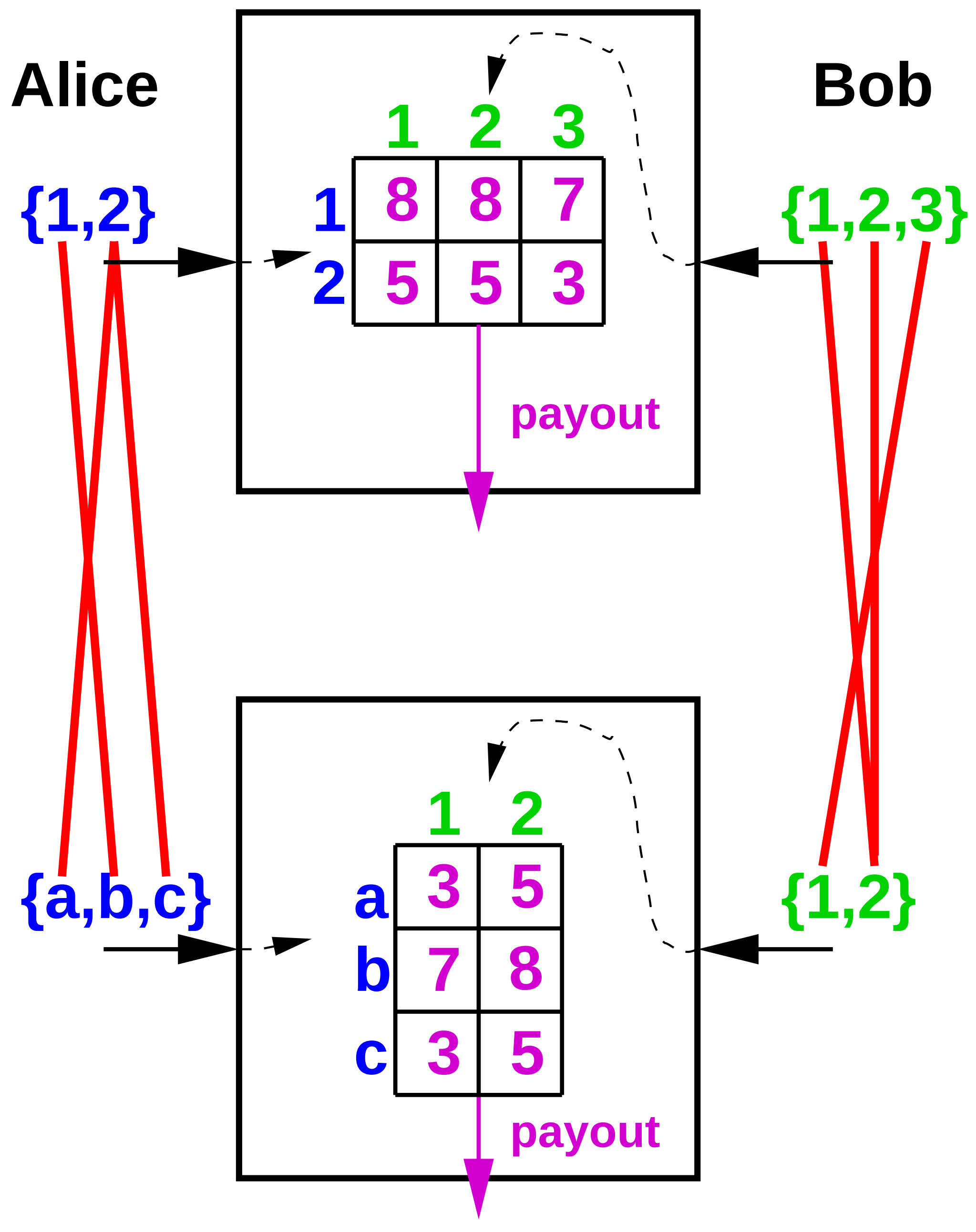

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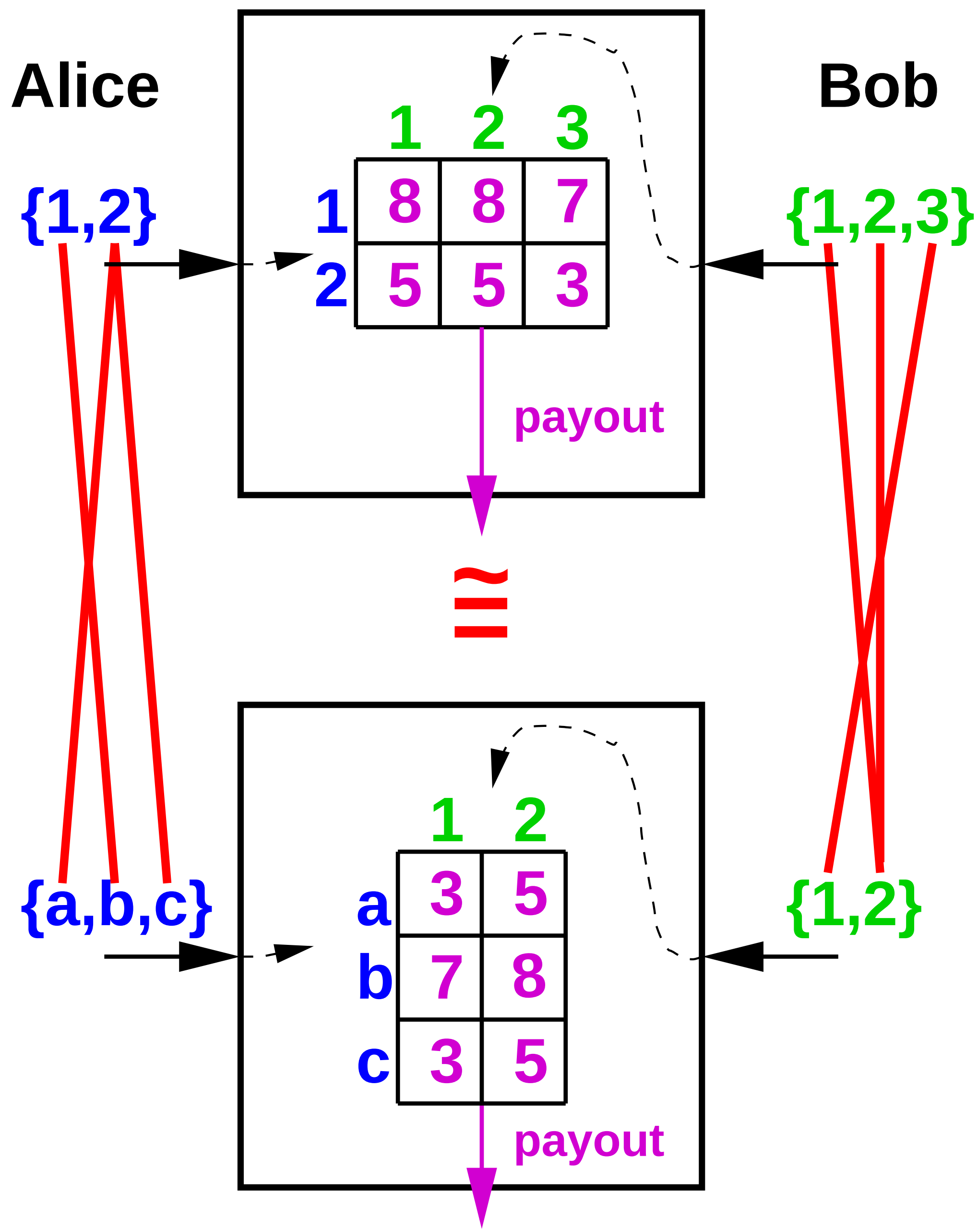

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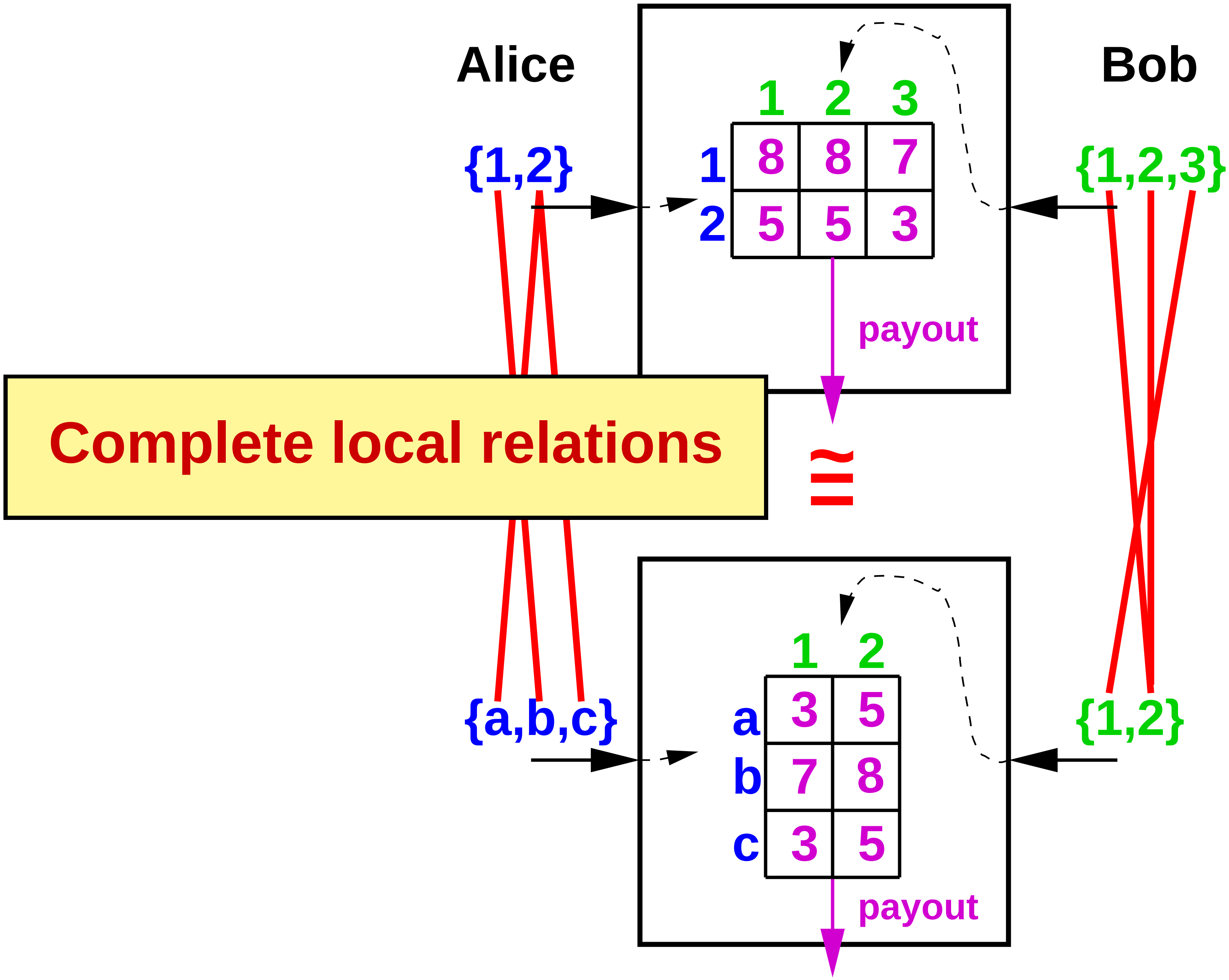
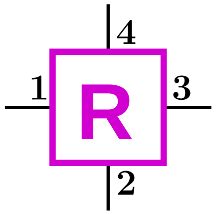


