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**ABSTRACT:**

Many computing systems consist of a possibly huge number of components that not only work independently but also communicate with each other from time to time. The catastrophic consequences of failures, such as loss of human lives, environmental damages, and financial losses, in many of these critical systems compel computer scientists and engineers to develop techniques for ensuring that these systems are implemented correctly despite of their complexity. Although a number of theories and software tools have been developed to support the formal description and verification of functional properties of systems, only in recent years the formal modeling and assessment of performance characteristics have received attention.

This thesis addresses the problem of providing a suitable linguistic support which enables designers to formally describe and evaluate system performance in the early stages of system design, in order to avoid cost increases due to the late discovery of inefficiency.

As a solution to such a problem, in this thesis we propose an integrated approach to modeling and analyzing functional and performance characteristics of systems which relies on formal description techniques extended with a stochastic representation of time. Such an approach, inspired to Olderog work on relating different views of concurrent systems, is based on both stochastically timed process algebras and stochastically timed Petri nets in order to profit from their complementary advantages: compositionality and explicit description of concurrency.

In order to develop the integrated approach in the Markovian case, in this thesis we propose a new stochastically timed process algebra called EMPA (Extended Markovian Process Algebra), inspired to Herzog TIPP and Hillston PEPA, which has a considerable expressive power as it allows for the description of not only exponentially timed activities but also immediate activities, priority and probability related features, and nondeterminism. In order to support the various phases and analyses of the integrated approach, EMPA is equipped with a suitable collection of semantics, mapping terms to integrated transition systems and their projections (functional transition systems and Markov chains) and generalized stochastic Petri nets, respectively, as well as a notion of integrated equivalence, relating terms describing systems with the same functional and performance properties. The major consequences of the restriction to exponentially distributed and zero durations are the possibility of defining the integrated semantics for EMPA in the interleaving style (thanks to the memoryless property of exponential distributions) and the capability of computing performance measures using Markov chains. The integrated equivalence, which is defined on the integrated semantics according to Larsen-Skou probabilistic bisimulation because of its relationship with the aggregation technique for Markov chains known as ordinary lumping, is proved to be for a large class of terms the coarsest congruence contained in the intersection of a purely functional equivalence and a purely performance equivalence, thus allowing for compositional reasoning and emphasizing the necessity, beside the convenience, of defining such an equivalence on the integrated semantics instead of its projections.

In this thesis EMPA is enriched with the capability of specifying performance measures at a high level of abstraction through rewards and the capability of describing value passing among system components. Rewards are directly specified within terms so the semantics and the integrated equivalence are modified accordingly in order to allow for an algebraic treatment of performance measures. Value passing is handled by providing suitable semantic rules that produce compact symbolic integrated transition systems on which

a symbolic extension of the integrated equivalence is defined in the style of Hennessy-Lin. We show that by means of value passing based expressions it is possible to deal with activities having a generally distributed duration.

Since the generation of the semantic models underlying EMPA terms as well as the checking of the integrated equivalence and the simulation of EMPA terms can be fully mechanized, in this thesis we implement the integrated approach in the Markovian case through a software tool called TwoTowers which compiles EMPA descriptions of systems into their underlying semantic models and invokes other tools such as CWB-NC, MarCA, and GreatSPN to analyze these models. Exploiting already existing tools is advantageous both from the point of view of the implementors, as the construction of TwoTowers becomes easier and faster, and from the point of view of the users, since they are provided with a full range of automated techniques implemented in widely used tools.

The thesis is concluded by presenting several case studies concerning communication protocols (alternating bit protocol, CSMA/CD, token ring, ATM switch, adaptive mechanism for packetized audio) and distributed algorithms (randomized algorithm for dining philosophers, mutual exclusion algorithms) modeled with EMPA and analyzed with TwoTowers. The purpose of such case studies is to demonstrate the adequacy of the integrated approach, the expressiveness of EMPA, and the importance of developing automated tools such as TwoTowers to help designers in modeling and analyzing complex systems in a formal way.

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