Abstract multi-party setting



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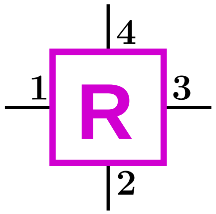

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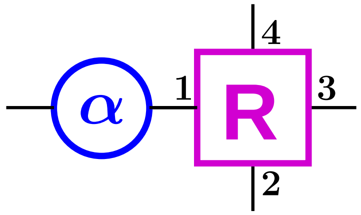
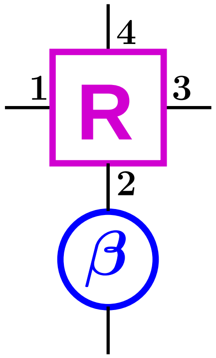


$$\alpha^{1}\mathbf{R}$$

Abstract multi-party setting [3]



β^2**R**

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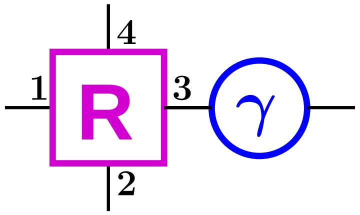

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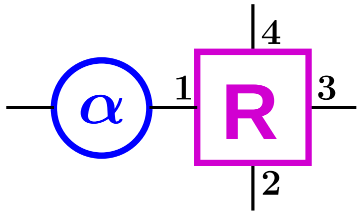

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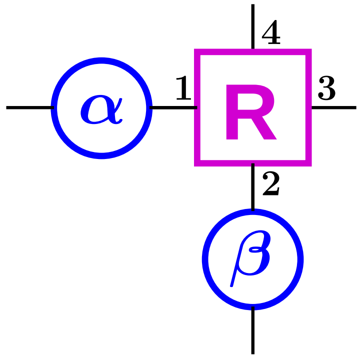

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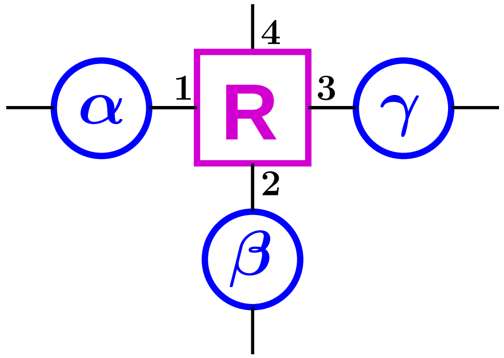

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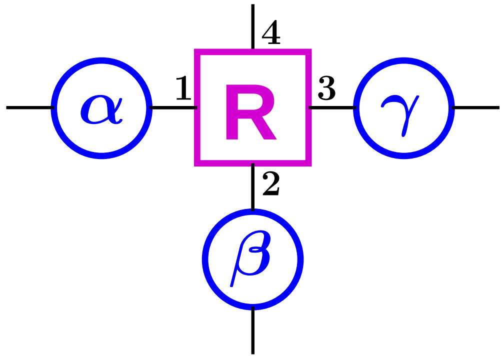

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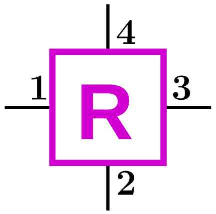

$\mathbf{R}$

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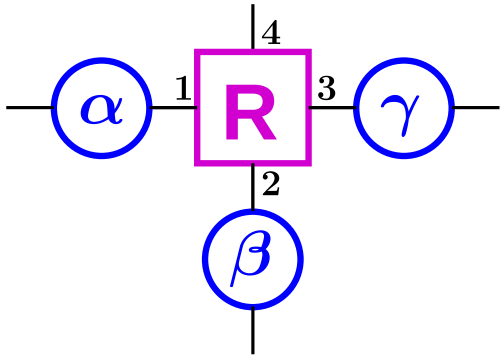

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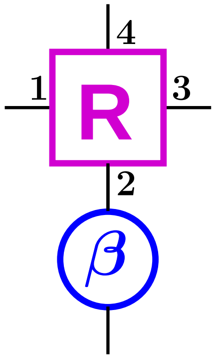

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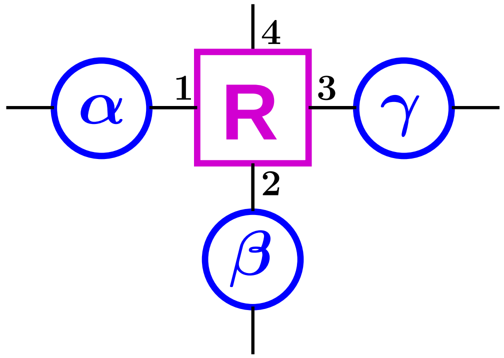

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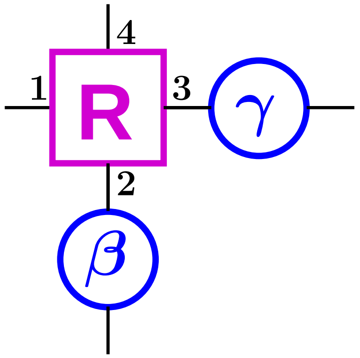

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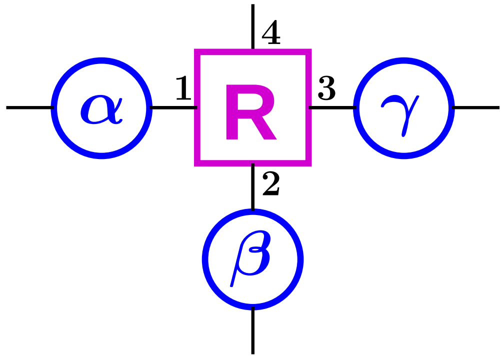

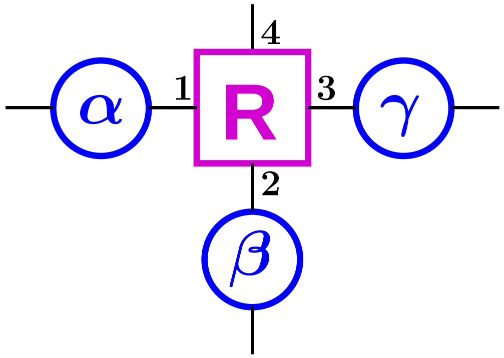

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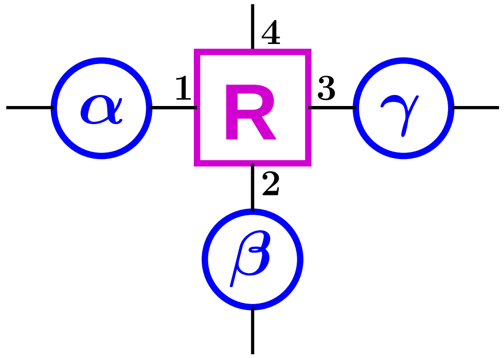

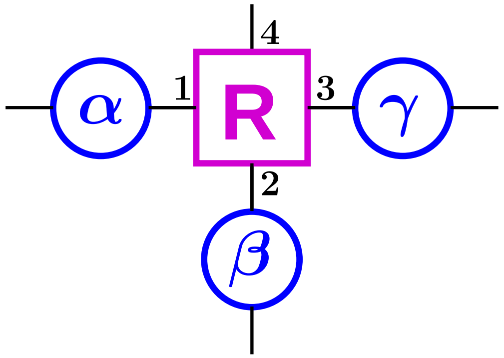

$$\gamma^{3}\beta^{2}\alpha^{1}\mathsf{R} \quad = \quad \alpha^{1}\gamma^{3}\beta^{2}\mathsf{R}$$

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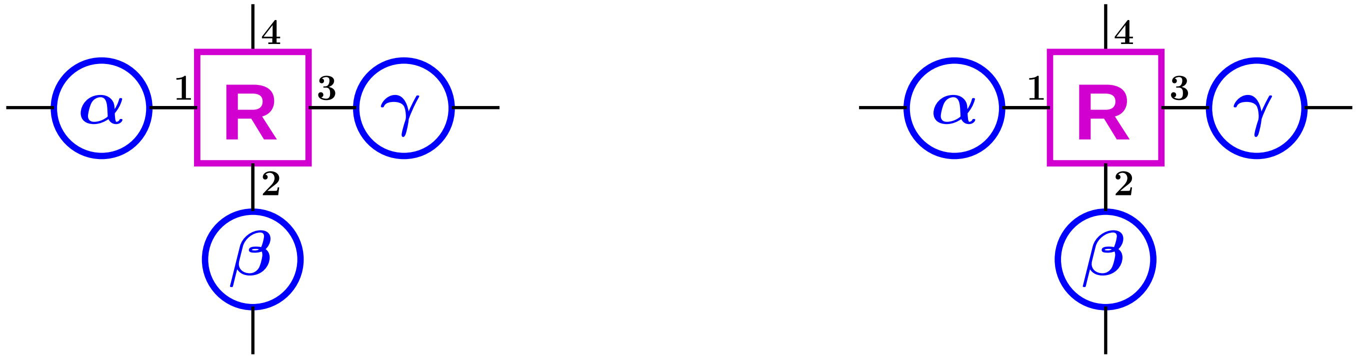

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Resource set Φ for interface set $\mathcal{I} = \{1, 2, 3, 4\}$, oper. $||$

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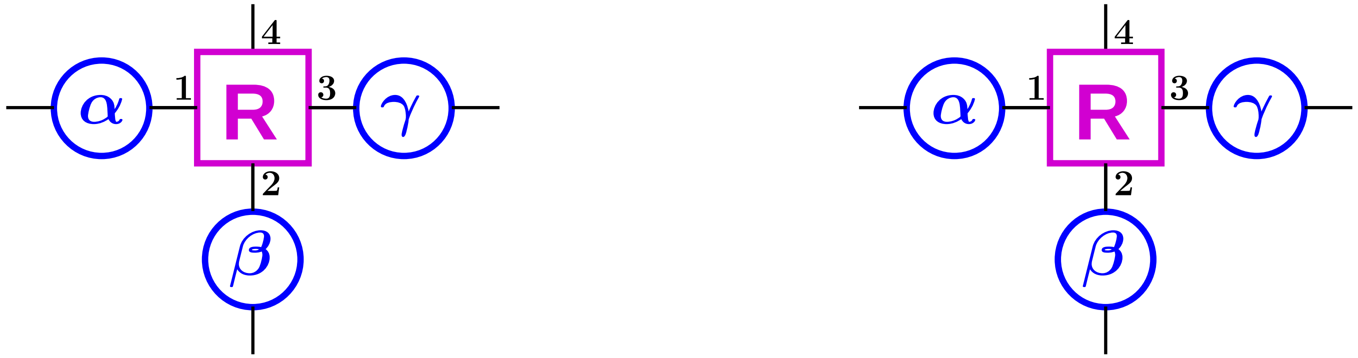

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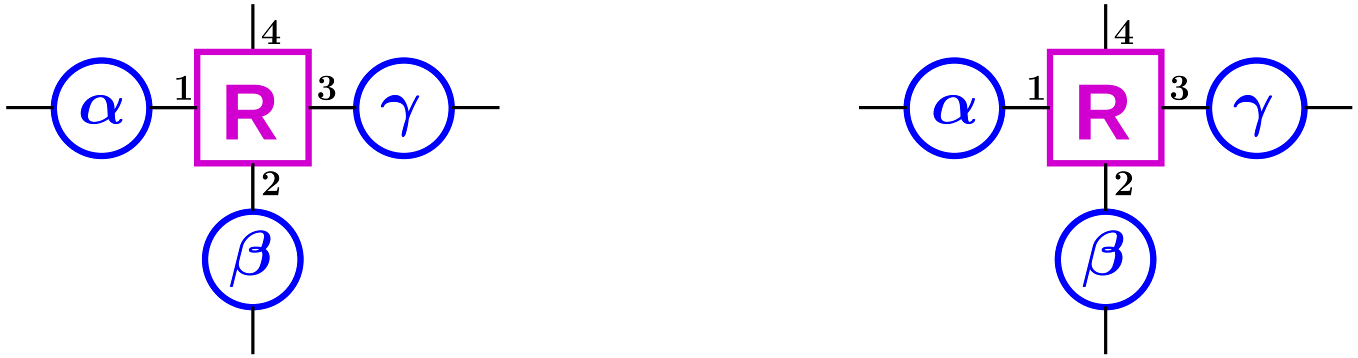

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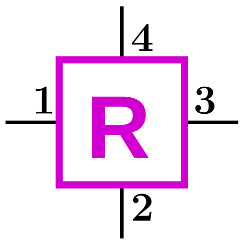
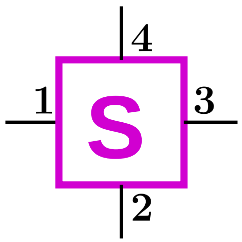
Algebraic laws:

- $\alpha^i\mathbf{R} \in \Phi$ for all $\mathbf{R} \in \Phi$, $\alpha \in \Sigma$, $i \in \mathcal{I}$
- $\alpha^i\beta^j\mathbf{R} \equiv \beta^j\alpha^i\mathbf{R}$ for all $i \neq j$

Resource isomorphisms





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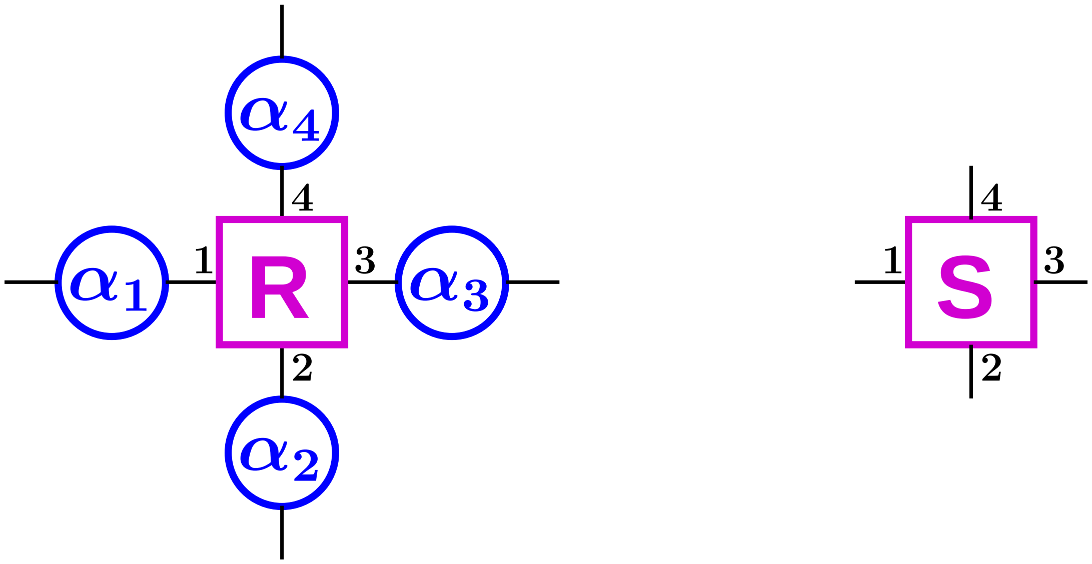

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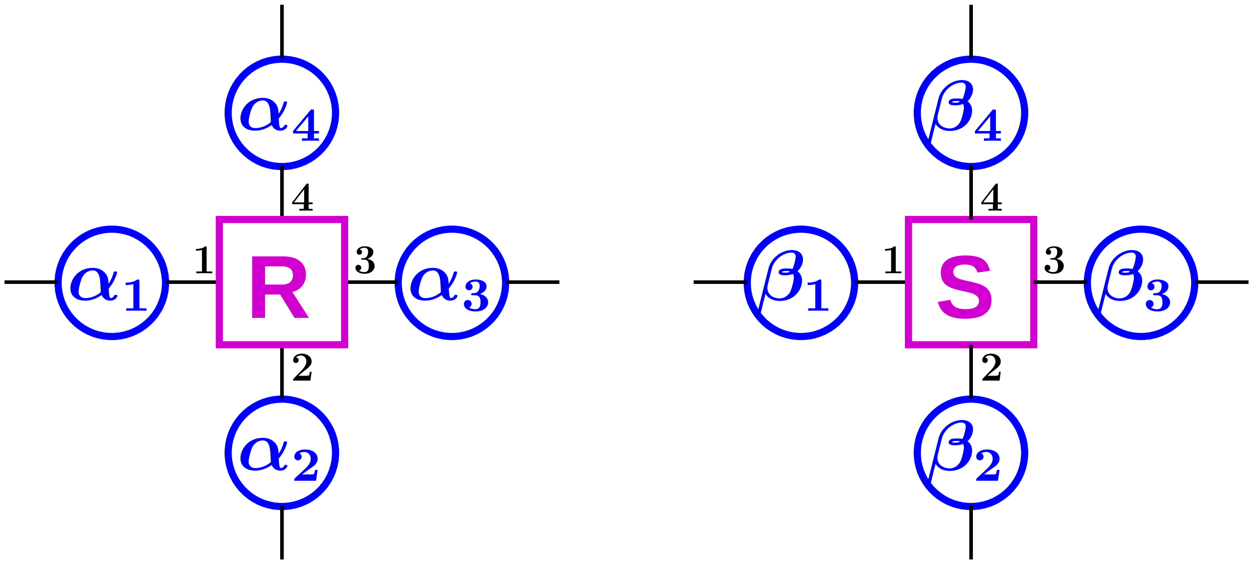

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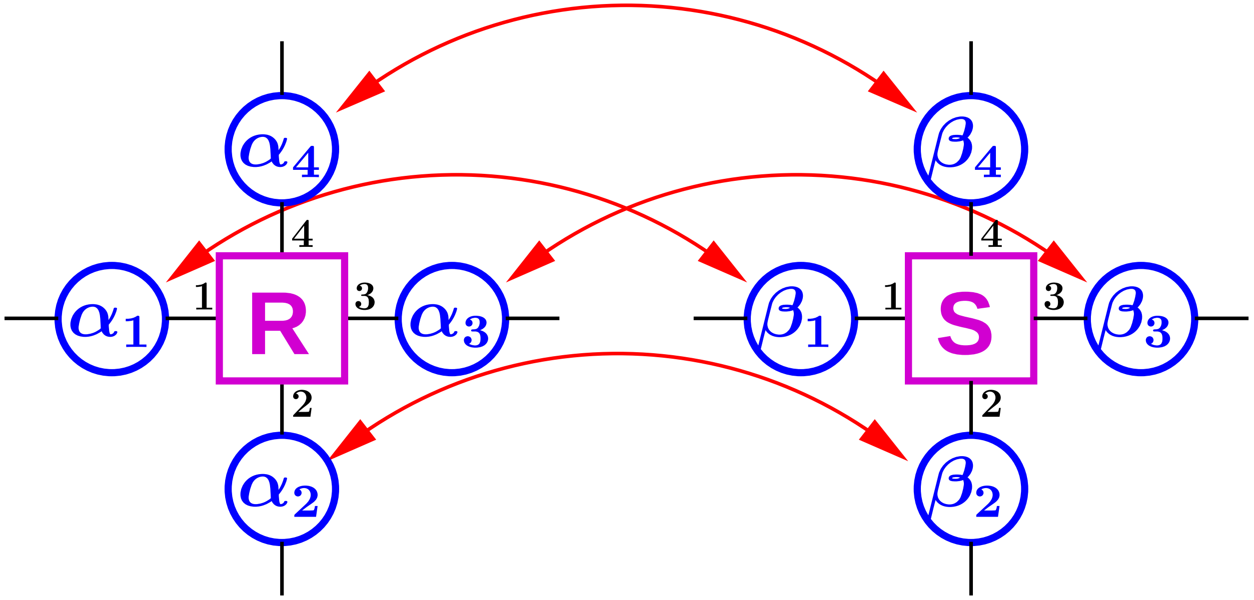

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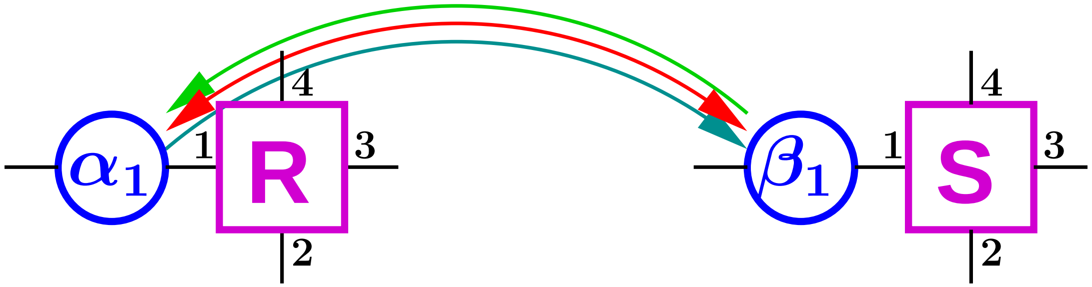

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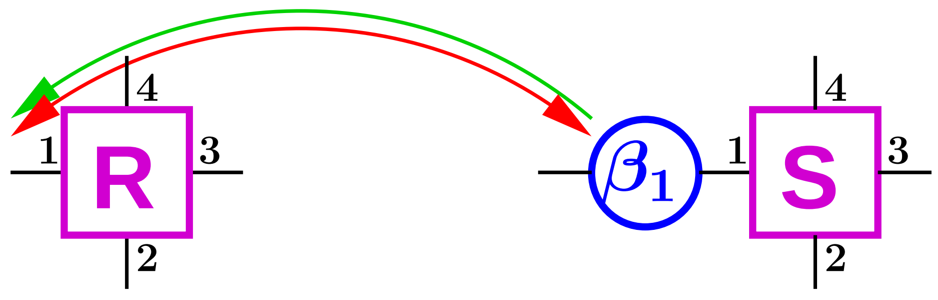

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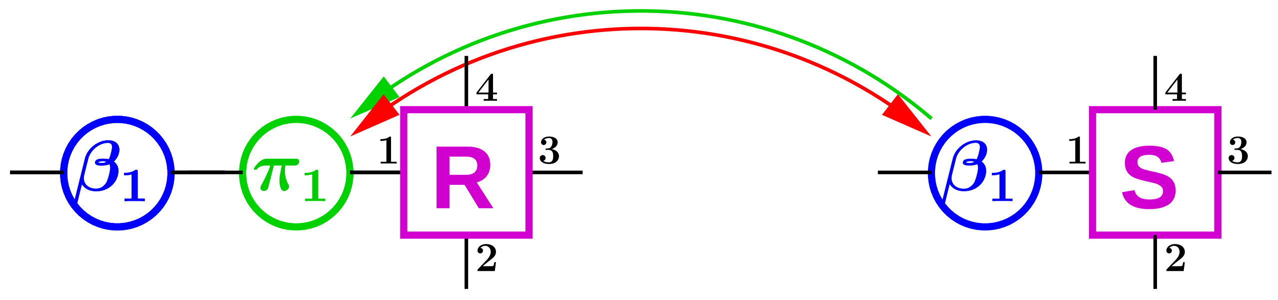

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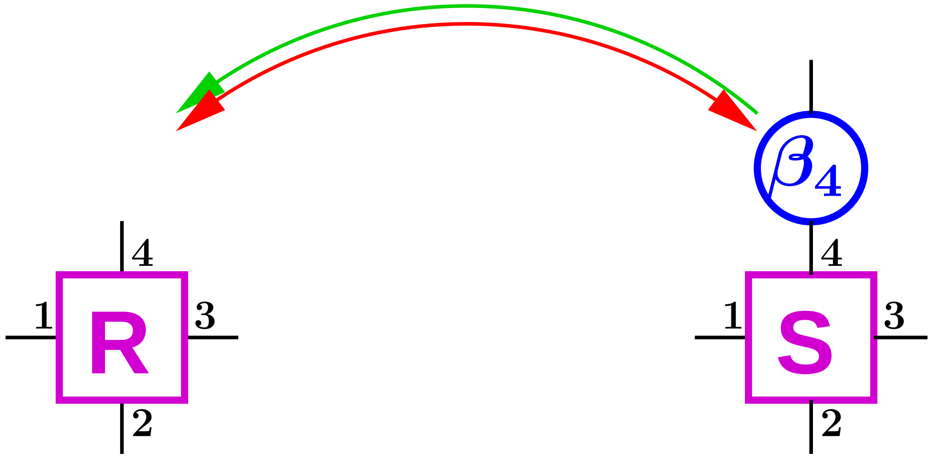

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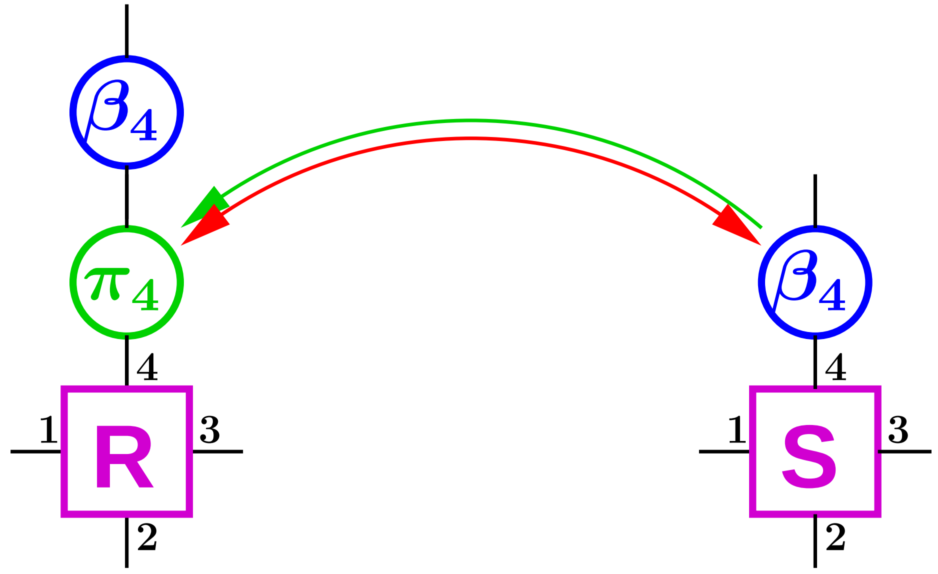

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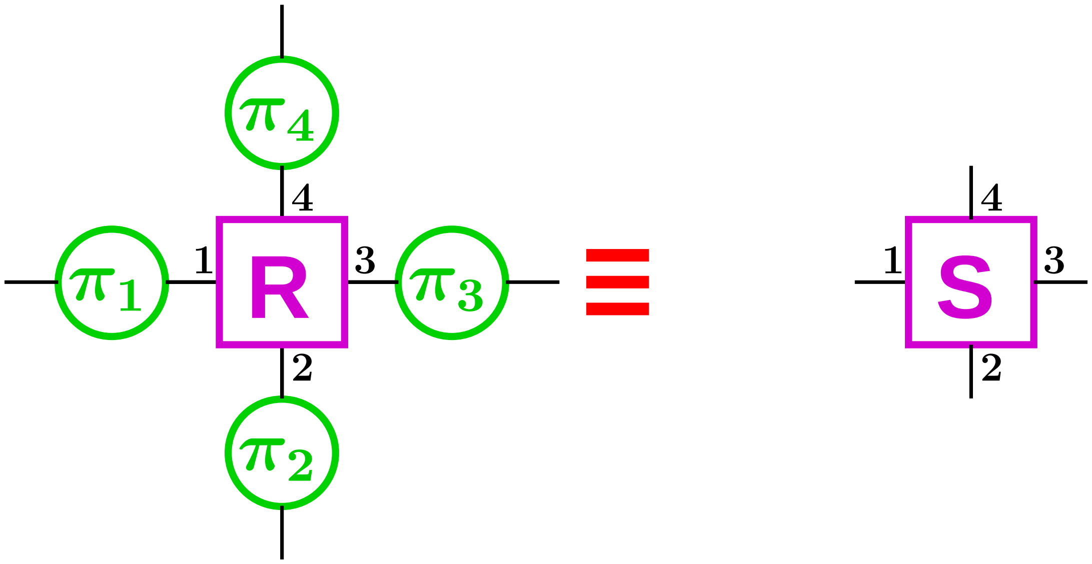

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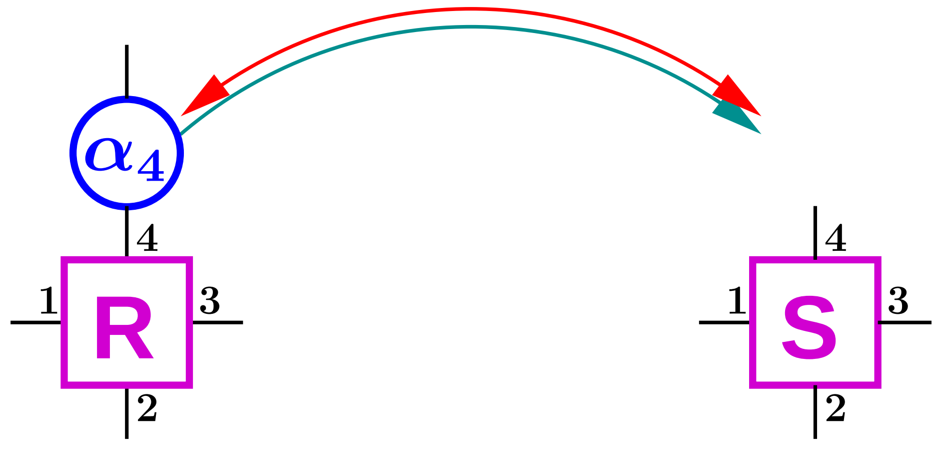

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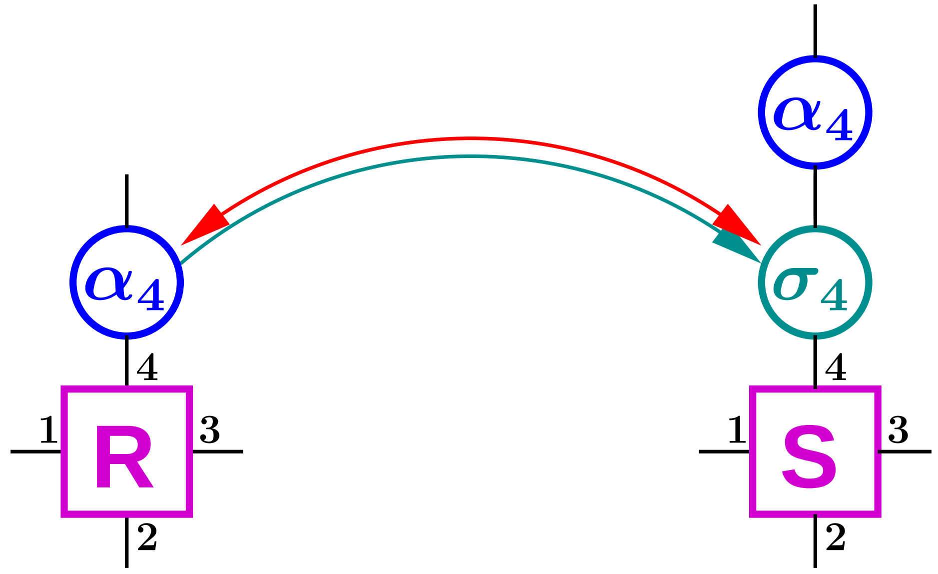

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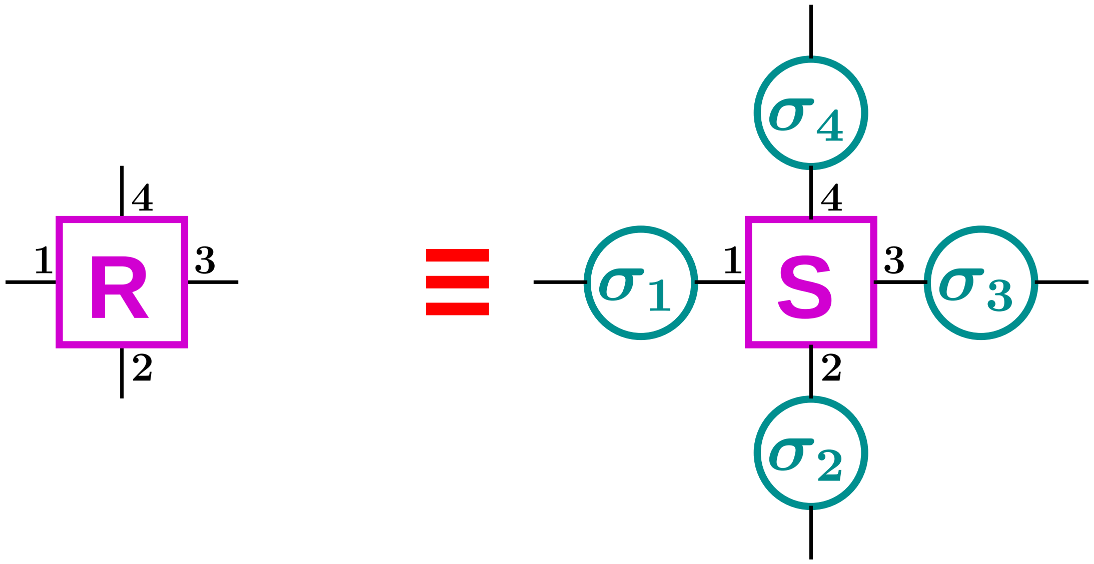

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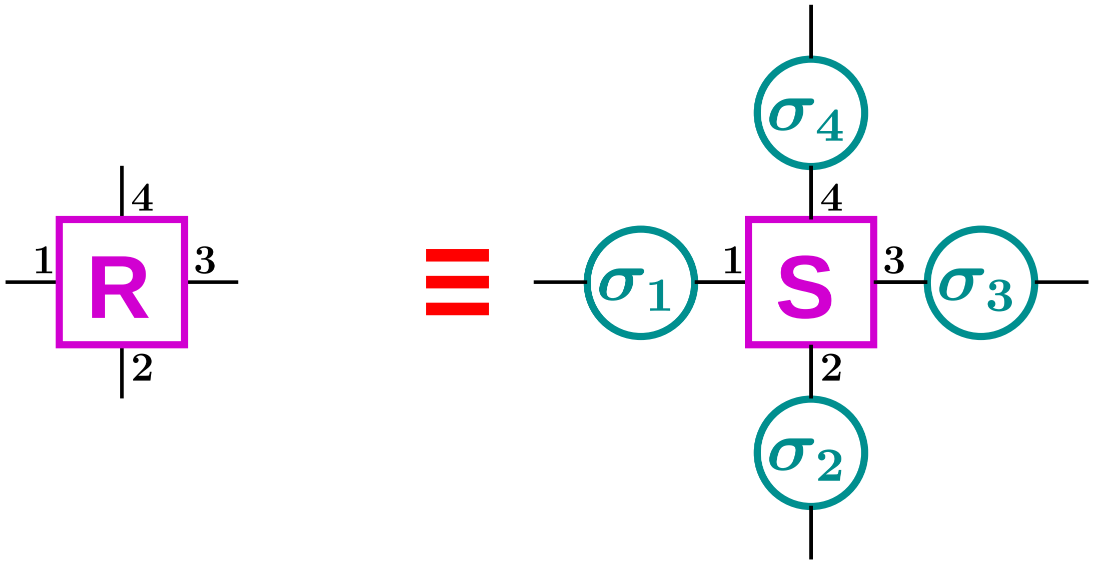

Definition: **R** *is isomorphic to* **S** via π, denoted **R** $\cong^{\pi}$ **S**, if

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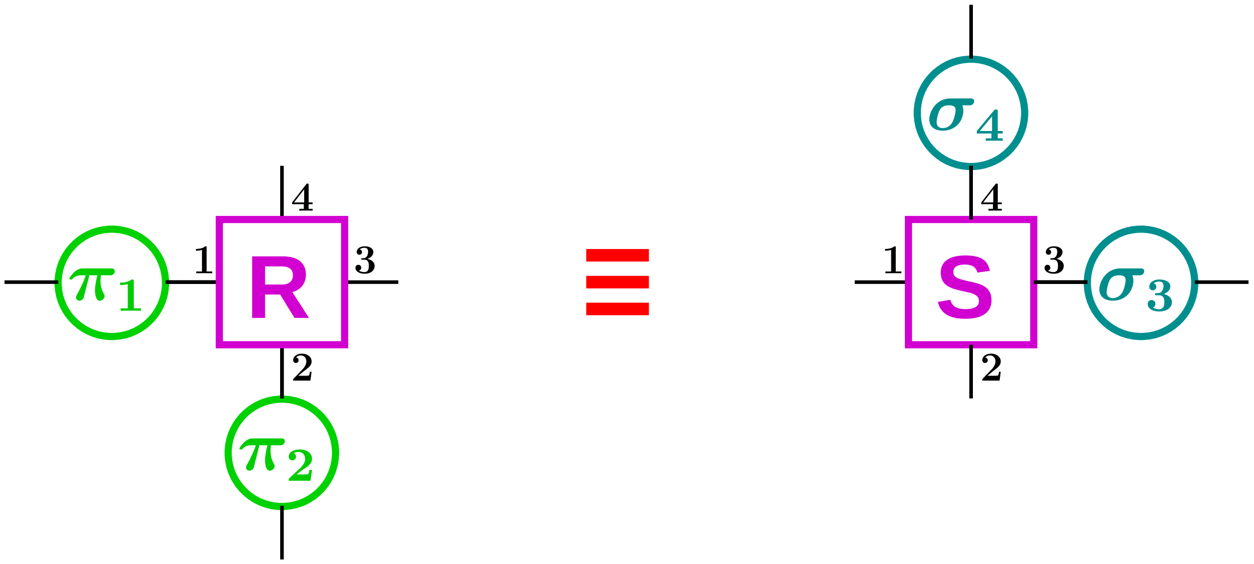

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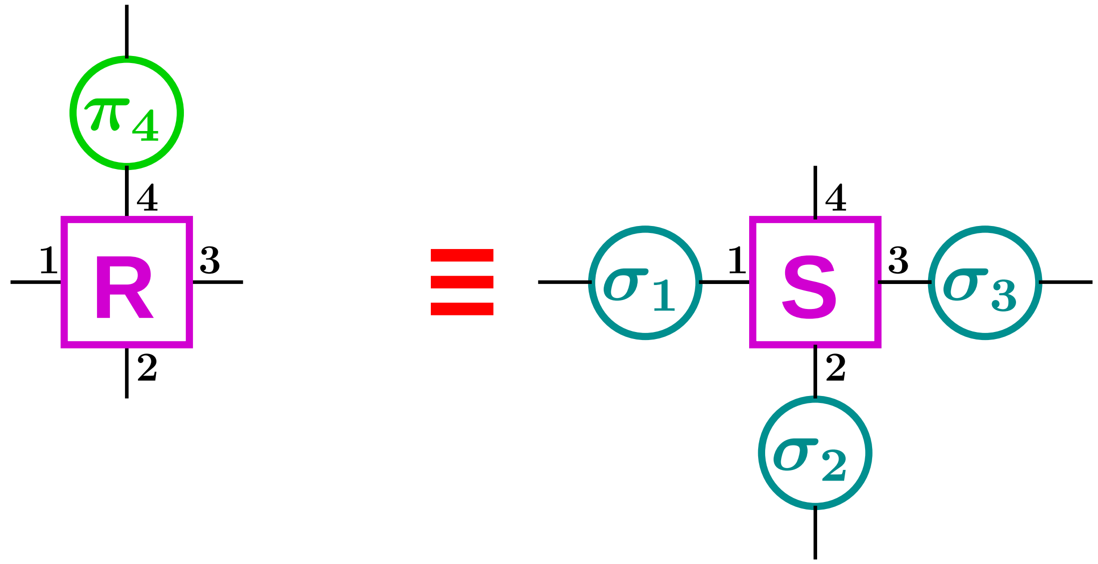

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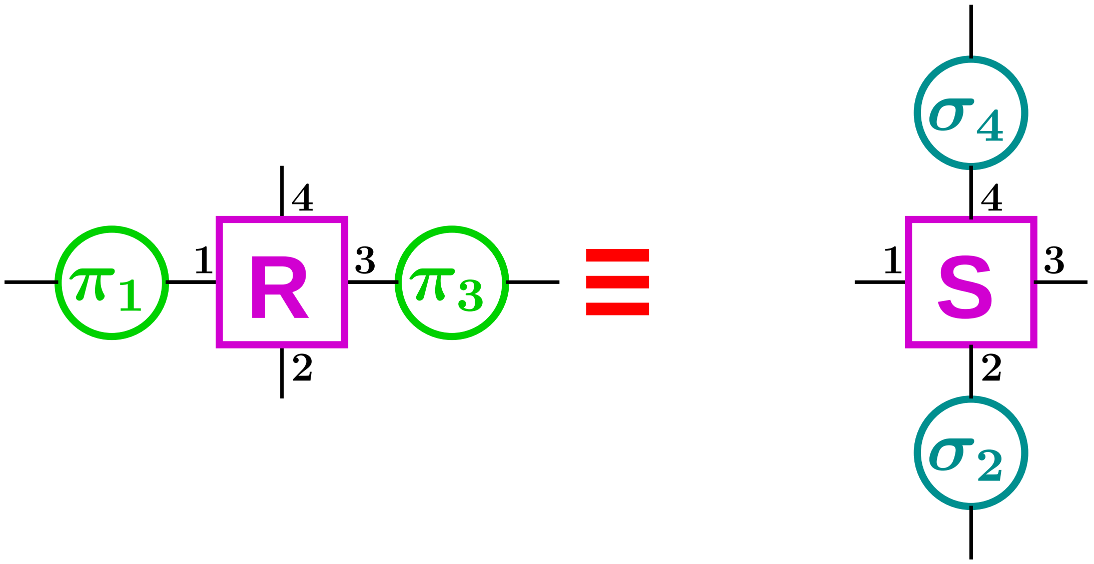

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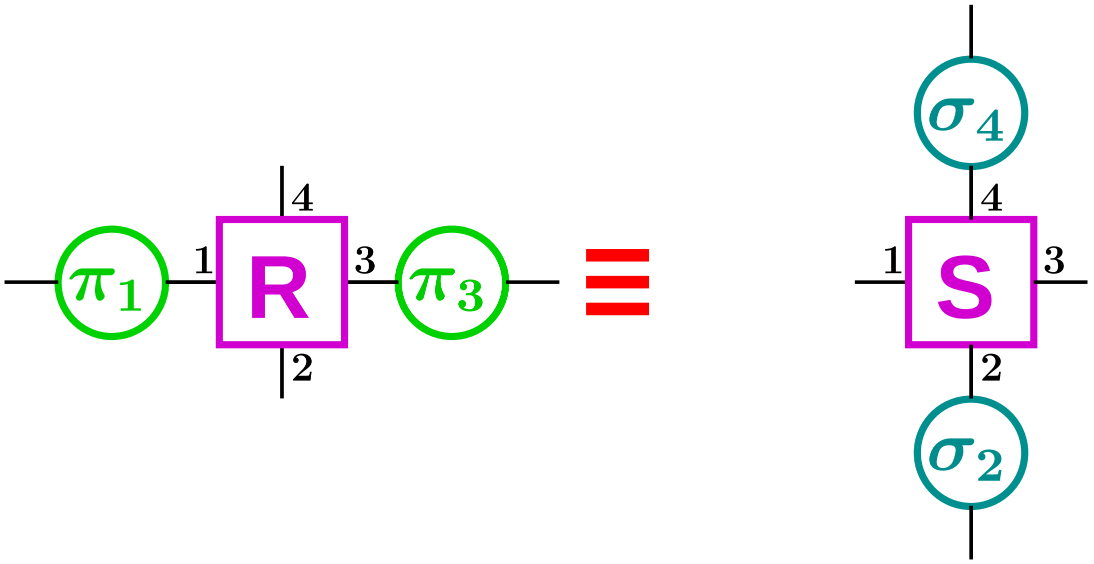

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$$R \cong^{\pi} S \;:\Longleftrightarrow\; \exists \sigma\, \forall \mathcal{P} \subseteq \mathcal{I} : \; \pi_{\mathcal{P}} R \equiv \sigma_{\overline{\mathcal{P}}} S$$

Example: 2-party resources [2]

$$\mathbf{R} \cong^{\pi} \mathbf{S} :\Longleftrightarrow \begin{cases} \pi_1 \mathbf{R} \pi_2 \approx \mathbf{S} \\ \pi_1 \mathbf{R} \approx \mathbf{S} \sigma_2 \\ \mathbf{R} \pi_2 \approx \sigma_1 \mathbf{S} \\ \mathbf{R} \approx \sigma_1 \mathbf{S} \sigma_2 \end{cases}$$

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$$\left. \vphantom{\begin{array}{l} a \\ b \\ c \end{array}} \right\} \Leftrightarrow \text{abstract UC}$$

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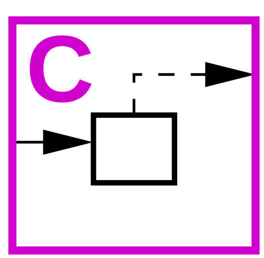

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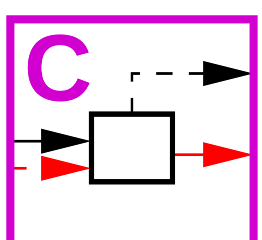

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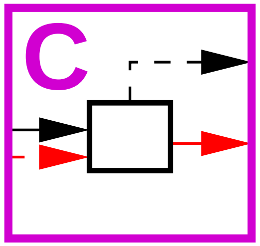

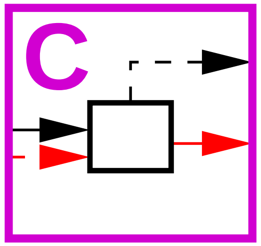

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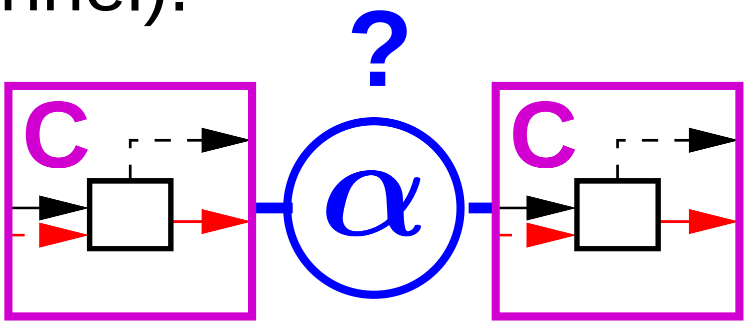

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Note: Isomorphism is the precisest possible relation between resources, but as such is completely rigid.

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Corollary: A delayed communication channel cannot be realized (from a communication channel).

Abstraction by Sets of Resources

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Theorem: $\mathcal{R} \sqsubseteq^{\pi} \mathcal{S}$ is a universally composable reduction.

Reductions

The reduction

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is called sequentially composable if

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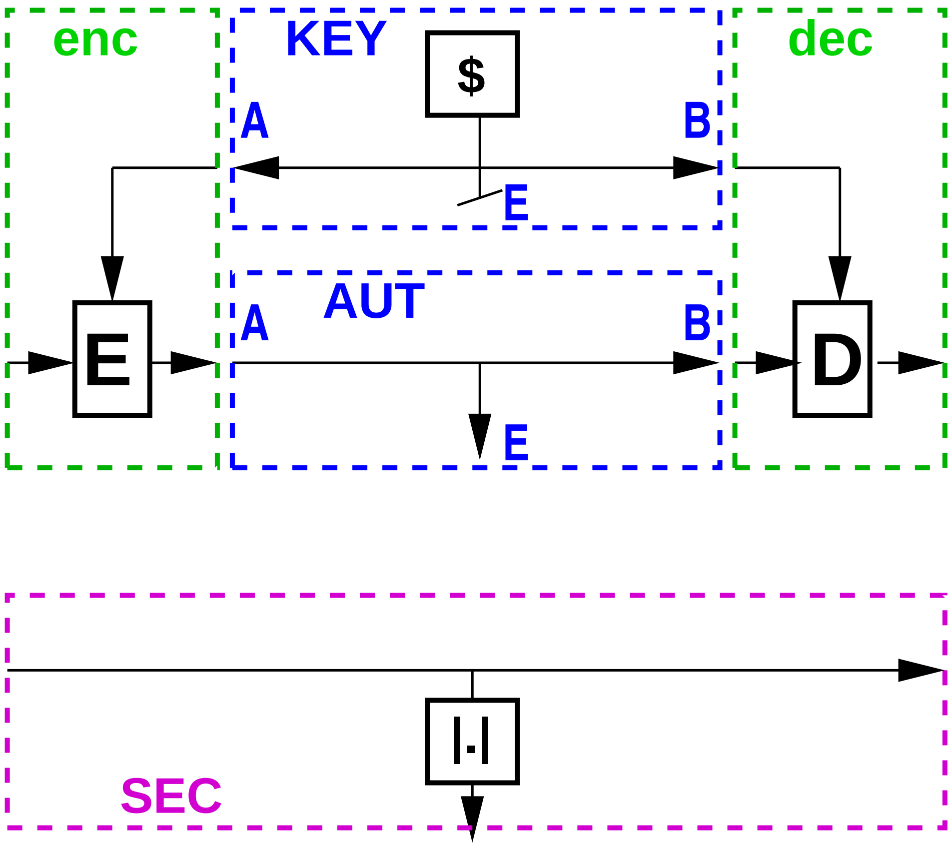
It is called universally composable if in addition:

2. $\mathbf{R} \xrightarrow{\mathsf{id}} \mathbf{R}$
3. $\mathbf{R} \xrightarrow{\alpha} \mathbf{S} \;\Rightarrow\; \mathbf{R} \| \mathbf{T} \xrightarrow{\alpha | \mathsf{id}} \mathbf{S} \| \mathbf{T}$

Example: Encryption



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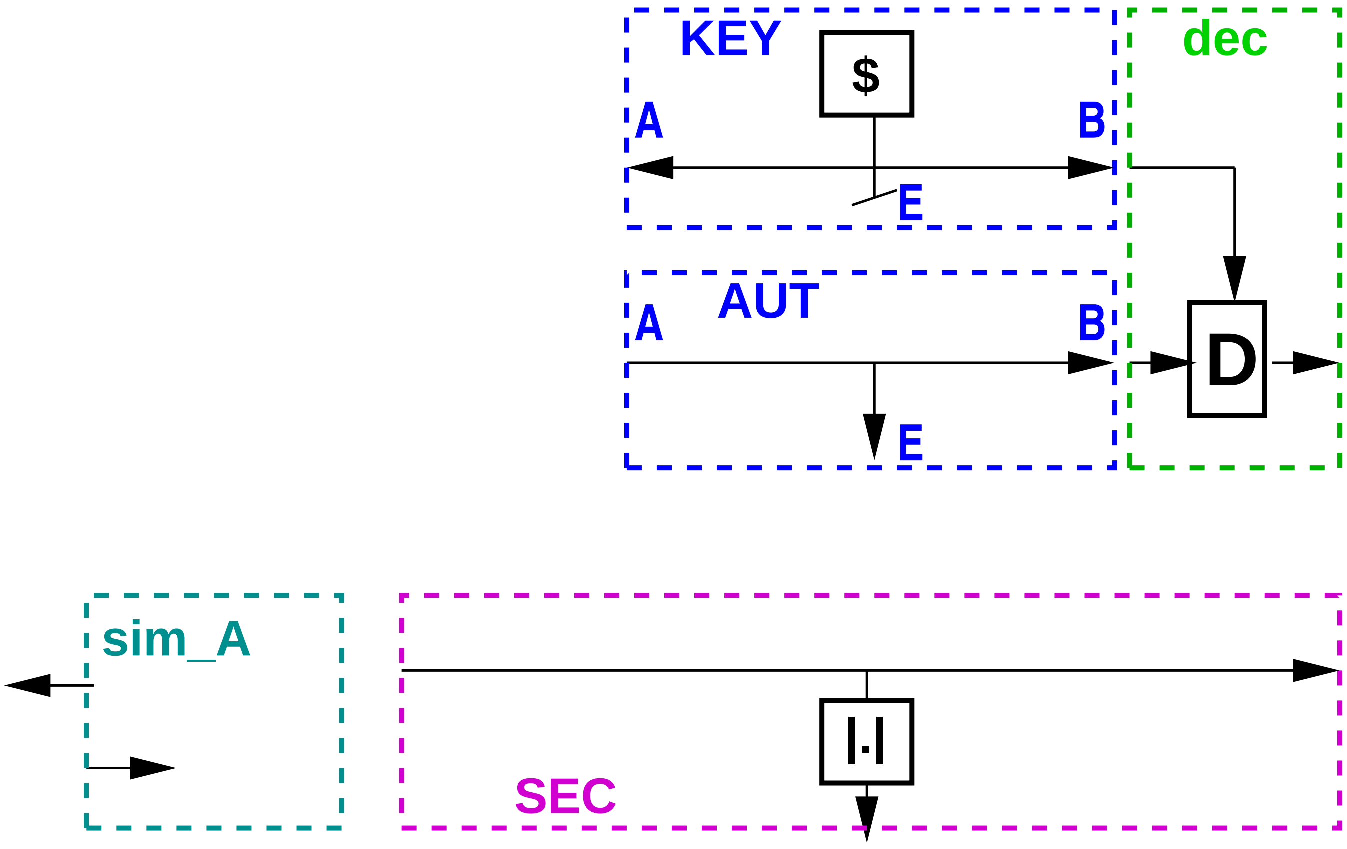

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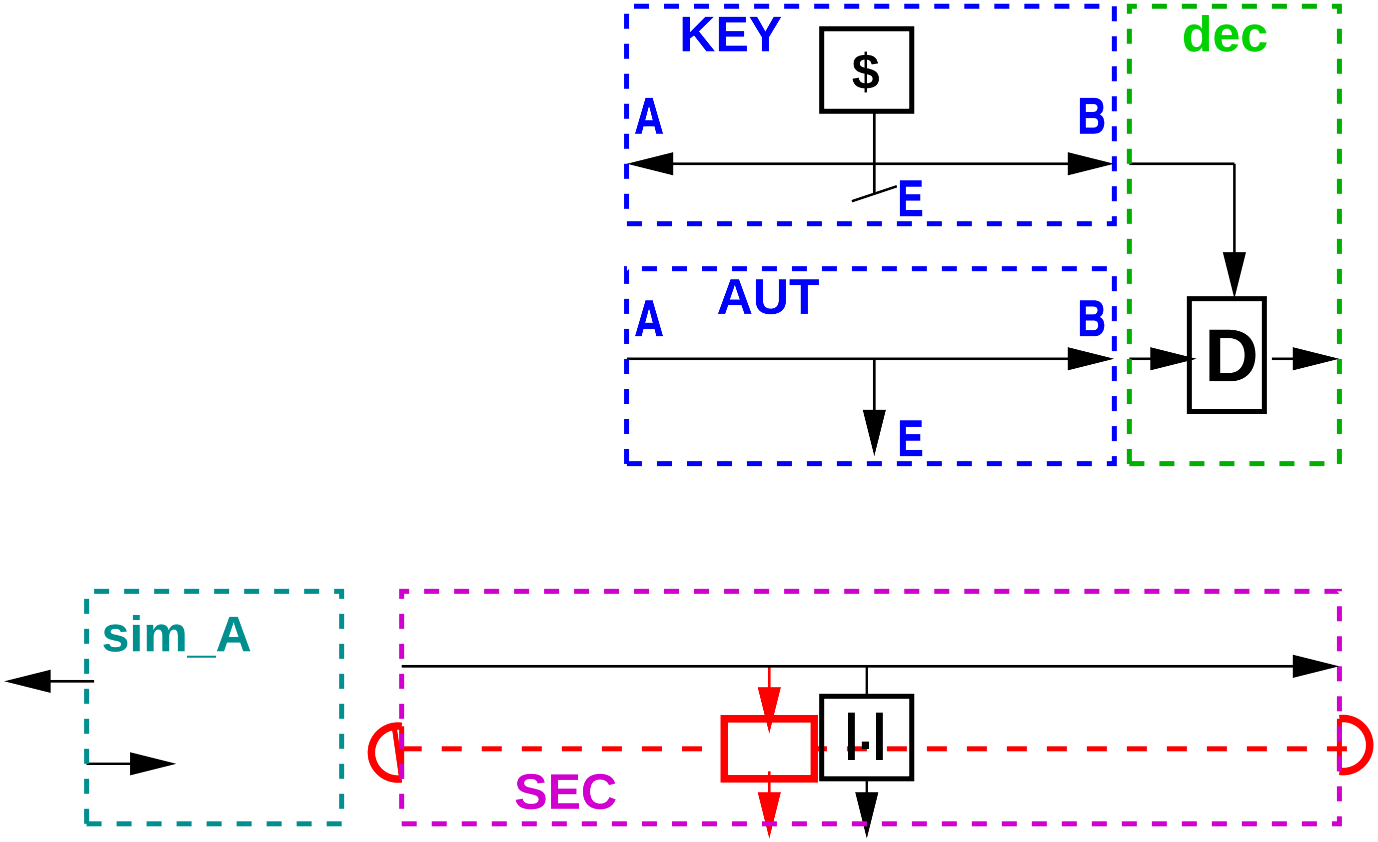

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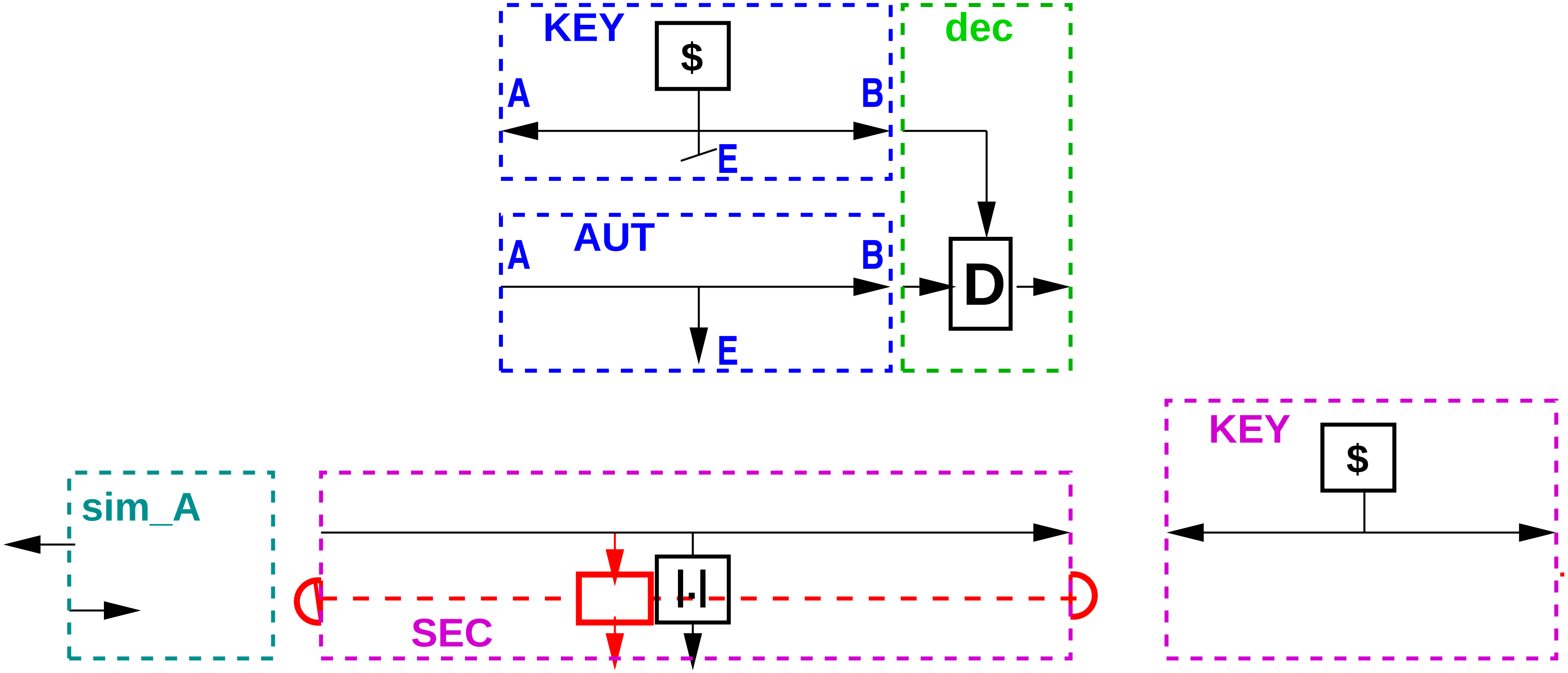

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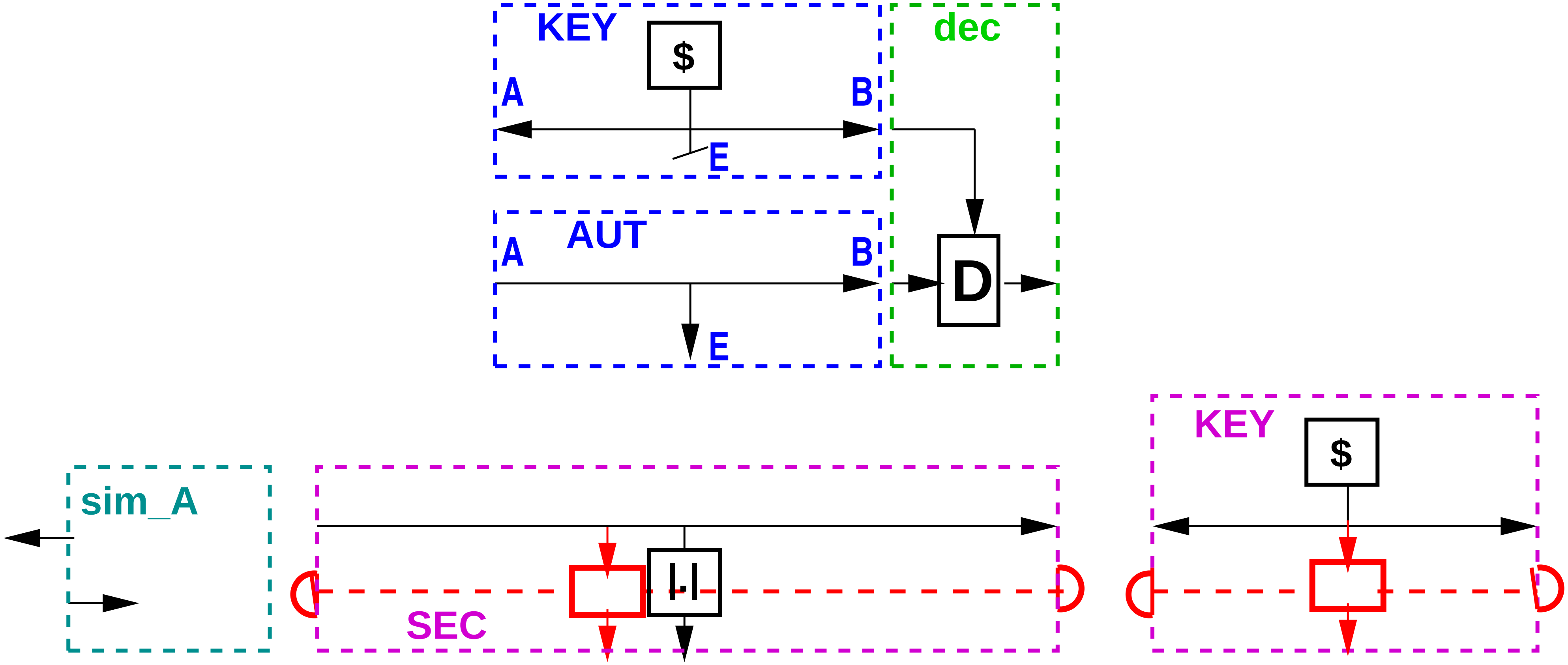

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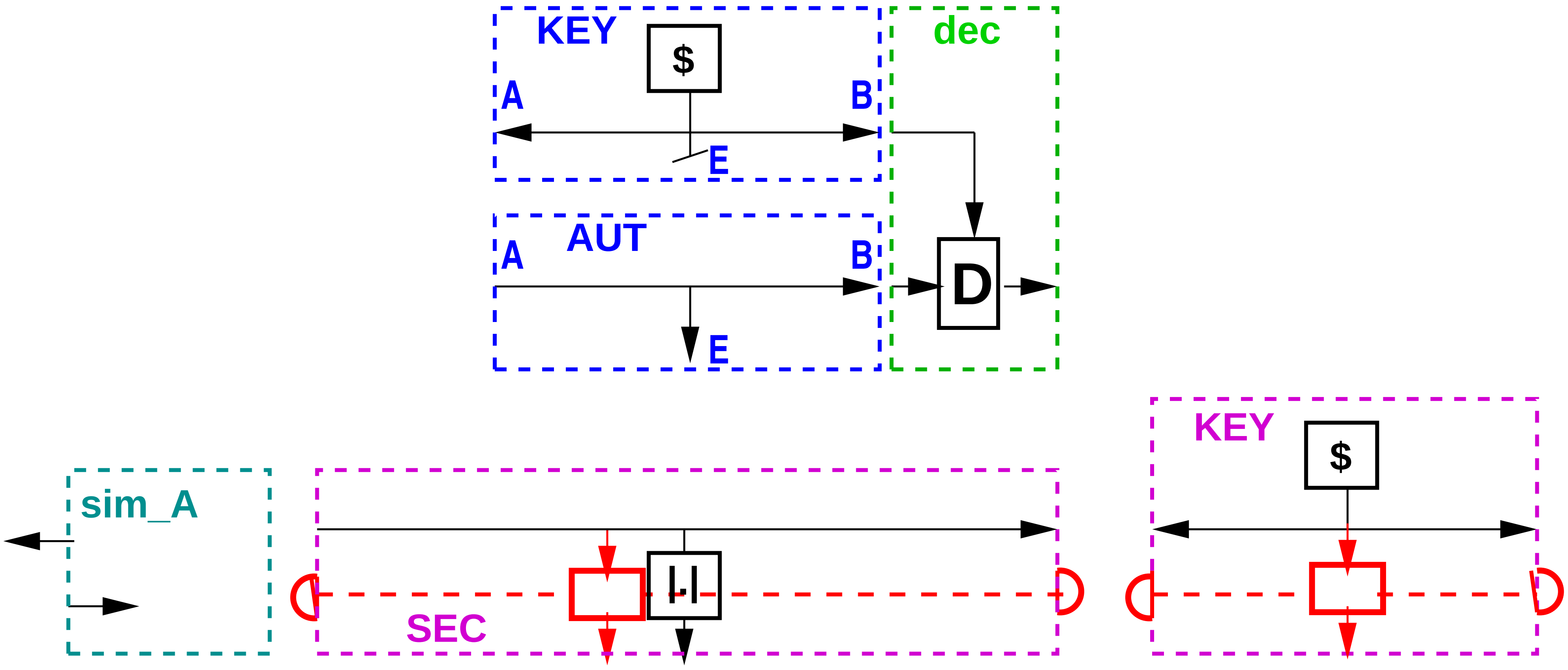

Theorem: An unleakable (uncoercible) secure communication channel cannot be realized from an authenticated channel and a secret key.

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Let's try to identify the right level of abstraction of what we do in cryptography.

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- general notion of interfaces: consistency domains